Computer Output #1

Null Hypothesis: Y has a unit root
Exogenous: None
Lag Length: 1 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		5.312408	1.0000
Test critical values:	1% level	-2.590340	
	5% level	-1.944364	
	10% level	-1.614441	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y)
Method: Least Squares
Date: 10/19/13 Time: 11:52
Sample (adjusted): 3 94
Included observations: 92 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(-1)	0.004872	0.000917	5.312408	0.0000
D(Y(-1))	0.342754	0.094485	3.627604	0.0005

R-squared	0.147427	Mean dependent var	51.82283
Adjusted R-squared	0.137954	S.D. dependent var	45.59209
S.E. of regression	42.33063	Akaike info criterion	10.35040
Sum squared resid	161269.4	Schwarz criterion	10.40522
Log likelihood	-474.1183	Hannan-Quinn criter.	10.37252
Durbin-Watson stat	2.096705		

Null Hypothesis: Y has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.820061	0.9939
Test critical values:	1% level	-3.503049	
	5% level	-2.893230	
	10% level	-2.583740	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y)
Method: Least Squares
Date: 10/19/13 Time: 11:53
Sample (adjusted): 3 94
Included observations: 92 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(-1)	0.002599	0.003170	0.820061	0.4144
D(Y(-1))	0.343837	0.094727	3.629753	0.0005
C	16.49044	22.01721	0.748979	0.4558

R-squared	0.152767	Mean dependent var	51.82283
Adjusted R-squared	0.133728	S.D. dependent var	45.59209
S.E. of regression	42.43425	Akaike info criterion	10.36585
Sum squared resid	160259.3	Schwarz criterion	10.44809
Log likelihood	-473.8293	Hannan-Quinn criter.	10.39904
F-statistic	8.023939	Durbin-Watson stat	2.107897
Prob(F-statistic)	0.000625		

Null Hypothesis: Y has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 2 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.983249	0.6023
Test critical values:	1% level	-4.062040	
	5% level	-3.459950	
	10% level	-3.156109	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y)
Method: Least Squares
Date: 10/19/13 Time: 11:55
Sample (adjusted): 4 94
Included observations: 91 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Y(-1)	-0.047957	0.024181	-1.983249	0.0505
D(Y(-1))	0.293021	0.104490	2.804304	0.0062
D(Y(-2))	0.157670	0.100263	1.572572	0.1195
C	235.8543	107.2270	2.199580	0.0305
@TREND("1")	2.705673	1.306515	2.070909	0.0414

R-squared	0.196223	Mean dependent var	52.47473
Adjusted R-squared	0.158838	S.D. dependent var	45.41148
S.E. of regression	41.64909	Akaike info criterion	10.34981
Sum squared resid	149179.6	Schwarz criterion	10.48777
Log likelihood	-465.9165	Hannan-Quinn criter.	10.40547
F-statistic	5.248705	Durbin-Watson stat	1.958721
Prob(F-statistic)	0.000786		

SAS Code for
Computer Output # 2

```
data MT;
input y;
datalines;
4958.900
4857.800
4850.300
4936.600
5032.500
4997.300
.
.
.
.

9518.200
9552.000
9625.500
;

proc arima data = MT;
  identify var = y(1);
  e p = 0 q = 0;
  e p = 1 q = 0;
  e p = 2 q = 0;
  e p = 0 q = 1;
  e p = 0 q = 2;
  e p = 1 q = 1;
  run;
```

