

MIDTERM EXAM
TAKE-HOME PART
KEY

Assignment of Points:

Q5.5 (2, 2, 3, 3) = 10

Q5.9 (2, 3, 2, 3) = 10

Q5.15 (2, 3, 3) = 8

Q5.18 (3, 3) = 6

Total = 34

Q 5.5

(a) Report briefly on how each of the variables influences the value of a home.

All the considered variables have statistically significant effects.

Per capita crime rate, nitric oxide concentration, proportion of owner-occupied units built prior to 1940, weighted distance to five Boston employment centers, full-value property-tax rate per \$10,000, pupil-teacher ratio by town have NEGATIVE effects on the median value of owner-occupied homes by district.

At the same time, average number of rooms per dwelling and index of accessibility to radial highways POSITIVELY affect the median value of owner-occupied homes.

(b) Find 95% interval estimates for the coefficients of CRIME and ACCESS

CRIME: $-0.1834 \mp 1.96 * 0.0365 = [-0.25494, -0.11186]$

ACCESS: $0.2723 \mp 1.96 * 0.0723 = [0.130592, 0.414008]$

(c) Test the hypothesis that increasing the number of rooms by one increases the value of a house by \$7,000.

Basically, we need to test that $\beta_{ROOMS} = 7$

Test statistic is: $t = (6.3715 - 7) / 0.3924 = -1.6016$, which is higher than -1.96. That means that we can accept the null hypothesis at the 5% level.

- (d) Test as an alternative hypothesis H_1 that reducing the pupil-teacher ratio by 10 will increase the value of a house by more than \$10,000

Now, we need to test $H_0: (-10)\beta_{PTRATIO} = 10$ or equivalently $H_0: \beta_{PTRATIO} = -1$ against the alternative hypothesis of $H_1: (-10)\beta_{PTRATIO} > 10$ or equivalently $H_1: \beta_{PTRATIO} < -1$

Test statistic is: $t = (-1.1768 + 1)/0.1394 = -1.2683$, which is greater than the 5% left-tail critical value of -1.645. Therefore, we reject the alternative hypothesis that reducing the pupil-teacher ratio by 10 will increase the value of a house by more than \$10,000.

Q 5.9

- (a) What is the marginal effect of experience on wages?

$$\beta_3 + 2\beta_4 \text{ * EXPER}$$

- (b) What signs do you expect for each of the coefficients β_2 , β_3 and β_4 ? Why?

The sign of β_2 is expected to be positive, since the education should have positive effect on a wage.

The sign of β_3 is expected to be positive, since the experience should have positive effect on a wage.

The sign of β_4 is expected to be negative, since the experience should have an inverted U-shaped relationship with a wage.

- (c) After many years of experience do wages start to decline? (Express your answer in terms of β 's.)

Presumably, after 30-35 years of experience wages are expected to decline. That's why β_4 is expected to have negative sign. Wages start to decline after $(-\beta_3/2\beta_4)$ years of experience.

- (d) The results from estimating the equation using 1000 observations in the file *cps4c_small.dat* are given in Table 5.9 on page 204. Find 95% interval estimates for

- (i) The marginal effect of education on wages.

$$2.2774 \pm 1.96 * 0.1394 = [2.004176, 2.550624]$$

- (ii) The marginal effect of experience on wages when EXPER = 4

$$0.6821 + 2 * (-0.0101) * 4 = 0.6013$$

95% interval estimate:

$$0.6013 \pm \sqrt{0.010987185 + 4 * (-0.000189259) * 4 + 4 * 0.000003476 * 16 * 1.96} \\ = [0.424014723, 0.778585277]$$

- (iii) The marginal effect of experience on wages when EXPER = 25

$$0.6821 + 2 * (-0.0101) * 25 = 0.1771$$

95% interval estimate:

$$0.1771 \pm \sqrt{0.010987185 + 4 * (-0.000189259) * 25 + 4 * 0.000003476 * 625 * 1.96} \\ = [0.123377226, 0.230822774]$$

- (iv) The number of years of experience after which wages decline

Wages start to decline after $(-\beta_3/(2\beta_4)) = 33.7673$ years of experience.

