MATH 6333
11p3p2c

(11.3.2) $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial t^2} + Q(x,t) \quad x>0$

$u(0,t) = 1 \quad u(x,0) = f$

$$u(x,t) = \int_0^t\int_0^\infty G \cancel{Q}^{\to 0} dx_0 dt_0 + \int_0^\infty G\, \cancel{u(x,0)}^{\to 0} dx_0 + k\int_0^t \underset{0 \text{ at } \infty}{G\nabla u} - \underset{1 \text{ at } 0}{u\nabla G} \Big|_0^\infty dt_0$$

$$= k\int_0^t \frac{\partial G}{\partial x_0}\Big|_{x_0=0} dt_0$$

Use Images to determine G.

$$G = \frac{H(t-t_0)}{\sqrt{4\pi k(t-t_0)}}\left(e^{\frac{-(x-x_0)^2}{4k(t-t_0)}} - e^{\frac{-(x+x_0)^2}{4k(t-t_0)}}\right)$$

$$\frac{dG}{dx_0}\Big|_{x_0=0} = \frac{H(t-t_0)\,x}{k\sqrt{4\pi k}\,(t-t_0)^{3/2}}\, e^{-\frac{x^2}{4k(t-t_0)}}$$

$$u = \frac{x}{\sqrt{4\pi k}}\int_0^t \frac{1}{(t-t_0)^{3/2}}\, e^{\frac{-x^2}{4k(t-t_0)}} dt_0$$

let $\eta = \frac{x}{\sqrt{4k(t-t_0)}}$

$$u = \frac{2}{\sqrt{\pi}}\int_{\frac{x}{\sqrt{4kt}}}^\infty e^{-\eta^2} d\eta$$

Complimentary Error Function.
Look up in Handbook of Functions.

(11.3.3) Images w/ two $\oplus$ sources. yawn!