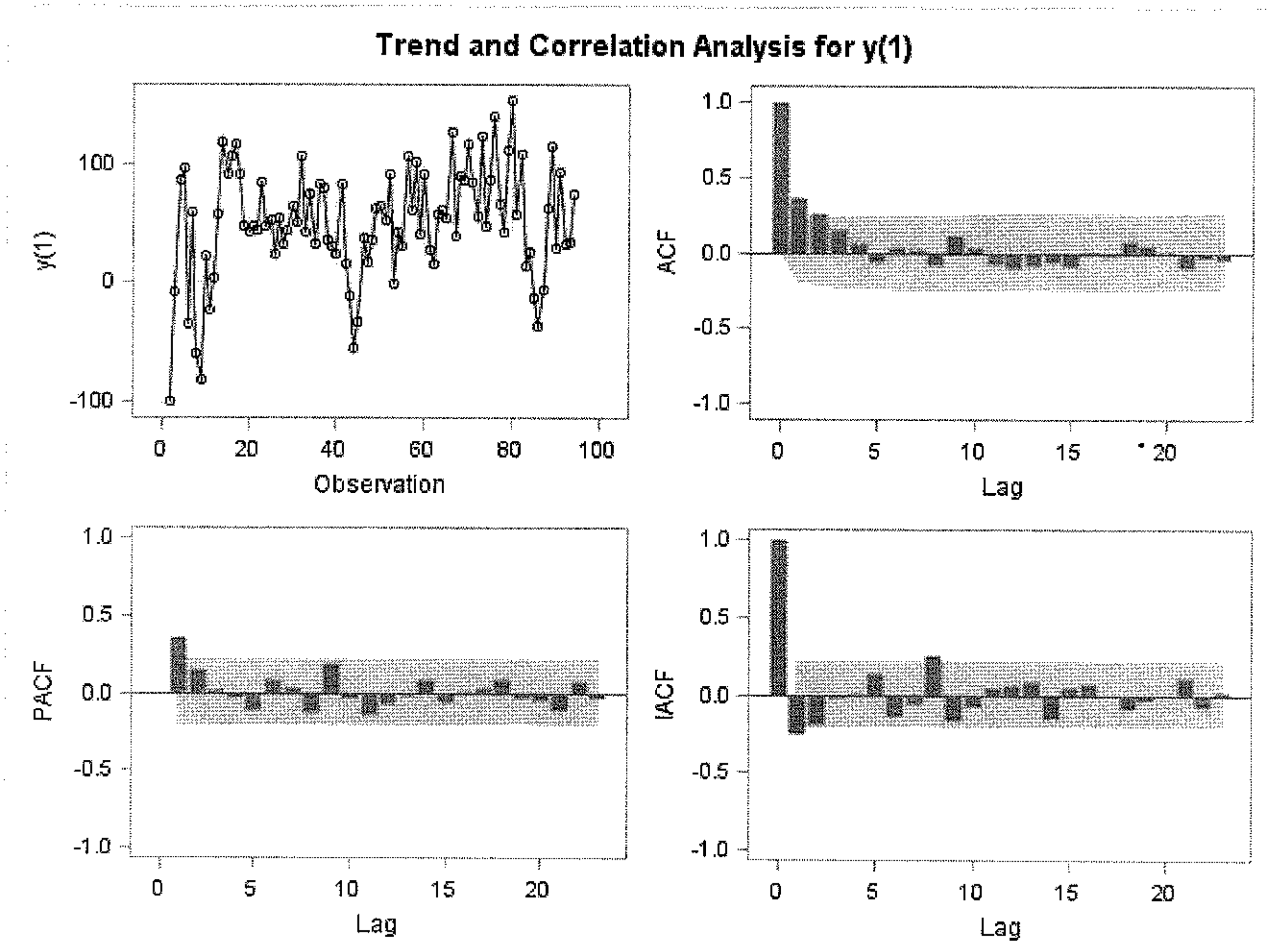
Computer Output #2

The SAS System

The ARIMA Procedure

Name of Variable = y	
Period(s) of Differencing	1
Mean of Working Series	50.17849
Standard Deviation	47.77749
Number of Observations	93
Observation(s) eliminated by differencing	1

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	22.15	6	0.0011	0.360	0.261	0.153	0.061	-0.053	0.029
12	26.08	12	0.0105	0.027	-0.082	0.112	0.038	-0.069	-0.103
18	28.50	18	0.0548	-0.079	-0.055	-0.078	0.004	-0.019	0.075



