ECO 5350 T. Fomby
Intro. Econometrics Fall 2008

Time Series Model Practice Exercise

Purpose: **To learn how to build an Autoregressive Distributed Lag (ARDL) Model of two time series that have unit roots in them.**

Go to the website of this course and download the EVIEWS program **ardl.wf1**. Use it to answer the various parts of this exercise.

Scenario: Let us suppose that the time series **y** represents the weekly sales of **100 randomly chosen franchise stores owned by ABC corporation** (in thousands of dollars). In these 100 randomly chosen stores, ABC decides to check out the usefulness of its weekly **newspaper** advertising expenditures **x** (in thousands of dollars) over a period of **250 weeks**. During this 250 week period **only** newspaper advertising is used by the 100 stores. What we need to do is build an autoregressive distributed lag model that describes how newspaper advertising expenditures affect the sales of these 100 stores. You will be able to do that by going through the following steps.

a) **STEP 1: Conduct unit root tests of the x and y series.** Use your EVIEWS program plot the advertising (x) series, print it out, and hand it in with this exercise. Do the same for the sales (y) series. Given these two graphs you should be able to choose the correct case of the ADF test to use. For these two series we should be using Case (1/2/3) with (no intercept – no trend / intercept – no trend / intercept and trend). Based on the Schwartz criterion, use the automatic lag length selection capability of EVIEWS to produce a Dickey-Fuller t-test for a unit root in the **advertising series (x)**. The Dickey-Fuller t-statistic for the x series is __________ with one-tailed p-value of __________. The null hypothesis of this test is
____. The alternative hypothesis of this test is _____________________________________. Given the results of this test on x, I conclude that ______________________________
____.

Now let us go to the **sales series (y)**. The Dickey-Fuller t-statistic for the y series is __________ with one-tailed p-value of __________. The null hypothesis of this test is ____. The alternative hypothesis of this test is _____________________________________. Given the results of this test on x, I conclude that ______________________________
____.

b) **STEP 2: Plot the stationary forms of the advertising and sales series before proceeding**. By the results of part a) above you should now know that the stationary form of the advertising and sales series are, respectively, dx = x – x(-1) and dy = y – y(-1), the differences (weekly changes in) the series. **Print out the graphs of these two series and hand them in with this exercise.** Do they appear to have a constant mean, constant variance, and constant covariance? Of course, one might have considered taking the log of the advertising expenditures (lx) and log of the sales series (ly) and then differenced them resulting in the weekly percentage change in the series as a possible stationary form (i.e. dlx and dly). **Print out the graphs of the dlx and dly series and hand them in with this exercise.** (To the discerning eye, you should notice that, though these series appear "almost" stationary, they are slightly downward-trending at the end of the dlx and dly series indicating the percentage change transformation for the x and y series is not quite right and thus we should stick with first differencing to achieve stationarity of the advertising and sales series.)

c) **STEP 3: Build the Autoregressive Core of the ARDL model.**

Use OLS to estimate the following autoregressive equations:

(i) $dy_t = \phi_0 + \phi_1 dy_{t-1} + a_t$

(ii) $dy_t = \phi_0 + \phi_1 dy_{t-1} + \phi_2 dy_{t-2} + a_t$

(iii) $dy_t = \phi_0 + \phi_1 dy_{t-1} + \phi_2 dy_{t-2} + \phi_3 dy_{t-3} + a_t$

(iv) $dy_t = \phi_0 + \phi_1 dy_{t-1} + \phi_2 dy_{t-2} + \phi_3 dy_{t-3} + \phi_4 dy_{t-4} + a_t$

In model (i): $\hat{\phi}_1$ = ______________, Q(16) = ___________ (p-value = _________),
Newey-West t-statistic for $\hat{\phi}_1$ = __________ (p-value = _________).

In model (ii): $\hat{\phi}_2$ = ______________, Q(16) = ___________ (p-value = _________),
OLS t-statistic for $\hat{\phi}_2$ = __________ (p-value = _________).

In model (iii): $\hat{\phi}_3$ = ______________, Q(16) = ___________ (p-value = _________),
OLS t-statistic for $\hat{\phi}_3$ = __________ (p-value = _________).

In model (iv): $\hat{\phi}_4$ = ______________, Q(16) = ___________ (p-value = _________),
OLS t-statistic for $\hat{\phi}_4$ = __________ (p-value = _________).

(Notice that in model (i), the Box-Pierce Q-statistic that you obtain from the "Residual Tests / Correlogram-Q-Statistics" view of the OLS estimation output has a Q(16) statistic with a p-value < 0.05 thus we need to use the Newey-West standard error in constructing the t-statistic for $\hat{\phi}_1$ in order to get a consistent test of the significance of the dy_{t-1} variable. Thereafter, the usual t-statistic on the last autoregressive term is OK as the Q-statistics of the residuals have p-values greater than 0.05. Thus, from this step you should see that the "core" autoregressive model is model (iii) with 3 autoregressive terms.)

d) **STEP 4: Use the Cross-Correlation Function to get the "beginning" specification of the distributed lag part of our ARDL model.**

Go to the group symbol of your ARDL.wf1 program. Here I have already generated a so-called **cross-correlation function (ccf)**. Print out this graph and turn it in with this exercise. On the left-hand-side of the ccf graph notice that the cross-correlation at lag 0 is insignificant, while the cross-correlations at lags 1, 2, and, possibility, lag 3 are statistically significant.

e) **STEP 5: Use the "top-down" approach to finalize our ARDL model for the advertising to sales relationship.**

The above ccf of STEP 4 above suggests the following **initial** model to estimate in EVIEWS: "dy c dy(-1) dy(-2) dy(-3) dx(-1) dx(-2) dx(-3). Starting with this model, successively eliminate the variable with the highest p-value greater than 0.05 until you have a final model whose variables are all statistically significant at the 0.05 level. In the below space write out in "conventional form" your final ARDL model:

f) **Some conclusions from the final model.**

Given your final model, answer the following questions: The number of weeks before advertising expenditure affects the sales of the 100 stores is ___________. A change in advertising expenditures has a (finite period effect / infinite but diminishing effect) on sales. If there had been no autoregressive term in the final model, a change in advertising expenditures would have a (finite period effect / infinite but diminishing effect) on sales. **The total cumulative effect** on sales in the 100 stores from **a temporary one-period increase** of $1,000 in newspaper advertising is $ ________________________. (Report your answer **in dollars**.) Be sure you show me how you got your answer.

Furthermore, suppose that the average profit rate for each dollar of sales for the 100 stores is $0.20. Is newspaper advertising profitable for ABC corporation? Explain your answer. (Notice we are abstracting from the fact that the cumulative effect of the temporary one-time $1,000 increase in advertising **occurs over time** and should be calculated as **a present value** with a certain discount factor. That is, the PV of the temporary one-time $1,000 increase in advertising on sales is less than the amount you have calculated above but, in this problem, let's not worry about PV comparisons.)

Finally, suppose that ABC corporation picks out randomly 100 other stores to try an **internet advertising campaign** on. Furthermore, suppose the final ARDL equation for this experiment was determined to be

$$d\hat{y}_t = 0.3 + 0.2dy_{t-1} + 6.0dz_{t-1} + 2.0dz_{t-2} \quad ,$$

where $dz_t = z_t - z_{t-1}$ represents the change in the internet advertising expenses (in thousands of dollars) from one week to the next of the 100 randomly chosen stores. Given this information, is internet advertising, again abstracting from any PV calculations, profitable for ABC corporation? Again, assume a 20% profit rate on sales. Explain your answer.