Negative-coupling resonances in pump-coupled lasers

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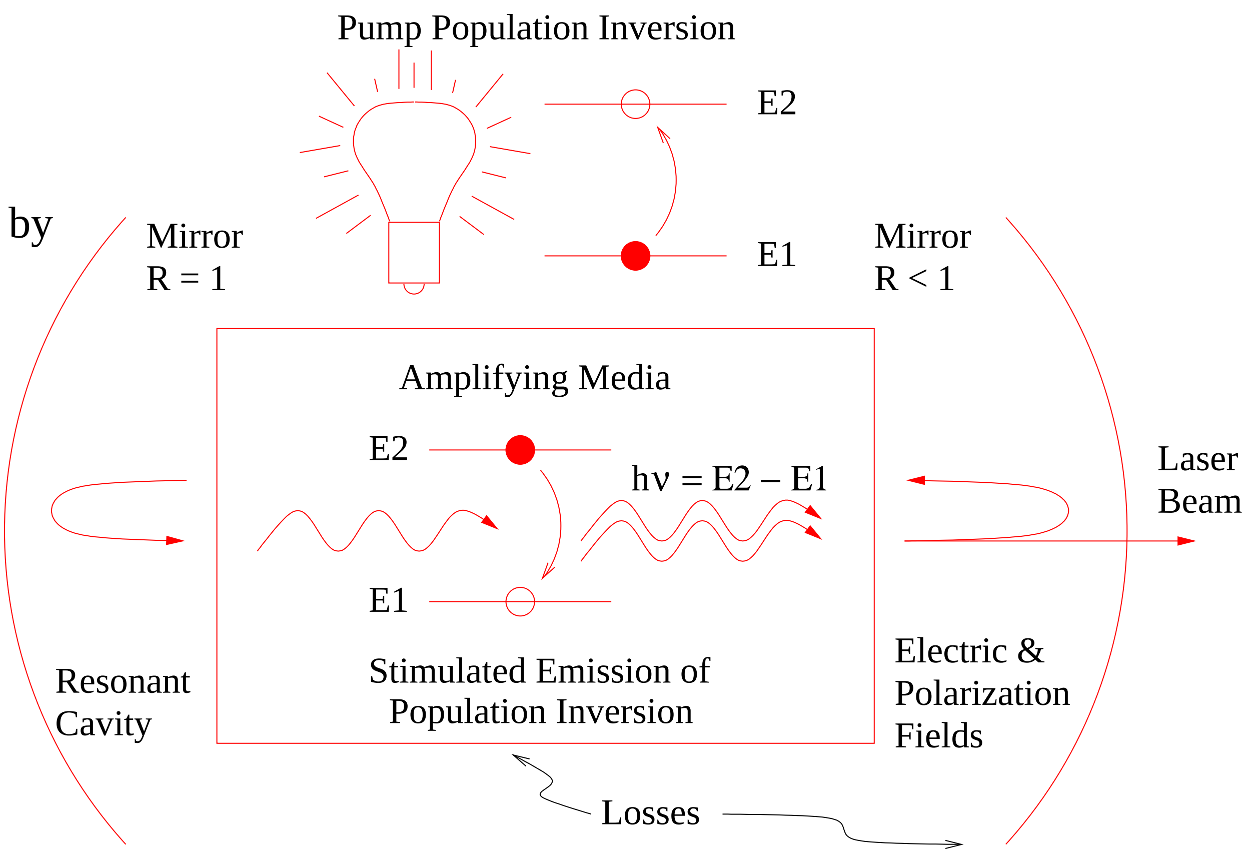
Abstract

We consider coupled lasers, where the intensity deviations from the steady state, modulate the pump of the other lasers. Most of our results are for two lasers where the coupling constants are of opposite sign. This leads to a Hopf bifurcation to periodic output for weak coupling. As the magnitude of the coupling constants is increased (negatively) we observe novel amplitude effects such as a weak coupling resonance peak and, strong coupling subharmonic resonances and chaos. In the weak coupling regime the output is predicted by a set of slow evolution amplitude equations. Pulsating solutions in the strong coupling limit are described by discrete map derived from the original model.