We can get 95% interval estimate using Delta method.

$$\begin{aligned}\text{Var}(-\hat{\beta}_3/2*\hat{\beta}_4) &= 0.010987185*(1/(2*0.0101))^2 + 2*(-0.000189259)*(1/(2*0.0101))* \\ &\quad (-0.6821/(2*0.0101)) + 0.000003476*(-0.6821/(2*0.0101))^2 = 27.56344813 \\ \text{se}(-\hat{\beta}_3/2*\hat{\beta}_4) &= \sqrt{27.56344813} = 5.250090297 \\ 95\% \text{ interval estimate: } &33.7673 \pm 5.250090297 = [28.5172097, 39.0173903]\end{aligned}$$

Q 5.15*

Reconsider the presidential voting data introduced in Exercise 2.14.

(a) Estimate the regression model

$$\text{VOTE} = \beta_1 + \beta_2 \text{GROWTH} + \beta_3 \text{INFLATION} + e$$

Report the results in standard format. Are the estimates for β_2 and β_3 significantly different from zero at a 10% significance level? Did you use one-tail tests or two-tail tests? Why?

ANSWER:

The estimated regression model is:

$$\begin{aligned}\widehat{\text{VOTE}} &= 52.16 + 0.6434\text{GROWTH} - 0.1721\text{INFLATION} \\ (\text{se}) &= (1.46) \qquad (0.1656) \qquad (0.4290)\end{aligned}$$

The hypothesis test results on the significance of the coefficients are:

$H_0: \beta_2 = 0$	$H_1: \beta_2 > 0$	$p\text{-value} = 0.0003$	significant at 10% level
$H_0: \beta_3 = 0$	$H_1: \beta_3 < 0$	$p\text{-value} = 0.3456$	not significant at 10% level

One-tail tests were used because more growth is considered favorable, and more inflation is considered not favorable, for re-election of the incumbent party.

(b) Assume the inflation rate is 4%. Predict the percentage vote for the incumbent party when the growth rate is (i) -3%, (ii) 0%, (iii) 3%.

ANSWER:

- (i) For $\text{INFLATION} = 4$ and $\text{GROWTH} = -3$, $\widehat{\text{VOTE}}_0 = 49.54$
- (ii) For $\text{INFLATION} = 4$ and $\text{GROWTH} = 0$, $\widehat{\text{VOTE}}_0 = 51.47$
- (iii) For $\text{INFLATION} = 4$ and $\text{GROWTH} = 3$, $\widehat{\text{VOTE}}_0 = 53.40$

(c) Test, as an alternative hypothesis, that the incumbent party will get the majority of the expected vote when the growth rates is (i) -3%, (ii) 0%, (iii) 3%. Use a 1% level of significance. If you were the president seeking re-election, why might you set up each of these hypotheses as an alternative rather than a null hypothesis?

ANSWER:

- (i) When $\text{INFLATION} = 4$ and $\text{GROWTH} = -3$, the hypotheses are
 $H_0: \beta_1 + 4\beta_3 \leq 50$ $H_1: \beta_1 + 4\beta_3 > 50$

The calculated t-value is $t = -0.399$. Since $-0.399 < 2.457 = t_{(0.99,30)}$, we do not reject H_0 . There is no evidence to suggest that the incumbent party will get the majority of the vote when $\text{INFLATION} = 4$ and $\text{GROWTH} = -3$.

- (ii) When $\text{INFLATION} = 4$ and $\text{GROWTH} = 0$, the hypotheses are
 $H_0: \beta_1 + 4\beta_3 \leq 50$ $H_1: \beta_1 + 4\beta_3 > 50$

The calculated t-value is $t = 1.408$. Since $1.408 < 2.457 = t_{(0.99,30)}$, we do not reject H_0 . There is no evidence to suggest that the incumbent part will get the majority of the vote when $INFLATION = 4$ and $GROWTH = 0$.

