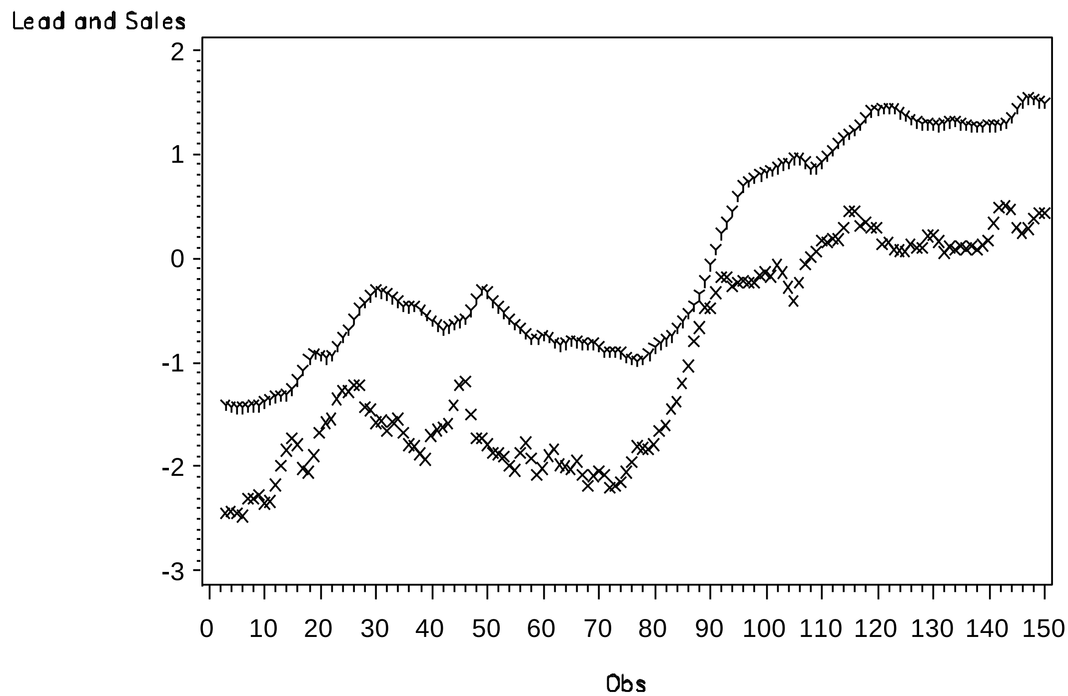
ECO 5375-701 Prof. Tom Fomby
Eco and Bus Forecasting Fall 2016

EXERCISE 10
KEY

Purpose: To **learn how to build an equal-lag-length VAR for a set of stationary time series and to use it in an out-of-sample forecasting experiment to determine if a supplementary variable is useful in helping us forecast a target variable.** This exercise is due **Tuesday, November 29**.

a) Recall the Series M data set that Box and Jenkins introduced in their seminal 1970 textbook. For a plot of the data run the SAS program BJ_M_Series.sas. Cut and Paste the second graph produced by this program, namely, the plot of the smoothed, standardized versions of the leading (x) and target (y) series. You can see that the two series seem to be related but the issue is if x is **related enough** to y so that x can be used to improve our forecasts of y.

"Smoothed" Standardized Leading Indicator vs. "Smoothed" Standardized Sales
X="Smoothed" Leading Indicator Y=Sales



b) To find out if x can be a useful "supplementary" variable for helping us forecast the target series y, we need to run an **out-of-sample forecasting experiment** (horserace) that compares a Box-Jenkins model's forecasting accuracy of y versus an equal-lag-length VAR's forecasting accuracy of y. Briefly describe to me the nature of an out-of-forecasting experiment and how it can be used to determine if x is a useful supplementary variable for forecasting y. **Answer:** You split your data into two parts – the **in-sample data set** and the **out-of-sample data set**. Here we decided to use observations 1 – 120 as the in-sample data set and observations 121 – 150 as the out-of-sample data set. Then to gauge the potential usefulness of the "supplementary" variable in helping one forecast a target variable, we run a **forecasting horserace** over the out-of-sample data between a **Box-Jenkins model** of the target series (the benchmark forecasting method) and **an equal-lag-length VAR** which contains both the target variable and the supplementary variable. If the forecasting accuracy for the target variable produced by the VAR is better than the forecasting accuracy offered by the Box-Jenkins model, we conclude that the supplementary variable is useful. In order to strengthen our conclusion on the usefulness of the supplementary variable, we conduct a **Diebold-Mariano test** for significant difference between the VAR and Box-Jenkins model, and if the VAR produces a statistically superior forecasting accuracy of the target variable, we have stronger conviction that the proposed supplementary variable is useful.

c) A useful SAS program for this part of the exercise is M Series-Unit Root.sas. Before we can run an out-of-sample forecasting experiment we need to determine the Box-Jenkins model for y and then, separately, an equal-lag-length VAR for x and y. As it turns out, diligent work by the student, vis-à-vis a unit root test would determine that the y series has a unit root in it and needs to be differenced in order to make it stationary. Also, by diligent work, the student can determine that a log transformation of the y-series is not needed before differencing. The same can be said for the x series. Using the first 120 observations (the in-sample data set) is can be determined by (a) using the sample ACF and PACF (b) the P-Q box, and (c) overfitting exercises that the best Box-Jenkins model for the y series is the ARIMA(1,1,1) model. Thus we will use it as our "benchmark" forecasting model for y in the out-of-sample experiment. Using the first 120 observations, report below the ARIMA(1,1,1) model that you have estimated for the y series. Be sure and include the standard errors of the estimates, the AIC and SBC goodness-of-fit measures of the fitted model, and the Box-Pierce-Ljung Q statistic for white noise residuals (lag = 24).

