$$S' = B - \frac{\beta S I}{N} + \gamma I - \mu S$$
$$I' = \quad + \beta \frac{SI}{N} - \gamma I - \mu I$$

a) B: a constant birth rate independent of population size

β: Transmission rate that accounts for contact rate & prob. of successful contact. Transmission is modeled by mass-action coupling $\sim SI$.

$\pm \gamma I$: Recovery from I and reinjection into S w/ mean time in I class as $\frac{1}{\gamma}$ and an exponential distribution of recovery times.

$-\mu S, -\mu I$: Natural death with mean lifetime $\frac{1}{\mu}$ and an exponential distribution of death times.

b) $N' = B - \mu N \Rightarrow N' + \mu N = B$
$$\Rightarrow N(t) = \frac{B}{\mu} + \left(N_0 - \frac{B}{\mu}\right) e^{-\mu t}$$

$$\lim_{t \to \infty} N(t) = \frac{B}{\mu} \equiv N_L$$

c) Let $x = \frac{S}{N_L}$ $y = \frac{I}{N_L}$

$$N_L x' = B - \beta N_L x N_L y \frac{1}{N_L} + \gamma N_L y - \mu N_L x$$
$$N_L y' = \quad + \beta N_L x N_L y \frac{1}{N_L} - \gamma N_L y - \mu N_L y$$

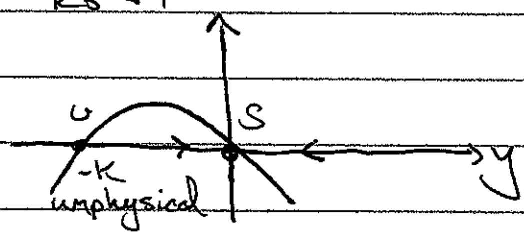
$$x' = \mu(1-x) - \beta x y + \gamma y$$
$$y' = \beta x y - (\gamma + \mu) y$$

w/ $x + y = 1 \Rightarrow x = 1 - y$
$$y' = \beta y (1-y) - (\gamma + \mu) y$$

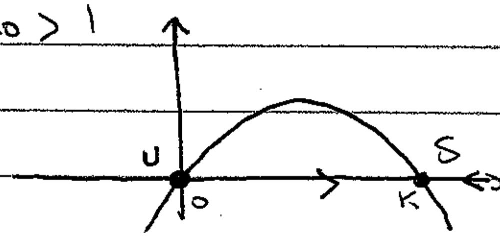
$$y' = (\beta - (\gamma + \mu))y - \beta y^2$$
$$= \beta\left(1 - \frac{(\gamma+\mu)}{\beta}\right)y - \beta y^2$$
$$= \beta\left(1 - \frac{1}{R_0}\right)y - \beta y^2$$
$$= \beta\left(1 - \frac{1}{R_0}\right)y\left(1 - \frac{1}{1-\frac{1}{R_0}}y\right)$$
$$= ry(1 - y/K)$$
$$r = \beta\left(1 - \frac{1}{R_0}\right)$$
$$K = 1 - \frac{1}{R_0}$$

d) $R_0 = \frac{\beta}{\mu+\gamma} = \frac{\text{input into } y \text{ class}}{\text{output from } y \text{ class}}$ -or- $\frac{\text{rate entering } y}{\text{rate leaving } y}$ or ...

e) $R_0 < 1$



$R_0 > 1$



Endemic state stable $R_0 > 1$