Conditional Least Squares Estimation

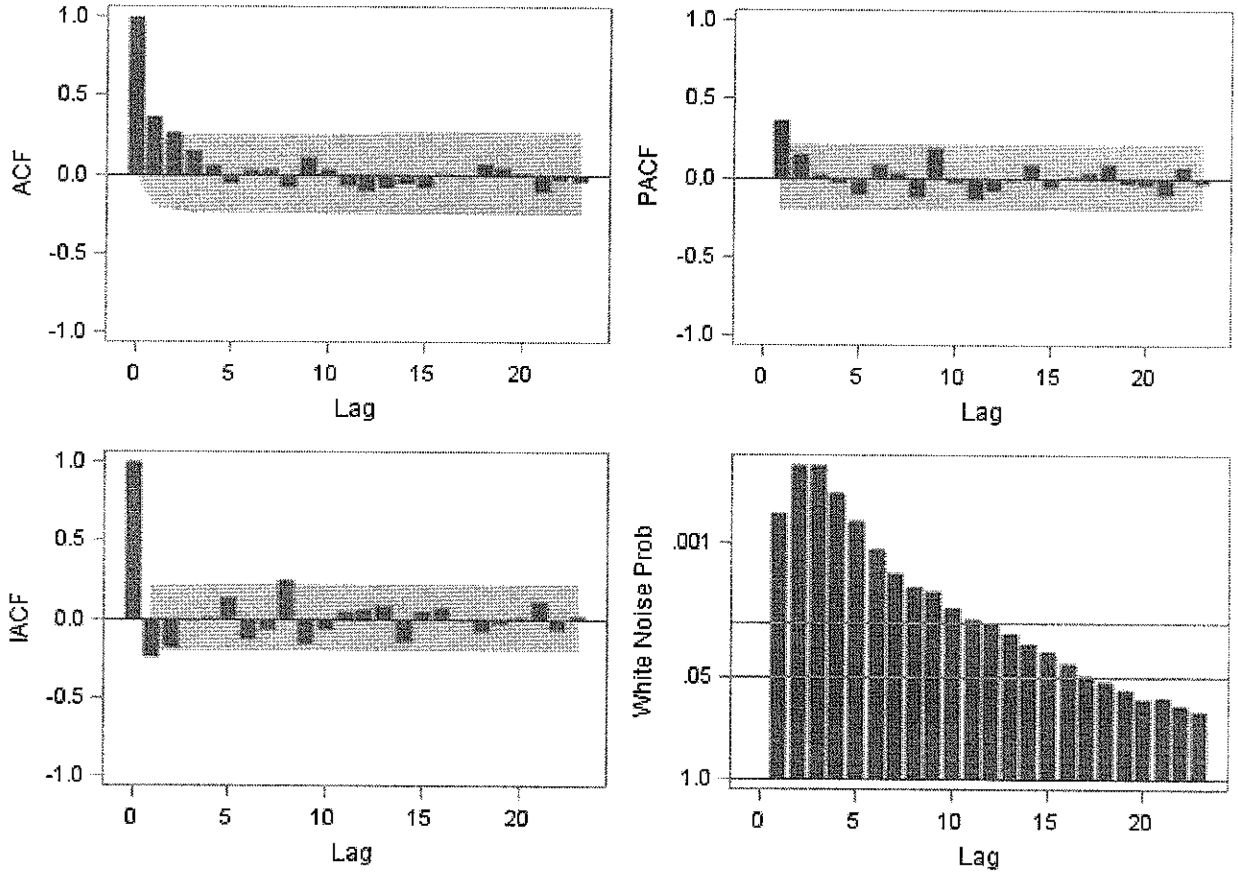
Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	50.17849	4.98115	10.07	<.0001	0

Constant Estimate	50.17849
Variance Estimate	2307.501
Std Error Estimate	48.03645
AIC	985.1017
SBC	987.6343
Number of Residuals	93

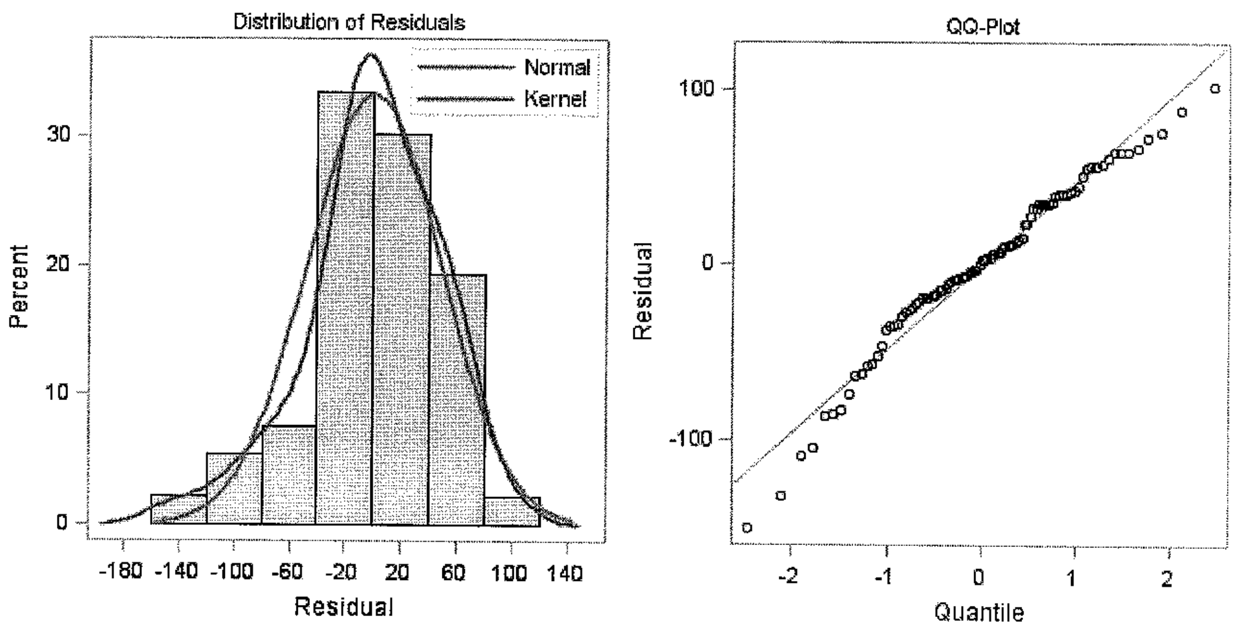
*** AIC and SBC do not include log determinant.**