Laser Physics

Light
Amplification by
Stimulated
Emission of
Radiation



1958 : Townes & Schawlow. Laser Theory. Received Nobel Prize.
1960 : Maiman. First operational laser. Ruby cyrstal as amplifier.

Rate equations with pump coupling

- $I = |E|^2$: Intensity, D : Inversion

$$\frac{dI_j}{dt} = (D_j - 1)I_j$$
$$\frac{dD_j}{dt} = \epsilon_j^2[A_j - (1 + I_j)D_j]$$

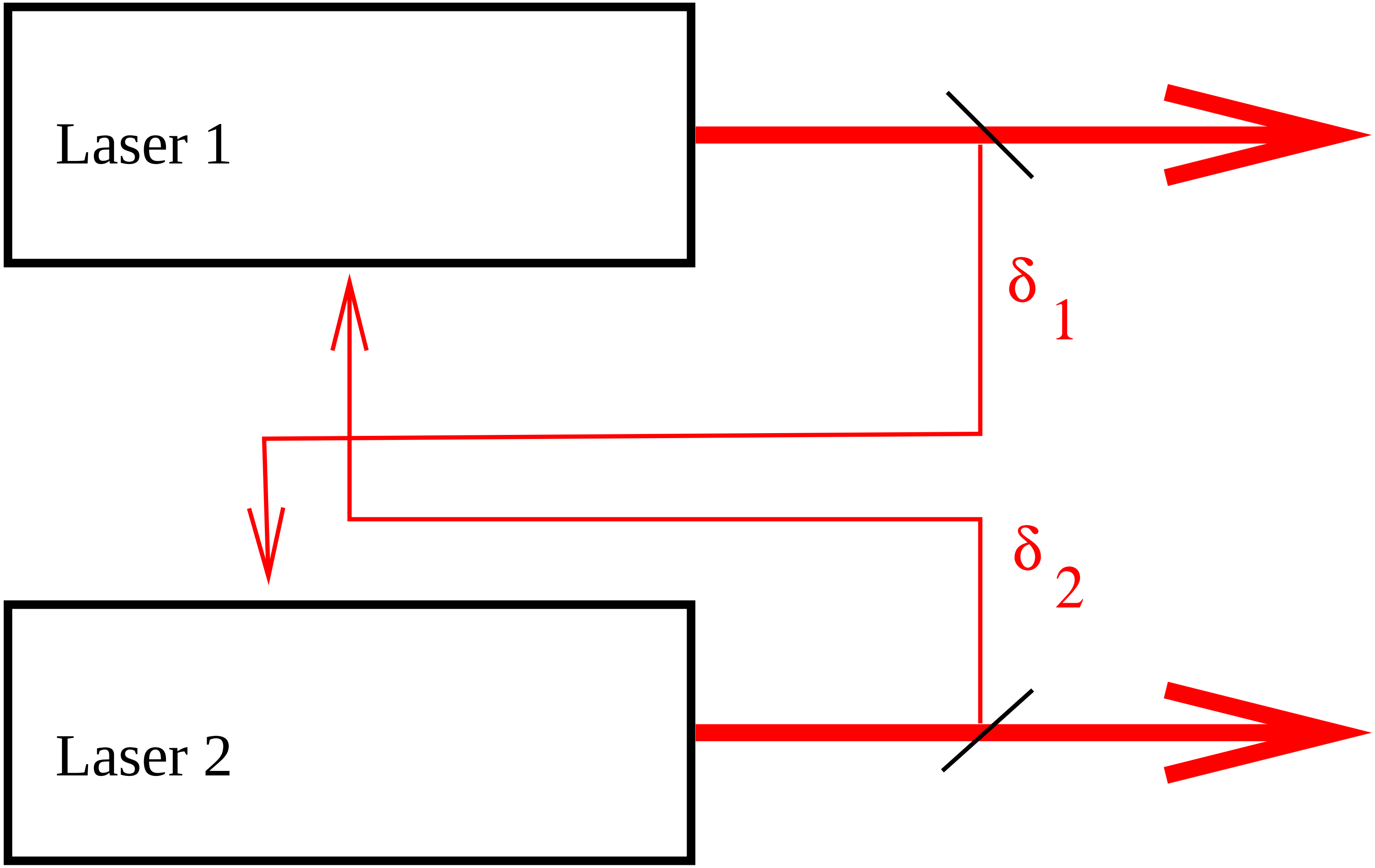
- Non-zero steady-state corresponds to CW output.

