Math 6391 h05-delayneuron.gif

$\mu = 0$

)  1, 2 or 3 equil.

b) $\frac{1}{3} r_0^3 + \left(\frac{1}{b} - 1\right) r_0 + \frac{a}{b} = 0$

c) $\lambda^2 + \lambda\left(r_0^2 - (1-\rho b)\right) + \rho\left(1 - b(1-r_0^2)\right) = 0$

$2\lambda = -\left(r_0^2 - (1-\rho b)\right) \pm \left[\left(r_0^2 - (1-\rho b)\right)^2 - 4\rho\left(1 - b(1-r_0^2)\right)\right]^{1/2}$

Stable focus: $r_0^2 - (1-\rho b) > 0 \Rightarrow r_0^2 > 1 - \rho b$

$\left(r_0^2 - (1-\rho b)\right)^2 - 4\rho\left(1 - b(1-r_0^2)\right) < 0$

$\Rightarrow r_0^2 < 1 + \rho b + 2\sqrt{\rho}$

$\mu \neq 0$

a) Same as above. Use maple to compute

b) $\lambda^2 + \lambda(\rho b - 1 + r_0^2) + \rho\left(1 + b(r_0^2 - 1)\right) + (-\lambda\mu - \rho b\mu)e^{-\lambda\tau} = 0$

c) let $\lambda = i\omega$

$\omega^2 - \rho\left(1 + b(r_0^2 - 1)\right) = \mu(\rho b \cos\omega\tau + \omega \sin\omega\tau)$

$\omega(\rho b - 1 + r_0^2) = \mu(\omega\cos\omega\tau - \rho b \sin\omega\tau)$

let $b_1 = \omega^2 - \rho\left(1 + b(r_0^2 - 1)\right)$

$b_2 = \rho b - 1 + r_0^2$

Then $\cos\omega\tau = -\frac{\omega^2}{\mu\rho b} \frac{(b_2\rho b - b_1) + b_1(\omega^2 + \rho^2 b^2)}{\omega^2 + \rho^2 b^2}$

$\sin\omega\tau = -\frac{\omega(b_2\rho b - b_1)}{\mu\rho b(\omega^2 + \rho^2 b^2)}$

- $\tan\omega\tau = f_1(\omega)$ eliminates μ
  Multiple solutions $\Rightarrow$ many values of ω.

- $\cos^2\omega\tau + \sin^2\omega\tau = 1 \Rightarrow \mu^2 = f_2(\omega)$
  Equation for μ in terms of ω.