

Name Mr. Key
ID. _____

ECO 5350
Intro. Econometrics

Prof. T. Fomby
Spring 2006

Mid-Term 1

Instructions: Put your name and student ID in the upper right-hand-corner of this exam. This exam is worth a total of 50 points. The breakout of these points by questions is as follows:

- Q1 = (2,1,1,1) = 5 points
- Q2 = (3,2) = 5 points
- Q3 = (2,2,2,4) = 10 points
- Q4 = 7 points
- Q5 = 15 points
- Q6 = (4,4) = 8 points

You have one hour and twenty minutes to take this test. Good luck.

hypotheses and produce economic predictions

1. Definitions:

- (2) a) Define the term **econometrics**. That field in economics where statistical methods are developed and used to test economic hypotheses.
- (1) b) Monthly observations on the U.S. unemployment rate from 1950 to the present represent what kind of a data set? *time series data set*
- (1) c) Observations on the real gross domestic products of 35 countries in 1998 represent what kind of a data set? *cross section data set*
- (1) d) Observations on the real gross domestic products of 35 countries from 1950 to the present represent what kind of data set? *panel data set*

2. Interpretation of Coefficients

- a) Consider the following estimated regression equation

$$bwght = 119.77 - 0.514cigs$$

birth weight when cigs = 10 is $119.77 - 0.514(10) = 114.63$ oz.

where *bwght* is infant birth weight in ounces and *cigs* is the average number of cigarettes the mother smoked per day during pregnancy. Explain to me the interpretation of b_2 in this model. What is the predicted birth weight of a child when *cigs* = 10 per day? Show your work below.

(3) $b_2 = -0.514$. It represents the change in birth weight that is expected to occur with a one unit change in cigarettes smoked per day.

- b) Using data from 1988 for houses sold in Andover, Massachusetts, from Kiel and McClain (1995), the following equation relates housing price (*price*) to the distance from a recently built garbage incinerator (*dist*):

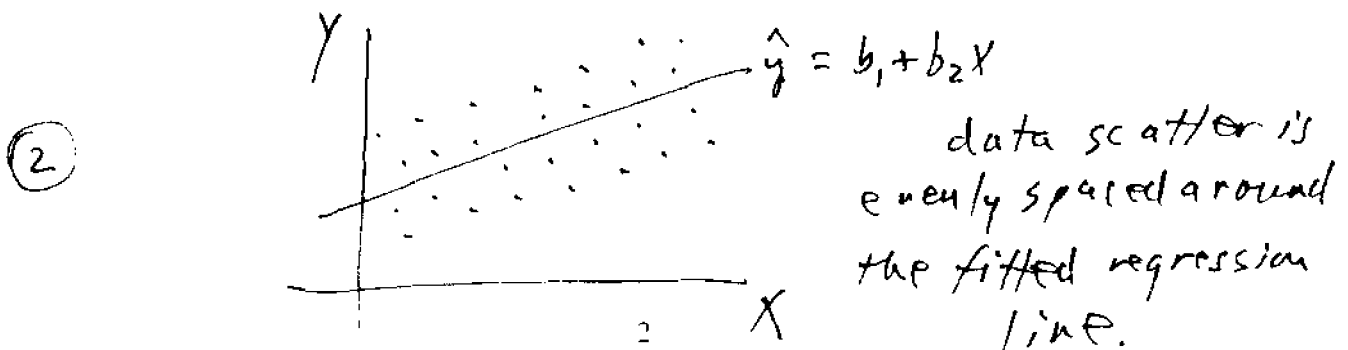
$$\log(\text{price}) = 9.40 + 0.312\log(\text{dist})$$

by 0.312 of one percent.

Explain to me the interpretation of b_2 in this model.

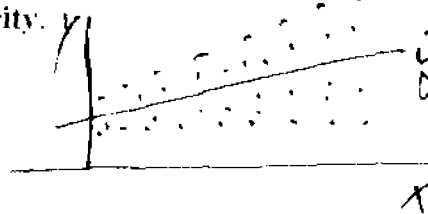
(2) with each one percent increase in the distance from the incinerator, the price of the home is expected to increase.

3. (a) In the space below draw me a scatter plot of data involving *y* and *x* that exhibits homoskedasticity.



(b) In the space below draw me a scatter plot of data involving y and x that exhibits heteroskedasticity.

(2)



The spread of the scatter of points is increasing (or decreasing) in x .

(c) In which of the above scatter plots would the method of least squares be more appropriately applied? Explain your answer.

(2)

The plot in part (a) above is appropriate for least squares estimation because one of the main-tained assumptions for least squares is homoskedasticity.