$$D_{j0} = 1, \quad I_{j0} = A_j - 1$$

- Investigate the effects of coupling two lasers through their pump:

$$A_j = A_{j0} + I_{j0}\delta_k(I_k - I_{k0}).$$

Pump-coupled Lasers



Rate equations with pump coupling

- Define new variables for the deviations from the cw state as

$$I_j = I_{j0}(1 + y_j), \quad D_j = 1 + \epsilon_j \sqrt{I_{j0}} x_j, \quad t_{new} = \epsilon_1 \sqrt{I_{10}} t_{old}.$$

- The new rate equations are

$$\begin{aligned}
\frac{dy_1}{dt} &= x_1(1 + y_1) \\
\frac{dx_1}{dt} &= -y_1 - \epsilon x_1(a_1 + b y_1) + \delta_2 y_2, \quad \delta_2 < 0 \\
\frac{dy_2}{dt} &= \beta x_2(1 + y_2) \\
\frac{dx_2}{dt} &= \beta[-y_2 - \epsilon \beta x_2(a_2 + b y_2) + \delta_1 y_1]
\end{aligned}$$

where a_j, b and β are dissipation & pump constants.

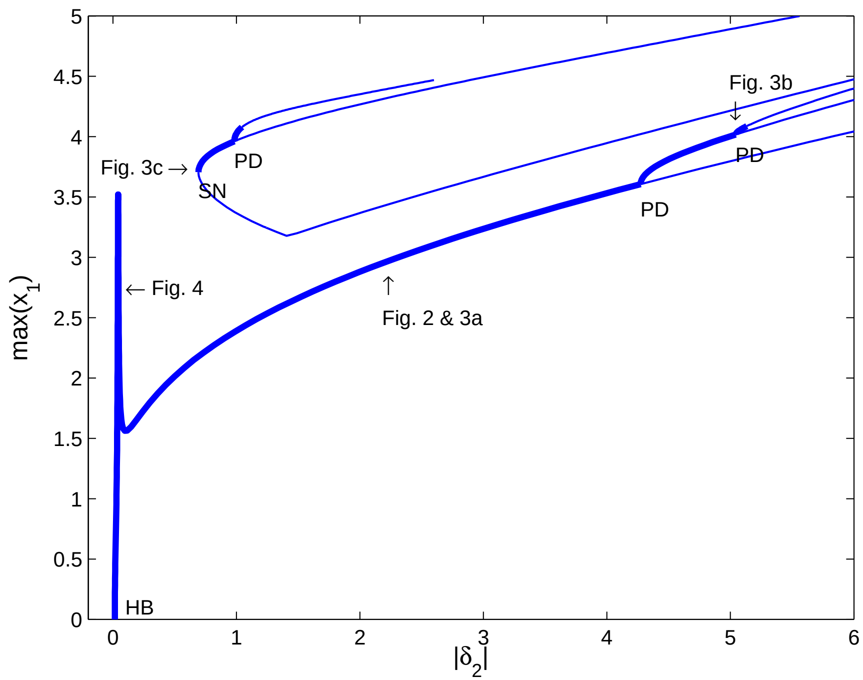
Hopf, Period-doublings and Chaos

For fixed δ_1 while varying δ_2:

- *Hopf bifurcation* at $\delta_2 = \delta_{2H}$.

$$\delta_1\delta_2 + \epsilon^2[a_1a_2 + 4\alpha^2\frac{a_1a_2}{(a_1+a_2)^2}] = 0$$

- *Period doubling* $\rightarrow$ chaos. *Saddle-node bifurcations* $\rightarrow$ subharmonic resonances.



