MATH 6333

6333-9p4p7

(9.4.7)

$u'' + 4u = \cos x$

$u'(0) = u'(\pi) = 0$

a) Let $u = u_H + u_P$

$u_H = c_1\cos 2x + c_2 \sin 2x$

let $u_P = A\cos x + B\sin x$

$u_P'' = -A\cos x - B\sin x$

$3A\cos x + 3B\sin x = \cos x$

$A = 1/3 \quad B = 0$

$u = c_1\cos 2x + c_2 \sin 2x + \frac{1}{3}\cos x$

$u' = -2c_1 \sin 2x + 2c_2\cos 2x - \frac{1}{3}\sin x$

$u'(0) = 2c_2 = 0 \quad c_2 = 0$

$u'(\pi) = 2c_2 = 0 \quad c_2 = 0$

$u = c_1\cos 2x + 1/3\cos x$

$c_1$ undetermined.

b) EFunction problem.

DE suggest sines & cosines

BCs suggest cosines

let $u = \sum_{n=0}^{\infty} U_n \cos nx$

$\sum_{n=0}^{\infty} U_n(-n^2+4)\cos nx = \cos x$

$n=1: \quad U_n = 1/3$

$n=2: \quad U_2 \cdot 0 = 0 \quad U_2 = ?$

$n \neq 1$ or $n \neq 2: \quad U_n = 0$

$u = U_2\cos 2x + 1/3\cos x$

$U_2$ undetermined.

c) F.A. needs completely homog solution. That is, both ODE & BCs homog.

$u_H'' + 4u_H = 0$

$u_H'(0) = u_H'(\pi) = 0$

$u_H \sim \cos 2x$.

$\Rightarrow \int_0^\pi \cos 2x \cos x \, dx = 0$

$0 = 0$

$\Rightarrow \infty$ solutions.

ASIDE: if try Modified G.F.

$G_M'' + 4G_M = \delta(x-x_0) - \frac{2}{\pi}\cos 2x_0 \cos 2x$

$G_M = \begin{cases} g_l = c_1\cos 2x - \frac{1}{2\pi}\cos 2x_0\, x\sin 2x \\ g_r = c_3\cos 2x + \frac{1}{2}\cos 2x_0 \sin 2x - \frac{1}{2\pi}\cos 2x_0\, x\sin 2x \end{cases}$

Continuity: $c_1 = c_3 + \frac{1}{2}\sin 2x_0$

JUMP: $0 = 0 \quad c_3 =$ undefined.

etc.