(iii) When $INFLATION = 4$ and $GROWTH = 3$, the hypotheses are

$$H_0: \beta_1 + 4\beta_3 \leq 50 \quad H_1: \beta_1 + 4\beta_3 > 50$$

The calculated t-value is $t = 2.950$. Since $2.950 > 2.457 = t_{(0.99,30)}$, we reject H_0 . We conclude that the incumbent part will get the majority of the vote when

$INFLATION = 4$ and $GROWTH = 3$.

As a president seeking re-election, you would not want to conclude that you would be reelected without strong evidence to support such a conclusion. Setting up re-election as the alternative hypothesis with a 1% significance level reflects this scenario.

Q 5.18

What is the relationship between crime and punishment? This important question has been examined by Cornwell and Trumbull using panel data from North Carolina. The cross sections are 90 countries, and the data are annual for the years 1981-1987. The data are in the file *crime.dat*.

Using data from 1987, estimate a regression relating the log of the crime rate LCRM RTE to the probability of an arrest PRBARR (the ratio of arrests to offenses), the probability of conviction PRB CONV (the ratio of convictions to arrests), the probability of a prison sentence PRBPIS (the ratio of prison sentences to convictions), the number of police per capita POLPC, and the weekly wage in construction WCON. Write a report of your findings. In your report, explain what effect you would expect each of the variables to have on the crime rate and note whether the estimated coefficients have the expected signs and are significantly different from zero. What variables appear to be the most important for crime deterrence? Can you explain the sign for the coefficient of POLPC?

Here is the SAS Output:

The SAS System					
The REG Procedure					
Model: MODEL1					
Dependent Variable: lcrm rte					
Number of Observations Read 90					
Number of Observations Used 90					
Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	16.11424	3.22285	25.33	<.0001
Error	84	10.68563	0.12721		

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Corrected Total	89	26.79987			
Root MSE		0.35667	R-Square	0.6013	
Dependent Mean		-3.54173	Adj R-Sq	0.5775	
Coeff Var		-10.07037			

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t
Intercept	1	-3.48210	0.35139	-9.91	<.0001
prbarr	1	-2.43318	0.32043	-7.59	<.0001
prbconv	1	-0.80768	0.11096	-7.28	<.0001
prbpris	1	0.33398	0.47002	0.71	0.4793
polpc	1	200.56635	43.58696	4.60	<.0001
wcon	1	0.00219	0.00083388	2.62	0.0104

Report of Findings:

It seems that all of the variables that were proposed to explain the log of the crime rate were statistically significant **except** for the variable PRBPRIS which is the ratio of prison sentences to convictions. This latter variable represents the strictness of the penal system as it relates to the probability that convictions actually wind up resulting in prison sentences. Evidently, this strictness (somewhat to my surprise) does not significantly affect the crime rate. Besides, the sign of the coefficient is positive and not negative as expected. On the other hand, the ratio of arrests to offenses (PRBARR – probability of being arrested) has an expected negative effect on the crime rate and the probability of a conviction giving rise to an arrest, PRBCONV (the diligence of the police force in rounding up violators of the law) has an expected negative effect on the crime rate. On the opposite side, the higher the weekly wage in construction, WCON, the higher the crime rate. This is somewhat unexpected. People who are earning good salaries would seem to be less prone to commit crimes but, on the other hand, they make for susceptible crime victims given the money such workers might be carrying with them. Interestingly, a higher number of police per capita, POLPC, tends to increase the crime rate, which on the face of things is counterintuitive. There is possibly an “endogenous” explanation of this effect. When crime rate in an area becomes high, there tends to be a response by the police department to increase the number of officers in the area. Given that this data is only taken from one year of

the panel of data, it is possible that the subsequent crime rate might be lower in years following 1987 as a result of having increased police presence in a crime-ridden area.