Small coupling: $\delta_2 = O(\epsilon)$

- Look for slowly-evolving small-amplitude solutions of the form

$$y_{j1}(t,T) = A_j(T)e^{it} + c.c.,$$

- Obtain the following slow-evolution equations for the amplitudes:

$$\begin{aligned} \frac{dA_1}{dT} &= -\frac{1}{2}a_1A_1 - \frac{1}{6}i|A_1|^2A_1 - \frac{1}{2}i\delta_2 A_2 \\ \frac{dA_2}{dT} &= -\frac{1}{2}a_2A_2 - \frac{1}{6}i|A_2|^2A_2 - \frac{1}{2}i\delta_1 A_1 + i\alpha A_2 \end{aligned}$$

- To analyze let $A_j(T) = R_j(T)e^{i\theta_j(T)}$ and $\psi = \theta_2 - \theta_1$.

Small coupling resonance

$$R_2^4 = 3\frac{\Delta_1^2}{\Delta_2}$$

$$\Delta_1 = (\delta_1\delta_2 + a_1a_2)\frac{(a_1 + a_2)^2}{a_1a_2} \text{ and } \Delta_2 = 1 + \frac{a_2\delta_2}{a_1\delta_1}$$

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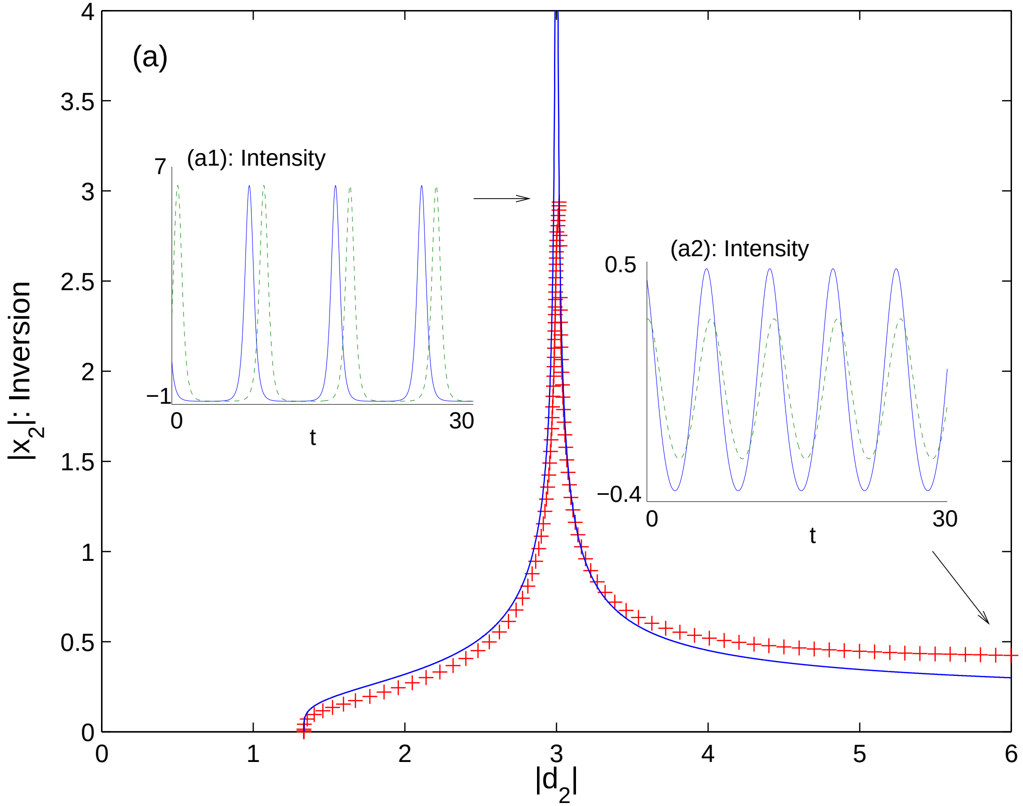
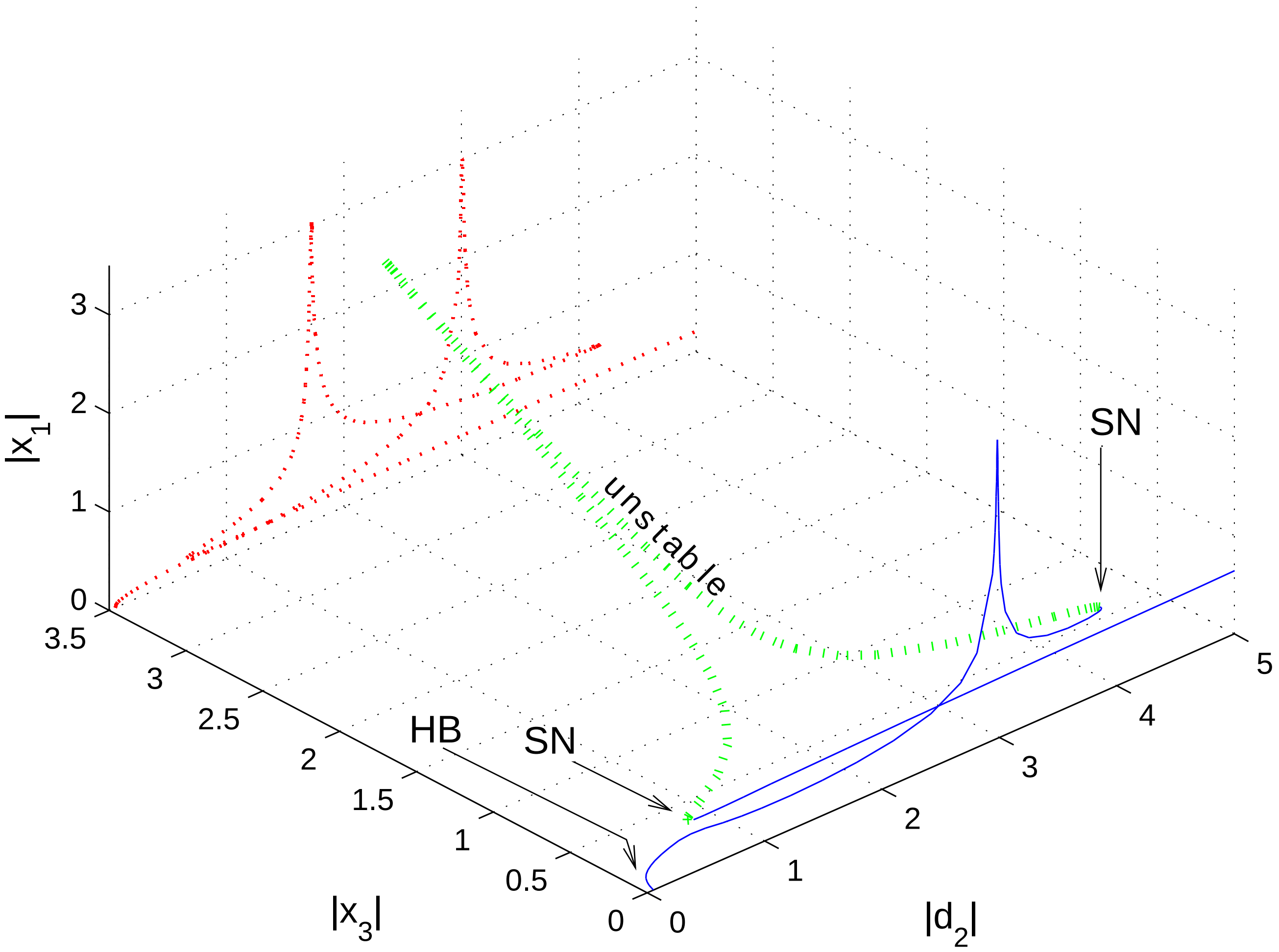
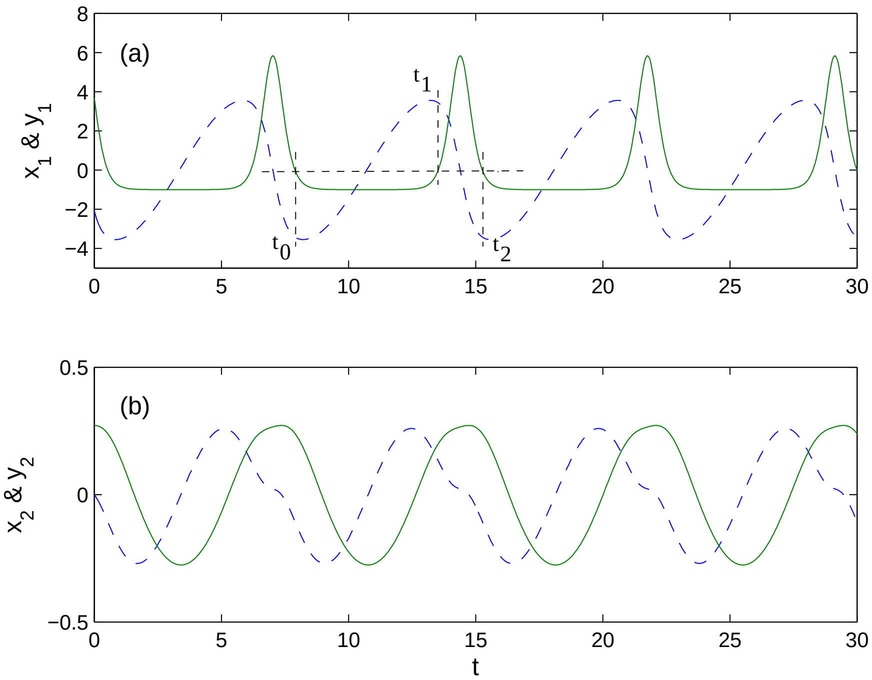


