

EXERCISE 2

Purpose: To learn some of the basic concepts of bivariate regression analysis. **This exercise is due on Tuesday, September 13.**

Consider the following data:

\mathbf{y}	\mathbf{x}
8	1
7	2
13	3
11	4
16	5
15	6

Using the above data, compute the following quantities:

1) $\bar{y} = \sum y_i / n$

2) $\bar{x} = \sum x_i / n$

3) $\hat{\beta}_1 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$

4) $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

5) $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

6) $\hat{u}_i = y_i - \hat{y}_i$

7) $SST = \sum(y_i - \bar{y})^2$ (Total Sum of Squares)

8) $SSE = \sum(\hat{y}_i - \bar{y})^2$ (Explained Sum of Squares)

9) $SSR = \sum \hat{u}_i^2$ (Residual Sum of Squares)

10) $R^2 = \frac{SSE}{TSS}$ (Coefficient of Determination)

11) Construct the following Analysis of Variance Table:

<u>Source</u>	<u>df</u>	<u>MS</u>	<u>F</u>	<u>p-value</u>
SSE	k	$\frac{SSE}{k}$	$\frac{MSE}{MSR}$	$\Pr(F_{k,n-k-1} > F_0)$
SSR	n-k-1	$\frac{SSR}{n-k-1}$		
SST	n-1			

The number of observations is denoted by n while k denotes the number of explanatory variables in your multiple regression model not counting the intercept term.

$$12) \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-k-1}$$

$$13) \hat{\sigma} = \sqrt{\hat{\sigma}^2} \quad (\text{Standard Error of Regression})$$

$$14) se(\hat{\beta}_0) = \sqrt{\frac{\hat{\sigma}^2 \sum x_i^2}{n \sum (x_i - \bar{x})^2}}$$

$$15) t_{\hat{\beta}_0} = \frac{\hat{\beta}_0}{se(\hat{\beta}_0)}$$

$$16) se(\hat{\beta}_1) = \sqrt{\frac{\hat{\sigma}^2}{\sum (x_i - \bar{x})^2}}$$

$$17) t_{\hat{\beta}_1} = \frac{\hat{\beta}_1}{se(\hat{\beta}_1)}$$

$$18) \Pr(\hat{\beta}_0 - t_{n-k-1,0.025} \cdot se(\hat{\beta}_0) < \beta_0 < \hat{\beta}_0 + t_{n-k-1,0.025} \cdot se(\hat{\beta}_0)) = 0.95$$

$$19) \Pr(\hat{\beta}_1 - t_{n-k-1,0.025} \cdot se(\hat{\beta}_1) < \beta_1 < \hat{\beta}_1 + t_{n-k-1,0.025} \cdot se(\hat{\beta}_1)) = 0.95$$

20) Suppose x = 2.5. Compute \hat{y} . (Prediction)