Autocorrelation Check of Residuals									
To Lag	**Chi-Square**	**DF**	**Pr > ChiSq**	**Autocorrelations**					
6	22.15	6	0.0011	0.360	0.261	0.153	0.061	-0.053	0.029
12	26.08	12	0.0105	0.027	-0.082	0.112	0.038	-0.069	-0.103
18	28.50	18	0.0548	-0.079	-0.055	-0.078	0.004	-0.019	0.075
24	30.39	24	0.1723	0.045	0.012	-0.102	-0.028	-0.045	0.002

Residual Correlation Diagnostics for y(1)



Residual Normality Diagnostics for y(1)



Model for variable y

Estimated Mean	50.17849
Period(s) of Differencing	1

Conditional Least Squares Estimation					
Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**	**Lag**
MU	48.88861	7.27282	6.72	<.0001	0
AR1,1	0.36274	0.09787	3.71	0.0004	1

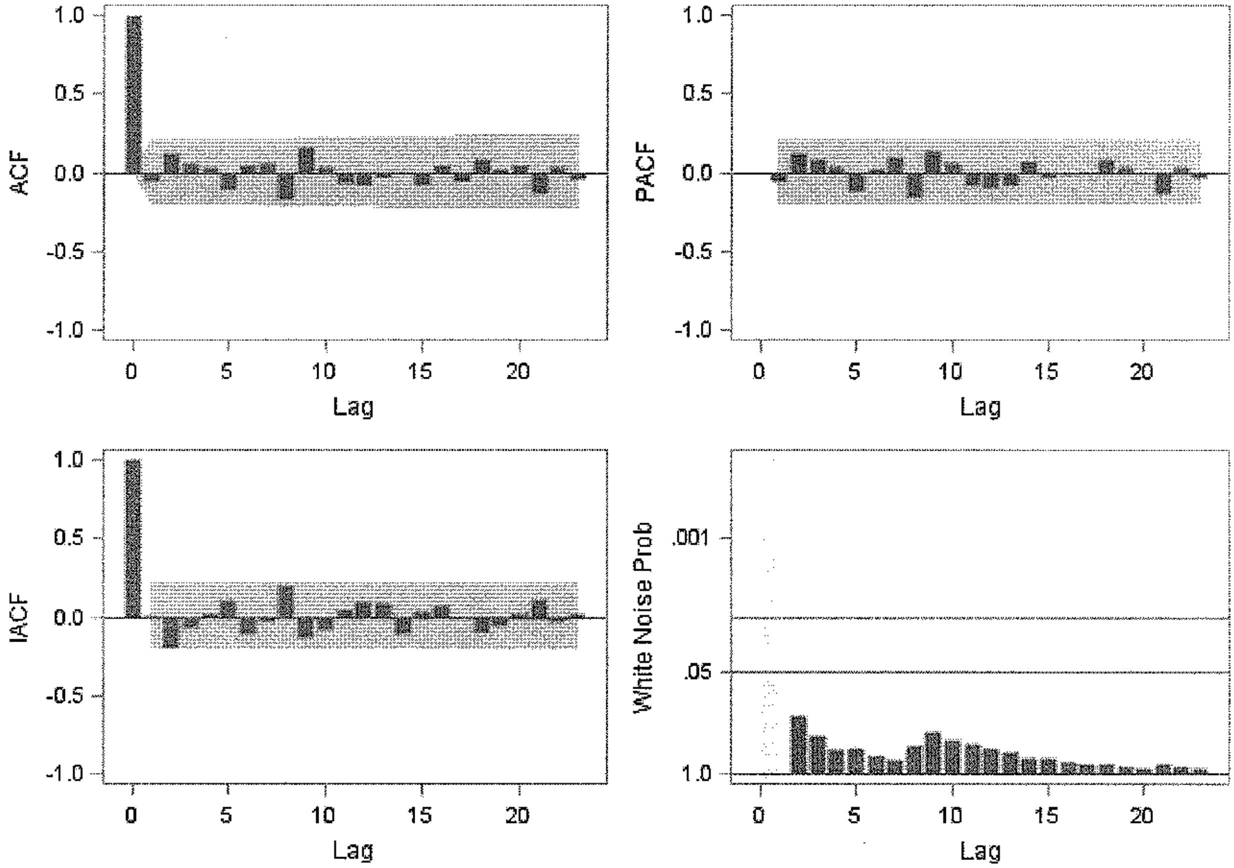
Constant Estimate	31.15467
Variance Estimate	2028.233
Std Error Estimate	45.03591
AIC	974.0883
SBC	979.1535
Number of Residuals	93