3 Lasers



Large coupling: $\delta_2 = O(1)$



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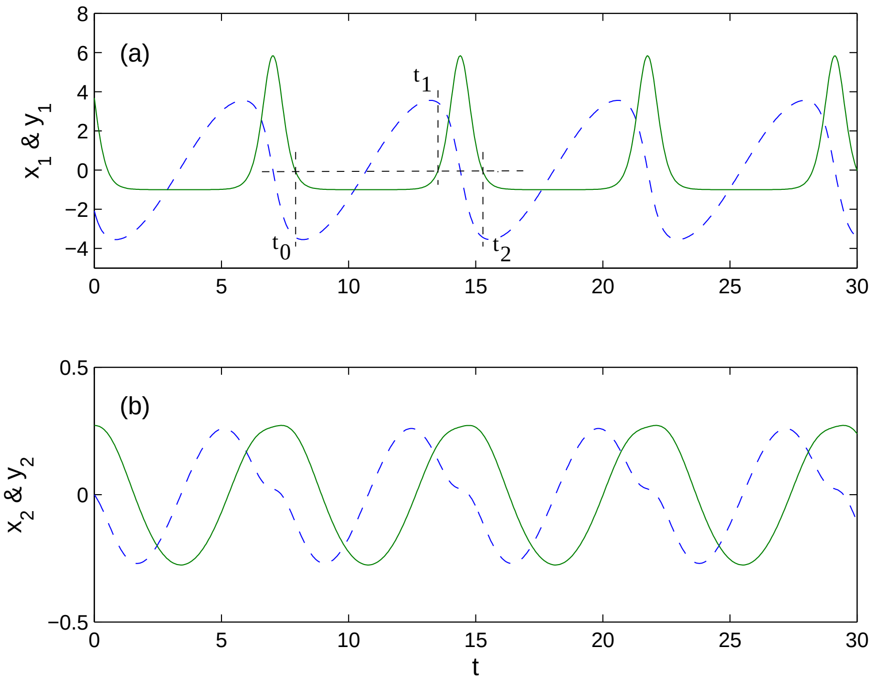