Answer:

Deviation-from-Mean Form:

$$\Delta y_t - \underset{(0.31389)}{0.456467} = \underset{(0.10275)}{0.85054}(\Delta y_{t-1} - 0.456467) + \hat{a}_t - \underset{(0.15270)}{0.62568}\hat{a}_{t-1}$$

AIC = 426.1963
SBC = 434.5337
Q(24) = 12.54 (p=0.9453)

Intercept Form:

$$\Delta y_t = 0.068224 + \underset{(0.10275)}{0.85054}\Delta y_{t-1} + \hat{a}_t - \underset{(0.15270)}{0.62568}\hat{a}_{t-1}$$

d) In the SAS program VARMAX1.sas we, among other things use the system-wide goodness-of-fit measures, SBC and AICC, to determine an optimal lag-length of a VAR using both the y and x series from the B-J Series M data set. Fill in the following blanks:

Minimum Information Criterion Based on SBC

Lag	MA 0
AR 0	**-1.265529**
AR 1	**-1.421625**
AR 2	**-1.555735**
AR 3	**-3.987073**
AR 4	**-4.348406**
AR 5	**-4.361527**
(Lag=5 is choice)	
AR 6	**-4.309591**

Minimum Information Criterion Based on AICC

Lag	MA 0
AR 0	**-1.311953**
AR 1	**-1.559855**
AR 2	**-1.784187**
AR 3	**-4.303901**
AR 4	**-4.751469**
AR 5	**-4.848346**
AR 6	**-4.877308**
AR 7	**-4.868746**
AR 8	**-4.921157**
(Lag=8 is choice)	
AR 9	**-4.838069**
AR 10	**-4.788212**

e) Given the information you report in part d) above, the SBC criterion suggests lag length __**5**___ while the AICC criterion suggests lag length ___**8**___.

f) We wound up choosing the lag length 8 because it appears that this model has white noise residuals. Fill in the following blanks:

Lag Length 5 Model

Portmanteau Test for Cross Correlations of Residuals

Up To Lag	DF	Chi-Square	Pr > ChiSq
6	4	**25.95**	<.0001
7	8	**32.16**	<.0001
8	12	**33.82**	0.0007
9	16	**37.34**	0.0019
10	20	**42.49**	0.0024
11	24	**48.06**	0.0025
12	28	**48.75**	0.0089

Lag Length 8 Model

Portmanteau Test for Cross Correlations of Residuals

Up To Lag	DF	Chi-Square	Pr > ChiSq
9	4	11.29	**0.0235**
10	8	12.71	**0.1222**
11	12	16.25	**0.1802**
12	16	17.16	**0.3752**

Notice in this case all of the probability values associated with the chi-square tests of white noise of the residuals are greater than 0.05 (except for the lag 9 statistic) and thus we conclude that the lag length 8 VAR has residuals that are white noise and thus the lag length 8 VAR is preferable.

f) We also conducted a Granger Causal test for y1 affecting y2 versus y2 affecting y1. Fill in the following blanks using the output from the VARMAX1.sas program. Briefly explain to meaning of this test as it relates to building a VAR for the Series M data set.

Granger-Causality Wald Test

Test	DF	Chi-Square	Pr > ChiSq
1	8	**4192.79**	<.0001
2	8	5.16	**0.7406**

Test 1: Group 1 Variables: y1
Group 2 Variables: y2

Test 2: Group 1 Variables: y2
Group 2 Variables: y1

Answer: Test 1 is testing if y2 Granger causes y1. In this case it does since the Chi-square statistic has a probability value that is less than 0.05. Test 2 is testing if y1 Granger causes y2. In this case it does not since the Chi-square statistic has a probability value that is greater than 0.05.

g) Using the VARMAX4.sas and VARMAX5.sas programs fill in the following blanks:

<u>VAR(8)</u>

MAE of ARIMA(1,1,1) model for y = ___**0.74658**__________
MAE for VAR(8) model, y variable = ___**0.17717**__________

<u>RVAR(8)</u>

MAE of ARIMA(1,1,1) model for y = ____**0.74658**_________
MAE for RVAR(8) model, y variable = ____**0.17633**_________

Extra Information: So one can see that, when using the MAE forecasting accuracy measure, the VAR(8) did better than the B-J model hence the supplementary variable is useful. Likewise, the RVAR(8) model did better than the B-J model and even better than the VAR(8) so not only does the supplementary variable appear to be useful but when using it through a restricted VAR it is even better. This is because, according to the Granger causal tests, Y2 is strictly exogenous and the restricted VAR appropriately incorporates this information uncovered by the Granger causal tests. (Y2 being strictly exogenous means that Y1 does not Granger cause Y1 but, to our benefit, Y2 does Granger cause Y1.)

h) In the SAS program M_Horserace.sas a Diebold-Mariano test is conducted on the statistical significance of the difference in the forecasting accuracies of the RVAR(8) model and the BJ model for y. Report below the appropriate test statistics and right-tail p-values for the significant difference in the following forecasting accuracy measures using RVAR(8):

MAE: t = __**6.57**__________ , p = __**<0.0001/2**_____ (right-tail p-value)
MSE: t = __**4.58**__________ , p = __**<0.0001/2**_____ (right-tail p-value)

Extra Information: In both the MAE and MSE tests the best ARMA model to use is ARMA(0,0). Using this model we do a one-side test of the significance of the mean (MU). So we can see from these two Diebold-Mariano tests that for both the MAE and MSE forecasting accuracy measures, the RVAR(8) provides statistically significant improvement when forecasting the target variable. We are pretty confident that our supplementary variable is helpful to us in forecasting the target variable.