*** AIC and SBC do not include log determinant.**

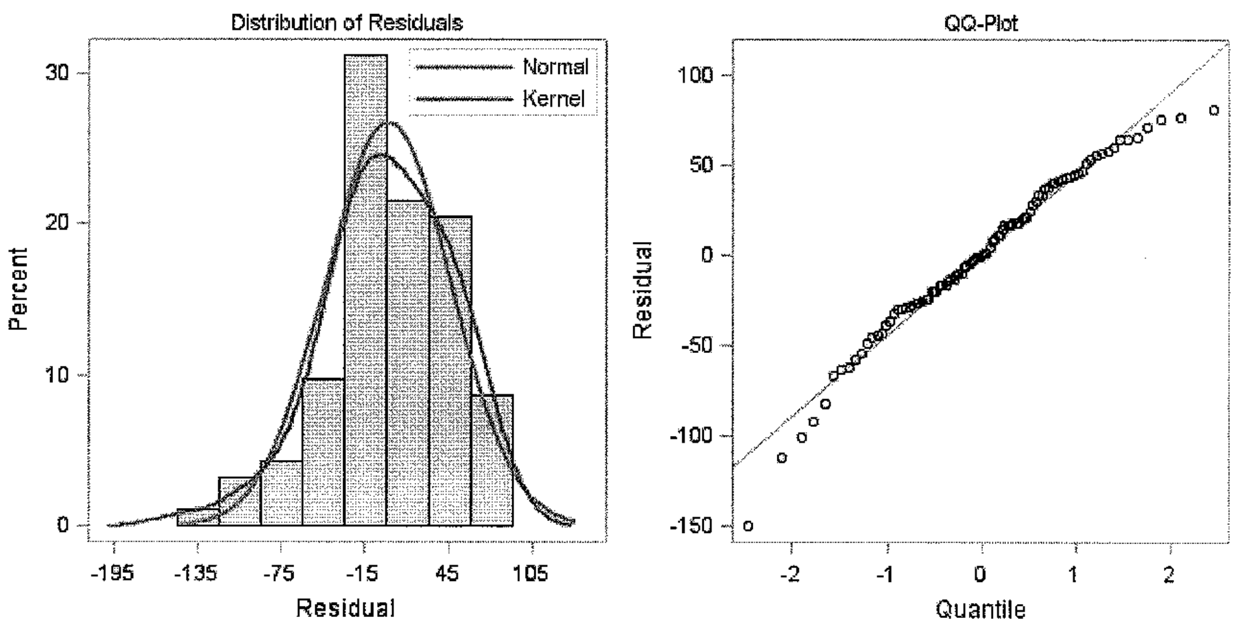
Correlations of Parameter Estimates		
Parameter	**MU**	**AR1,1**
MU	1.000	-0.021
AR1,1	-0.021	1.000

Autocorrelation Check of Residuals									
To Lag	**Chi-Square**	**DF**	**Pr > ChiSq**	**Autocorrelations**					
6	3.83	5	0.5741	-0.055	0.126	0.064	0.040	-0.107	0.052
12	10.88	11	0.4532	0.060	-0.164	0.163	0.032	-0.061	-0.072
18	13.21	17	0.7222	-0.032	-0.006	-0.075	0.050	-0.058	0.088
24	16.23	23	0.8452	0.026	0.041	-0.126	0.028	-0.047	0.056