- L1: Matched Asymptotics. Pulse = inner region.
- L2: Multiple Scales. Damped oscillations in outer. Kick by pulse in inner.

Map

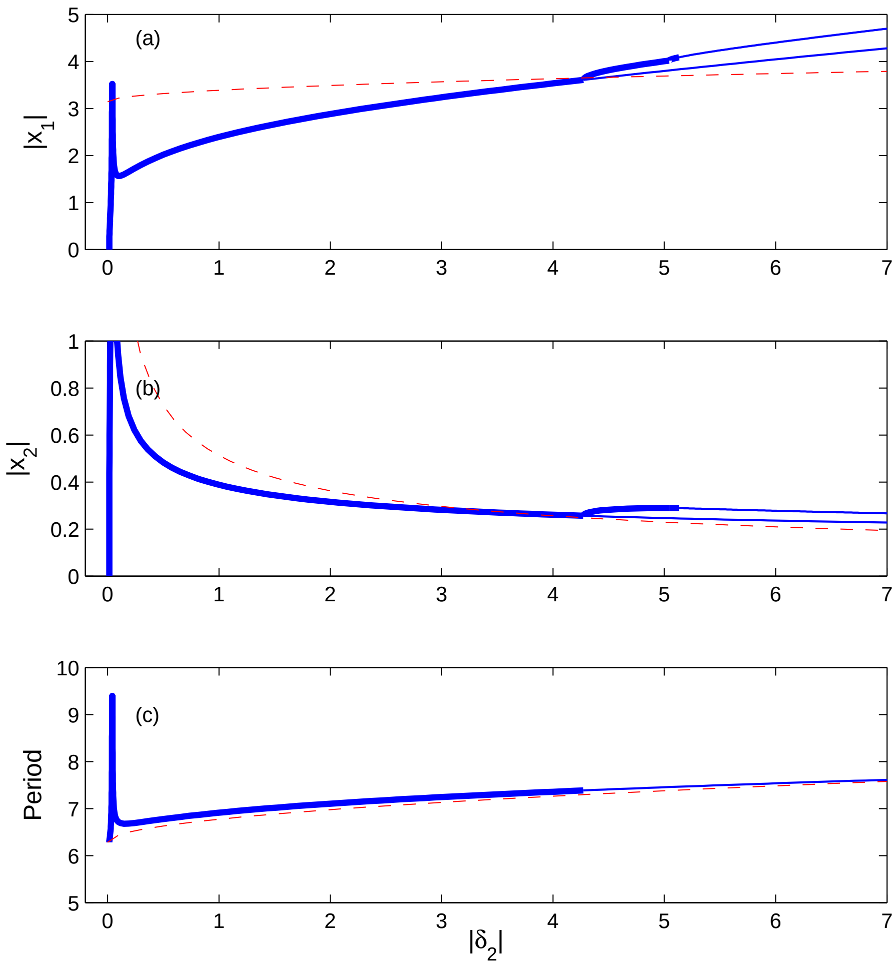
$$G(t) = (x_1 - \frac{1}{\gamma})e^{-\gamma t} + \frac{1}{\gamma} + \frac{\delta_2}{\alpha^2 + \omega^2} e^{-\gamma t}$$
$$\cdot \left[(\alpha y_2 - \omega x_2)(e^{\alpha t}\cos(\omega t) - 1) + (\omega y_2 + \alpha x_2) e^{\alpha t} \sin(\omega t) \right]$$

