MATH 6333

6333-9p4p2-3.pdf

9.4.2 ... and some more

$Lu=f \quad B(u)|_a=\alpha \quad B_2(u)|_b=B$

$Lu_H=0 \quad B_1(u_H)|_a=0 \quad B_2(u_H)|_b=0$

Green's Identity

$$\int_a^b u\,\underset{0}{\cancel{Lu_H}} - u_H\,\underset{f}{\cancel{Lu}}\,dx = \underbrace{J(u\,u_H)\Big|_a^b}_{\neq 0 \text{ due to BCs on } u}$$

F.A. $\int_a^b u_H f\,dx = -J\Big|_a^b$

Must be same

E-Function Prob.

$L\phi_n = -\lambda_n\sigma\phi_n \quad B_1(\phi_n)|_a = B_2(\phi_n)|_b = 0$

Green's Identity

$$\int_a^b u\,\underset{-\lambda_n\sigma\phi_n}{\cancel{L\phi_n}} - \phi_n\,\underset{f}{\cancel{Lu}}\,dx = \underbrace{J(u\,\phi_n)\Big|_a^b}_{\neq 0 \text{ due to BCs on } u}$$

$\int_a^b u(-\lambda_n\sigma\phi_n) - \phi_n f\,dx = J\Big|_a^b$

let $u = \sum U_m\phi_m$

$f = \sigma\sum F_m\phi_m$

$$\sum\left(U_m(-\lambda_m) - F_m\right)\int_a^b \underset{\|\phi_n\|^2}{\cancel{\sigma\phi_n\phi_m}}\,dx = J\Big|_a^b$$

$U_n(-\lambda_n) - F_n = \dfrac{J|_a^b}{\|\phi_n\|^2}$

$U_n = \dfrac{F_n + J|_a^b/\|\phi_n\|^2}{\lambda_n}$

If $u_H \neq 0$ then $\phi_1 \neq 0$ w/ $\lambda_1 = 0$

$\therefore\ F_1 + \dfrac{J|_a^b}{\|\phi_1\|^2} = 0$

Same as F.A. above

$\Longleftrightarrow$ Same idea

Let $u = v + r$

Choose $r \ni B_1(r)|_a = \alpha$

and $B_2(r)|_b = B$.

$Lv = f - Lr \quad B_1(v)|_a = B_2(v)|_b = 0$

$Lv_H = 0 \quad B_1(v_H)|_a = B_2(v_H)|_b = 0$

Note $v_H = u_H$.

Green's Identity w/ v & v_H

F.A. $\int_a^b v_H(f - Lr)\,dx = 0$

$\int_a^b u_H(f - Lr)\,dx = 0$

$\int_a^b u_H f\,dx = \int_a^b u_H Lr\,dx$

Green's Identity

$$\int_a^b v\,\underset{-\lambda\sigma\phi_n}{\cancel{L\phi_n}} - \phi_n\,\underset{\bar f = f - Lr}{\cancel{Lv}}\,dx = \underbrace{J(v\,\phi_n)\Big|_a^b}_{=0 \text{ due to BCs on } v}$$

$\int_a^b v(-\lambda_n\sigma\phi_n) - \phi_n\bar f\,dx = 0$

let $v = \sum V_m\phi_m$

$\bar f = \sigma\sum \bar F_m\phi_m$

$$\sum\left(V_m(-\lambda_m) - \bar F_m\right)\int_a^b \sigma\phi_n\phi_m\,dx = 0$$

$V_n = \dfrac{\bar F_n}{\lambda_n}$

If $u_H \neq 0$ then $\phi_1 \neq 0$ w/ $\lambda_1 = 0$

$\therefore\ \bar F_n = 0$

$\bar F_n = \int_a^b (f - Lr)\phi_1\,dx$

$= \int_a^b f\phi_1\,dx - \int_a^b Lr\phi_1\,dx$

Same F.A. as above.