Delayed-mutual coupling dynamics of lasers: scaling laws and resonances

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We consider a model for two lasers that are mutually coupled optoelectronically by modulating the pump of one laser with the intensity deviations of the other. While coupling causes oscillatory output at the laser’s relaxation frequency, significantly long delay introduces additional complexity as external cavity modes also become unstable. We derive the bifurcation and relevant scaling laws for both the internal and external modes and compare them to recent experimental results. There also exists a novel resonance phenomena where for a specific value of the coupling there is a strong amplitude response. For delayed-mutual coupling there is a sharp parameter-space boundary that determines whether the resonance response is hysteretic.
Mutually-coupled Lasers with Delay

Synchronization and amplitude instabilities in coupled systems with non-zero communication time

- Coupled, weakly-damped Hamiltonian systems.
- Delay coupling -> Delay-DE
- Coupled Lasers
  Communication & synchronization
  Optical computing
- Related systems
  Mechanics, Population Dynamics, ...
Laser Physics

1958: Townes & Schawlow. Laser Theory. Received Nobel Prize.
Pump-coupled Lasers

- Couple intensity deviations to pump (incoherent).
- Independent control.
- Delay due to transit time through fiber loop and coupling circuit.

Experiment
- DC Bias: "Pump"
- LD: Laser Diodes
- L: Fiber Loop
- PD: Photodiodes
- A: Electronic Amplifier
- Attn: Variable Attenuator
Rate Equations with Pump Coupling

$I = |E|^2$ : Intensity, $D$ : Inversion

\[
\frac{dI_j}{dt} = (D_j - 1)I_j
\]

\[
\frac{dD_j}{dt} = \epsilon_j^2[A_j - (1 + I_j)D_j]
\]

Non-zero steady-state corresponds to CW output.

$D_{j0} = 1, \quad I_{j0} = A_j - 1$

Investigate the effects of coupling two lasers through their pump:

$A_j = A_{j0} + I_{j0}\delta_k(I_k - I_{k0})$. 

TWC, Delay-coupled lasers – p.6
Scaled Rate Equations

Define new variables for the deviations from the cw state as

\[ I_j = I_{j0}(1 + y_j), \quad D_j = 1 + \epsilon_j \sqrt{I_{j0}} x_j, \quad t_{new} = \epsilon_1 \sqrt{I_{10}} t_{old}. \]

The new rate equations are

\[
\begin{align*}
\frac{dy_1}{dt} &= x_1(1 + y_1) \\
\frac{dx_1}{dt} &= -y_1 - \epsilon x_1(a_1 + b y_1) - \delta_2 y_2(t - \tau), \\
\frac{dy_2}{dt} &= \beta x_2(1 + y_2) \\
\frac{dx_2}{dt} &= \beta [-y_2 - \epsilon \beta x_2(a_2 + b y_2) - \delta_1 y_1(t - \tau)]
\end{align*}
\]

\( a_j, b : \text{dissipation constants.} \quad \beta \approx 1 : \text{laser frequency detuning.} \)

\( \delta = \text{coupling strength.} \quad \tau : \text{delay due to fiber \& circuit} \)
No delay, $\tau = 0$
Hopf, Period-doublings and Chaos

Without delay, $\tau = 0$: For fixed $\delta_1$ while varying $\delta_2$:

- **Hopf bifurcation** at $\delta_2 = \delta_{2H}$ with $\omega \approx 1$

  \[
  \delta_1 \delta_2 + \epsilon^2 [a_1 a_2 + 4\alpha^2 \frac{a_1 a_2}{(a_1 + a_2)^2}] = 0
  \]

- **Period doubling** $\rightarrow$ chaos. **Saddle-node bifurcations** $\rightarrow$ subharmonic resonances.

![Diagram showing bifurcations](image.png)
Resonance: No Delay

\[ |A|^4 = 3 \frac{\Delta_1^2}{\Delta_2} \]

\[ \Delta_1 = (\delta_1 \delta_2 + a_1 a_2) \frac{(a_1 + a_2)^2}{a_1 a_2} \text{ and } \Delta_2 = 1 + \frac{a_2 \delta_2}{a_1 \delta_1} \]
Delay, $\tau \neq 0$
Internal vs. External Modes

\[ \omega = \frac{\pi}{2 \tau} \]

Laser Cavity  |  External Cavity  
(Coupling Circuit)

- Hopf bifurcation

\[ \Delta = \delta_1 \delta_2 : \text{coupling strength.} \]

\[ \Delta_H (\text{internal}) < \Delta_H (\text{external}) \]
Multiple-scale Analysis of DDEs