$$\int_0^P G(t)dt = 0$$

$$x_2 \mapsto e^{-\frac{1}{2}\epsilon a_2 P}[x_2 \cos(\omega P) - y_2 \sin(\omega P)] + \delta_1 2G(P)$$
$$y_2 \mapsto e^{-\frac{1}{2}\epsilon a_2 P}[x_2 \sin(\omega P) + y_2 \cos(\omega P)]$$

$$x_1 \mapsto -G(P) + \frac{2}{3}\epsilon b G(P)^2$$

Fixed points = periodic solutions



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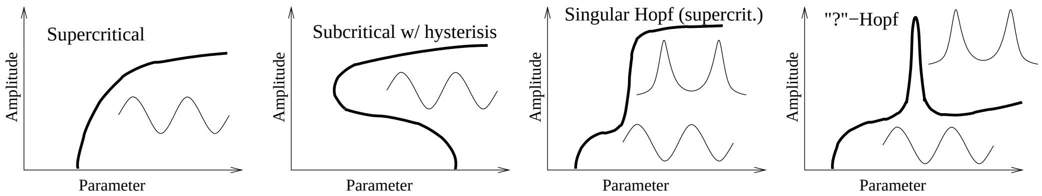

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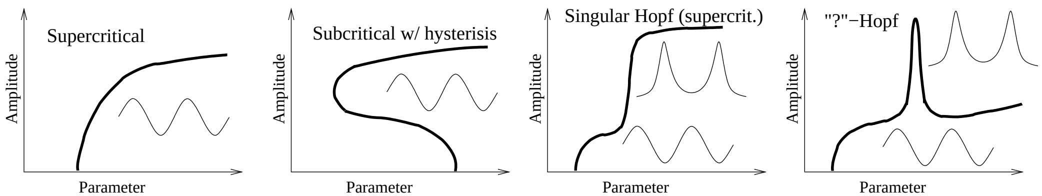

- Pulsating/Harmonic for strong coupling.
  Map → fixpoints.
  Similar to *single laser with periodic forcing*.
  Pulsations become larger.
  Harmonic solutions become smaller.

Fixed Points

$$\begin{aligned} \max[x_1] &= \pi + \sqrt{\frac{3a_2}{2a_1}\delta_1|\delta_2|}, \\ \max[x_2] &= \max[y_2] = \sqrt{\pi^2 \frac{2a_1}{3a_2}\frac{\delta_1}{|\delta_2|}}, \\ P &= 2\max[x_1]. \end{aligned}$$

L1: Pulsing, L2: Pulsing

- Derive subharmonic-Melnikov equations

$$\int_0^T -a_j x_j^2 + \delta_k x_j y_k dt = 0$$

- Use unperturbed Hamiltonian problem to evaluate integrals. Use matched asymptotics to construct pulsating solutions.
- Subharmonic-Melnikov equations conditions become:

$$(1 + \frac{a_2 \delta_2}{a_1 \delta_1}) T^2 = 0.$$

- For periodic solutions with $T \neq 0$, $\delta_2 = \delta_{2S}$.

L1: Pulsing, L2: Harmonic

- Localized solutions.
- Derive subharmonic-Melnikov equations

$$\int_0^T -a_j x_j^2 + \delta_k x_j y_k dt = 0$$

- Poincare'-Lindstedt for L1.
  Multiple scales for L2.
- Reproduce local bifurcation result.