MATH 6333

6333 - 8p3p1

(8.3.1) (f) $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + Q(x,t) \qquad u(x,0) = f(x)$

$\frac{\partial u}{\partial x}(0,t) = 0 \qquad \frac{\partial u}{\partial x}(L,t) = 0$

E. Value Prob: $\frac{\partial^2 \phi}{\partial x^2} + \lambda\phi = 0 \qquad \frac{\partial \phi}{\partial x}(0) = \frac{\partial \phi}{\partial x}(L) = 0$

$\phi_n(x) = \cos\frac{n\pi}{L}x \qquad \lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad n = 0, 1, 2 \ldots$

let $u(x,t) = \sum_{n=0}^{\infty} U_n(t)\cos\frac{n\pi}{L}x$

$Q(x,t) = \sum_{n=0}^{\infty} \overline{Q}_n(t)\cos\frac{n\pi}{L}x$

$f(x) = \sum_{n=0}^{\infty} F_n\cos\frac{n\pi}{L}x$

$\frac{dU_n}{dt} = -k\left(\frac{n\pi}{L}\right)^2 U_n + \overline{Q}_n(t) \qquad U_n(0) = F_n$

$n \neq 0 \quad U_n(t) = F_n e^{-k(\frac{n\pi}{L})^2 t} + \int_0^t \overline{Q}_n(s)e^{-k(\frac{n\pi}{L})^2(t-s)}ds$

$n = 0 \quad U_0(t) = \int_0^t \overline{Q}_0(s)ds + F_0$

(8.3.1g) Suppose $Q(x,t) = Q(x)$

Then $Q(x) = \sum_{n=0}^{\infty} \overline{Q}_n \cos\frac{n\pi}{L}x$

a constant

$n \neq 0 \quad U_n(t) = F_n e^{-k(\frac{n\pi}{L})^2 t} + \overline{Q}_n\int_0^t e^{-k(\frac{n\pi}{L})^2(t-s)}ds$

$= F_n e^{-k(\frac{n\pi}{L})^2 t} + \overline{Q}_n\frac{1}{k(\frac{n\pi}{L})^2}\left(1 - e^{-k(\frac{n\pi}{L})^2 t}\right)$

$n = 0 \quad U_0(t) = \int_0^t \overline{Q}_0\, ds = \overline{Q}_0 t + F_0$

As $t \to \infty$ the exponentials decay however $\overline{Q}_0 t$ becomes unbounded. The non-zero mean of the forcing term is the culprit.