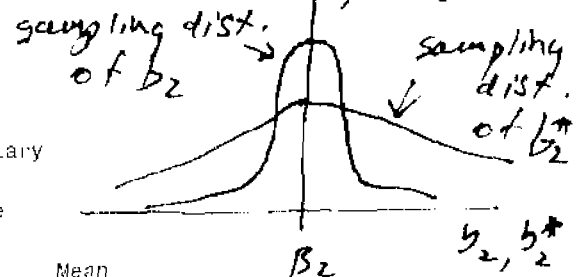
(d) Briefly state the Gauss-Markov Theorem. In a diagram below represent the implication of the Gauss-Markov theorem by drawing two competing sampling distributions.

(4)

Among the class of unbiased, linear estimators of β_2 , say $b_2^* = c_1 y_1 + c_2 y_2 + \dots + c_N y_N$, the estimator with the least variance is the least squares estimator, b_2 .

4. Fill in the blanks in the following regression output:

The REG Procedure
Model: MODEL1
Dependent Variable: lsalary
Analysis of Variance



| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|-----------------|-----|----------------|-------------|---------|--------|
| Model | 1 | 0.85091 | 0.85091 | 2.33964 | 0.1284 |
| Error | 175 | 63.79531 | 0.36369 | | |
| Corrected Total | 176 | 64.64622 | | | |

(4)

| | | | |
|----------------|---------|----------|---------|
| Root MSF | 0.60378 | R-Square | 0.01316 |
| Dependent Mean | 6.56285 | Adj R-Sq | 0.0075 |
| Coeff Var | 9.17195 | | |

$$R^2 = \frac{SSR}{SST} = \frac{0.85091}{64.64622} = 0.01316$$

(1)

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > t |
|-----------|----|--------------------|----------------|---------|---------|
| Intercept | 1 | 6.50550 | 0.06794 | 95.68 | <.0001 |
| centen | 1 | 0.00972 | 0.00636 | 1.5283 | 0.1284 |

(2)

$$t_{b_2} = \frac{b_2}{se(b_2)} = \frac{0.00972}{0.00636} = 1.5283$$

$$\therefore se = \frac{6.500550}{95.68} = 0.06794$$

5. Using the following data, fill in the appropriate blanks:

| X | Y | X ² | XY | $\hat{e}_i = y_i - \hat{y}_i$ | \hat{e}_i^2 |
|---|---|----------------|----|-------------------------------|---------------|
| 1 | 6 | 1 | 6 | 6.0 - 5.9 = 0.1 | 0.01 |
| 2 | 4 | 4 | 8 | 4 - 4.3 = -0.3 | 0.09 |
| 3 | 3 | 9 | 9 | 3 - 2.7 = 0.3 | 0.09 |
| 4 | 1 | 16 | 4 | 1 - 1.1 = -0.1 | 0.01 |

① $\bar{X} = \frac{10}{4} = 2.5$ ① $\bar{Y} = \frac{14}{4} = 3.5$ ① $\sum XY = 8.75$

① $b_2 = \frac{\sum X_i Y_i - N \bar{X} \bar{Y}}{\sum X_i^2 - N \bar{X}^2} = \frac{27 - 4(8.75)}{30 - 4(2.5)^2} = \frac{-8}{5} = -1.6$

① $b_1 = \bar{Y} - b_2 \bar{X} = 3.5 - (-1.6)(2.5) = 3.5 + 4.0 = 7.5$

oops!
Forgot to take square root.

① $var_{\hat{\beta}_2} = \frac{\hat{\sigma}^2}{\sum X_i^2 - N \bar{X}^2} = \frac{\sum (Y_i - \hat{Y}_i)^2 / (N-2)}{\sum X_i^2 - N \bar{X}^2} = \frac{0.20/2}{5} = 0.02$

$se(b_2) = \sqrt{var(b_2)} = \sqrt{0.02} = 0.1414$

① $t_{obs} = \frac{b_2}{se(b_2)} = \frac{-1.6}{0.1414} = -11.3154$ [I accepted your answer of $\frac{-1.6}{.02} = -80$]

However, I adjusted your answer for it.

① When $X_{i1} = 2.5$ then $\hat{Y}_{i0} = b_1 + b_2 X_{i1} = 7.5 - 1.6(2.5) = 3.5$

① The 95% confidence interval for β_2 is $[-2.2084, -0.9915]$

Note: $Pr(b_2 - se(b_2) t_{\alpha/2, N-2} < \beta_2 < b_2 + se(b_2) t_{\alpha/2, N-2}) = 1 - \alpha$

I accepted your answer of $[-1.66666, -1.51394]$

① Therefore, when I test $H_0: \beta_2 = 0$ versus $H_1: \beta_2 \neq 0$ at the 5% level of statistical significance, I conclude that $H_1: \beta_2 \neq 0$

and β_2 is apparently not equal to zero.