- Introduce slow time $T = \epsilon t$.
- Multiscale expansion of delay:

$$y_j(t - \tau) = y(t - \tau, T - \epsilon\tau)$$

$$= y_j(t - \tau, T) - \epsilon\tau \frac{\partial}{\partial T} y_j(t - \tau, T) + \ldots$$
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- Multiscale expansion of delay:

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&= y_j(t - \tau, T) - \epsilon \tau \frac{\partial}{\partial T} y_j(t - \tau, T) + \ldots
\end{align*}
\]

- Internal modes, $\omega \approx 1$: $\delta_j = O(\epsilon)$.
  - $O(1)$ problem: ODE.
  - Solvability condition: DDE.
Multiple-scale Analysis of DDEs

- Introduce slow time $T = \epsilon t$.
- Multiscale expansion of delay:

\[
y_j(t - \tau) = y(t - \tau, T - \epsilon \tau) = y_j(t - \tau, T) - \epsilon \frac{\partial}{\partial T} y_j(t - \tau, T) + \ldots
\]

- Internal modes, $\omega \approx 1$: $\delta_j = O(\epsilon)$.
  - $O(1)$ problem: ODE.
  - Solvability condition: DDE.

- External modes, $\omega \approx m\pi/(2\tau)$: $\delta_j = O(1)$.
  - $O(1)$ problem: DDE. Assume periodic solutions.
  - Solvability condition: DDE.
Slow Evolution of Periodic Solutions

Internal Modes, $\omega \approx 1$

\[ \frac{\partial A_j}{\partial T} = -\frac{a_j}{2} A_j(T) - \frac{i}{6} |A_j(T)|^2 A_j(T) + \frac{i}{2} d_k A_k(T - \tau_{\epsilon,k}) e^{-i\tau_k} \]

\[ |A| \sim (\Delta - \Delta_H)^{1/4}, \quad \Delta \sim \delta_1 \delta_2 \]
Slow Evolution of Periodic Solutions

- **Internal Modes, $\omega \approx 1$**

  \[
  \frac{\partial A_j}{\partial T} = -\frac{a_j}{2} A_j(T) - \frac{i}{6} |A_j(T)|^2 A_j(T) + \frac{i}{2} d_k A_k(T - \tau_{\epsilon, k}) e^{-i\tau_k}
  \]

  \[
  |A| \sim (\Delta - \Delta_H)^{1/4}, \quad \Delta \sim \delta_1 \delta_2
  \]

- **External Modes, $\omega \approx m\pi / (2\tau)$**
  (periodic solutions in $t$ and $T$)

  \[
  \frac{\partial A}{\partial T} = (p_l + iq_l) A + (p_n + iq_n) A|A|^2,
  \]

  \[
  |A| \sim (\Delta - \Delta_H)^{1/2}
  \]
Bifurcation Results

Internal Mode, $\omega \approx 1$: $|A| \sim (\Delta - \Delta_H)^{1/4}$

dots: numerical simulation. curve: analytical result
Bifurcation Results

External Mode, $\omega \approx m\pi/(2\tau)$: $|A| \sim (\Delta - \Delta_H)^{1/2}$

dots: numerical simulation. curve: analytical result
Bifurcation Results

Internal & External Modes

dots: numerical simulation. curve: analytical result
Experimental Results

- Observe External Modes with period $\propto$ delay.
- $\Delta = \delta_1 \delta_2$
- $|A| \sim (\Delta - \Delta_H)^{1/2}$
- Low-pass filter in coupling circuit attenuates internal mode.
Internal-Mode Resonance

- Bifurcation is singular when $a_2\delta_2 = a_1\delta_1$.
- Control of individual coupling strengths.
- Balance between damping and coupling.
- Sharp parameter boundary for hysteresis.
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- Control of individual coupling strengths.
- Balance between damping and coupling.
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Summary

- Incoherent coupling (intensity to pump) with delay
- Coupled, weakly-damped Hamiltonian systems
- Amplitude instabilities
- Independent control of coupling
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- Incoherent coupling (intensity to pump) with delay
  Coupled, weakly-damped Hamiltonian systems
  Amplitude instabilities
  Independent control of coupling

- Scaling laws
  Bifurcation depends on $\Delta = \delta_1 \delta_2$
  Experimentally observe secondary-instability $\rightarrow$ external modes
  Due to low-pass filter in feedback loop
  Multiple-scale analysis: $|A| \sim \pm (\Delta - \Delta_H)^{1/2}$
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- Resonance
  Amplitude peak for specific coupling
  Requires phase-shift provided by delay.
  Sharp parameter boundary for hysteresis
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- Incoherent coupling (intensity to pump) with delay
  Coupled, weakly-damped Hamiltonian systems
  Amplitude instabilities
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- Future work
  Sharp parameter boundary
  Mode competition for long delays
  In-phase vs. out-of-phase as function of delay
References


Experimental Results
Resonance: 3 Lasers