Lecture 10

Interpretation of Coefficients

1. Level-Level equation:

$$y = \beta_0 + \beta_1 x + u$$

Interpretation of β_1

$$\Delta y = \beta_1 \Delta x$$

2. Level-Log equation:

$$y = \beta_0 + \beta_1 \log(x) + u$$

log = log base e (natural log)

Interpretation of β_1

$$\Delta y = \left(\frac{\beta_1}{100}\right) \cdot \% \Delta x$$

3. Log-Level equation:

$$\log(y) = \beta_0 + \beta_1 x + u$$

Interpretation of β_1

$$\% \Delta y = 100 \beta_1 \cdot \Delta x$$

4. <u>Log-Log equation</u>:

$$\log(y) = \beta_0 + \beta_1 \log(X) + u$$

Interpretation of β_1

$$\% \Delta y = \beta_1 \cdot \% \Delta X$$

Examples:

A. $\hat{y} = 10 + 5X$, $\hat{\beta}_1 = 5$, $\hat{\beta}_0 = 10$

For every one unit increase (decrease) in X, you have a 5 unit increase (decrease) in y.

B. $\hat{y} = 10 + 200 \log(X)$

For every one percent ~~change~~ increase (decrease) in X, you have a $\frac{200}{100} = 2$ unit ~~change~~ increase (decrease) in y.

C. $\hat{\log}(y) = 10 + 0.05X$

For every one unit increase (decrease) in X, you have a $(0.05)100 = 5\%$ increase (decrease) in y.

D. $\hat{\log}(y) = 10 + 0.80 \log(X)$

For every one percent increase (decrease) in X, you have a 0.80 of one percent increase (decrease) in y.

See Table 2.3 in Wooldridge.

It is helpful to know that:

$$100 \cdot [\log(y_2) - \log(y_1)] \doteq \frac{y_2 - y_1}{y_1} \cdot 100 = \%\text{ change}$$

in y in going from y_1 to y_2

Homoskedasticity vs. Heteroskedasticity in the errors of a regression equation

Homoskedasticity: $Var(u|X) = \sigma^2$ for all X.

See Figure 2.8 in your textbook

Heteroskedasticity: $Var(u|X) \neq \sigma^2$ for all X.

See Figure 2.9 in your textbook.

Ordinary Least Squares is the appropriate method for estimating β_0 and β_1 when there is homoskedasticity in the errors of your regression model. However, OLS is not the appropriate estimation method when the errors are heteroskedastic. Instead weighted least squares (sometimes called generalized least squares) should be used.