Thus, X_2 is a statistically significant explainer of the variation in y . This is because $\beta_2 = 0$ is not inside the 95% confidence interval for β_2 .

Note: I am giving you 5 points on this question no matter what!

6. Consider the **Computer Output** you have been given. Recall the Alesina and Summers model where we have a regression equation that explains a developing country's future rate of inflation as a function of the independence of its central bank. Suppose we have a country whose central bank independence measure is 1.80. Use the computer output to produce a prediction of the country's subsequent inflation rate and also provide me with a 95% confidence interval for your prediction. Show your work below so that you will obtain full credit for your answer.

$$se(\text{prediction error}) = \sqrt{(0.89865)^2 + (0.27304)^2}$$

(2)

$$= 0.9392$$

$$\hat{y}_0 = 9.44019 - 1.63558(1.80)$$

(2)

$$= 6.49615 \quad (= \text{the reported intercept of the transformed model})$$

95% prediction confidence interval given that $X = 1.80$:

$$\hat{y}_0 \pm se(\text{prediction error}) \cdot t_{N-2, \alpha/2}$$

$$t_{14, 0.025} = 2.145$$

$$6.49615 \pm 0.9392(2.145)$$

$$6.49615 \pm 2.014584$$

(4)

$$[4.481566, 8.510734]$$

```

/* x = average index of central bank independence (1 = little independence
    4 = very independent)
y = average Inflation 1955 - 1988
Source: Alesina and Summers (1993) Jo. of Money,
Credit, and Banking */

```

```

data in;
  input x y;
cards;
1.5 8.5
1 7.6
2 6.4
1.75 7.3
2 6.7
2 6.1
2.5 6.5
2 4.1
2 6.1
2 6.1
2.5 4.5
2.5 4.2
2.5 4.9
3.5 4.1
4 3
4 3.2
;

```

```

proc reg data = in;
  model y = x;

run;

```

```

/* Here we use the transformed model to obtain the point
prediction when x = 1.80 (it is the estimate of the intercept
in the transformed model) and the ingredients for the construction
of the standard error of the prediction error. We use the
Standard Error of the Transformed Regression (RMSE in SAS) or the Mean
Square of the Error in the ANOVA table) to construct the standard error of the
prediction error. se(prediction error) = sqrt(RMSE^2 + se(intercept)^2)=
sqrt(mean square error + se(intercept)^2). */

```

```

data in;
  set in;
  xstar = x - 1.80;

proc reg data=in;
  model y = xstar;

run;

```

The REG Procedure
 Model: MODEL1
 Dependent Variable: y

Number of Observations Read 16
 Number of Observations Used 16

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|-----------------|----|----------------|-------------|---------|--------|
| Model | 1 | 28.07829 | 28.07829 | 34.77 | <.0001 |
| Error | 14 | 11.30609 | 0.80758 | | |
| Corrected Total | 15 | 39.38437 | | | |

Root MSE 0.89865 R-Square 0.7129
 Dependent Mean 5.58125 Adj R-Sq 0.6924
 Coeff Var 16.10129

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > t |
|-----------|----|--------------------|----------------|---------|---------|
| Intercept | 1 | 9.44019 | 0.69194 | 13.64 | <.0001 |
| x | 1 | -1.63558 | 0.27738 | -5.90 | <.0001 |

The REG Procedure
 Model: MODEL1
 Dependent Variable: y

Number of Observations Read 16
 Number of Observations Used 16

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|-----------------|----|----------------|-------------|---------|--------|
| Model | 1 | 28.07829 | 28.07829 | 34.77 | <.0001 |
| Error | 14 | 11.30609 | 0.80758 | | |
| Corrected Total | 15 | 39.38437 | | | |

| | | | |
|----------------|----------|----------|--------|
| Root MSE | 0.89865 | R-Square | 0.7129 |
| Dependent Mean | 5.58125 | Adj R-Sq | 0.6924 |
| Coeff Var | 16.10129 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > t |
|-----------|----|--------------------|----------------|---------|---------|
| Intercept | 1 | 6.49615 | 0.27304 | 23.79 | <.0001 |
| xstar | 1 | -1.63558 | 0.27738 | -5.90 | <.0001 |

Statistical Tables

Table 1 Area Under the Standard Normal Distribution

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3079 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4773 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4983 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |

Source: This table was generated using the SAS® function PROBNO RM.

Table 2 Right-Tail Critical Values for the t -distribution

| <i>DF</i> | $\alpha = .10$ | $\alpha = .05$ | $\alpha = .025$ | $\alpha = .01$ | $\alpha = .005$ |
|-----------|----------------|----------------|-----------------|----------------|-----------------|
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 31 | 1.309 | 1.696 | 2.040 | 2.453 | 2.744 |
| 32 | 1.309 | 1.694 | 2.037 | 2.449 | 2.738 |
| 33 | 1.308 | 1.692 | 2.035 | 2.445 | 2.733 |
| 34 | 1.307 | 1.691 | 2.032 | 2.441 | 2.728 |
| 35 | 1.306 | 1.690 | 2.030 | 2.438 | 2.724 |
| 36 | 1.306 | 1.688 | 2.028 | 2.434 | 2.719 |
| 37 | 1.305 | 1.687 | 2.026 | 2.431 | 2.715 |
| 38 | 1.304 | 1.686 | 2.024 | 2.429 | 2.712 |
| 39 | 1.304 | 1.685 | 2.023 | 2.426 | 2.708 |
| 40 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 50 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 |
| 60 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 70 | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 |
| 80 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 |
| 90 | 1.291 | 1.662 | 1.987 | 2.368 | 2.632 |
| 100 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 |
| 110 | 1.289 | 1.659 | 1.982 | 2.361 | 2.621 |
| 120 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 |
| ∞ | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 |

Source: This table was generated using the SAS® function TINV.

Table 3 Right-Tail Critical Values for the F-Distribution

| v ₁ | Upper 5% Points | | | | | | | | | | | | | | | | | | |
|----------------|-----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-------|--------|-------|--------|--------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 15 | 20 | 24 | 30 | 40 | 60 | 120 | ∞ |
| 1 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 243.91 | 245.95 | 248.01 | 249.05 | 250.1 | 251.14 | 252.2 | 253.25 | 254.31 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.43 | 19.45 | 19.45 | 19.46 | 19.47 | 19.48 | 19.49 | 19.50 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 | 8.68 | 8.64 | 8.62 | 8.59 | 8.57 | 8.55 | 8.53 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 | 5.69 | 5.66 | 5.63 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 | 4.46 | 4.43 | 4.40 | 4.37 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.77 | 3.74 | 3.70 | 3.67 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.34 | 3.30 | 3.27 | 3.23 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.04 | 3.01 | 2.97 | 2.93 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.83 | 2.79 | 2.75 | 2.71 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 | 2.77 | 2.74 | 2.70 | 2.66 | 2.62 | 2.58 | 2.54 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.72 | 2.65 | 2.61 | 2.57 | 2.53 | 2.49 | 2.45 | 2.40 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.43 | 2.38 | 2.34 | 2.30 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 | 2.46 | 2.42 | 2.38 | 2.34 | 2.30 | 2.25 | 2.21 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 | 2.39 | 2.35 | 2.31 | 2.27 | 2.22 | 2.18 | 2.13 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.20 | 2.16 | 2.11 | 2.07 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.35 | 2.28 | 2.24 | 2.19 | 2.15 | 2.10 | 2.06 | 2.01 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.31 | 2.23 | 2.19 | 2.15 | 2.10 | 2.06 | 2.01 | 1.96 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.27 | 2.19 | 2.15 | 2.11 | 2.06 | 2.02 | 1.97 | 1.92 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 | 1.98 | 1.93 | 1.88 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.99 | 1.95 | 1.90 | 1.84 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 | 1.92 | 1.87 | 1.81 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.15 | 2.07 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.78 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.13 | 2.05 | 2.01 | 1.96 | 1.91 | 1.86 | 1.81 | 1.76 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.89 | 1.84 | 1.79 | 1.73 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 | 1.82 | 1.77 | 1.71 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.13 | 2.06 | 1.99 | 1.93 | 1.88 | 1.84 | 1.79 | 1.73 | 1.67 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.12 | 2.04 | 1.96 | 1.91 | 1.87 | 1.82 | 1.77 | 1.71 | 1.65 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.12 | 2.04 | 1.96 | 1.90 | 1.85 | 1.80 | 1.75 | 1.69 | 1.63 |
| 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.10 | 2.03 | 1.94 | 1.90 | 1.85 | 1.81 | 1.75 | 1.70 | 1.64 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.09 | 2.01 | 1.93 | 1.89 | 1.84 | 1.79 | 1.74 | 1.68 | 1.62 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.00 | 1.92 | 1.84 | 1.79 | 1.74 | 1.69 | 1.64 | 1.58 | 1.51 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.92 | 1.84 | 1.75 | 1.70 | 1.65 | 1.59 | 1.53 | 1.47 | 1.39 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.95 | 1.91 | 1.83 | 1.75 | 1.66 | 1.61 | 1.55 | 1.50 | 1.43 | 1.35 | 1.25 |
| ∞ | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.75 | 1.67 | 1.57 | 1.52 | 1.46 | 1.39 | 1.32 | 1.22 | 1.00 |

Source: This table was generated using the SAS® function FINV. v₁ = numerator degrees of freedom; v₂ = denominator degrees of freedom.