Residual Correlation Diagnostics for y(1)



Residual Normality Diagnostics for y(1)



Model for variable y

Estimated Mean	48.88861
Period(s) of Differencing	1

Autoregressive Factors	
Factor 1:	1 - 0.36274 B**(1)

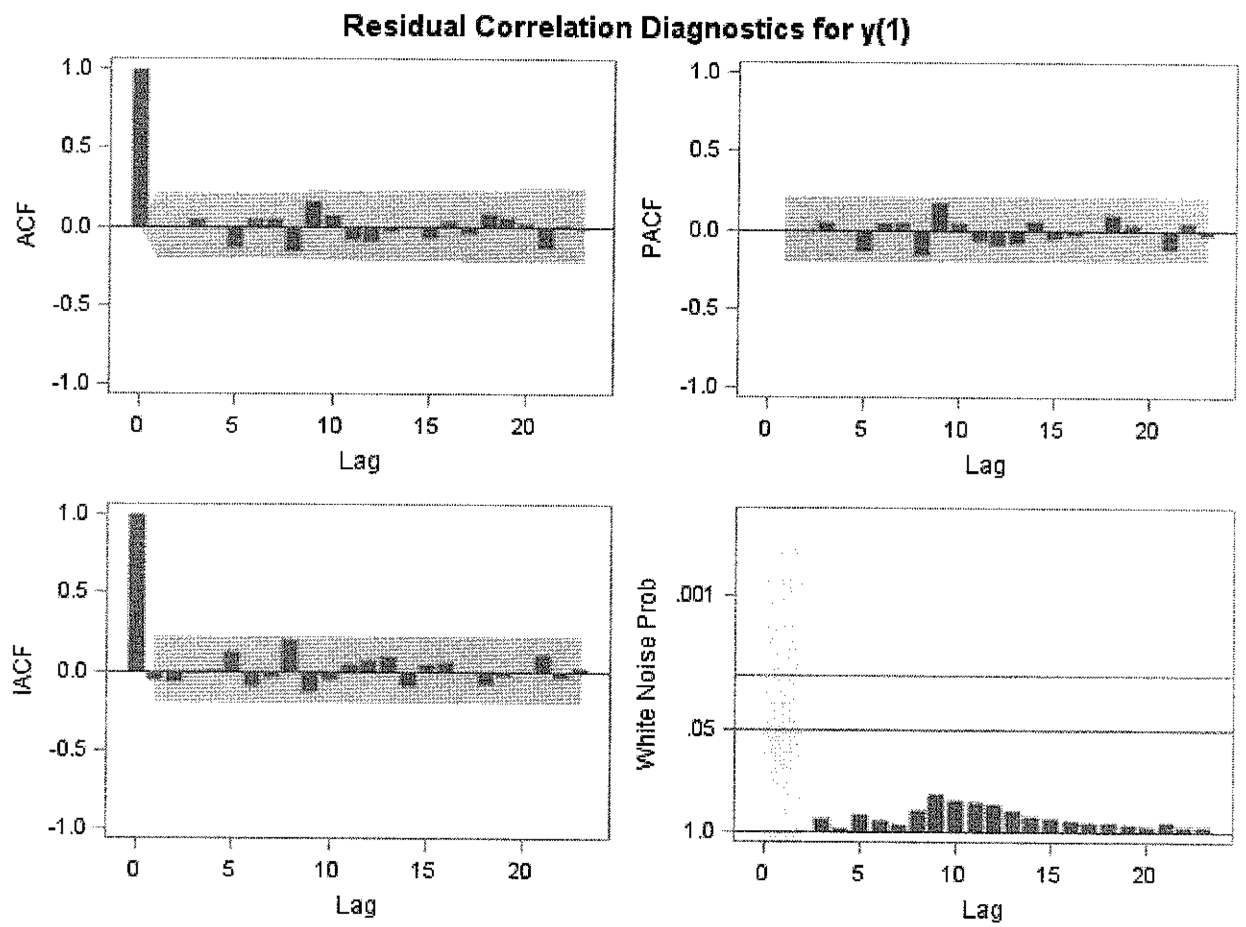
Conditional Least Squares Estimation					
Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**	**Lag**
MU	47.76943	8.49179	5.63	<.0001	0
AR1,1	0.30844	0.10446	2.95	0.0040	1
AR1,2	0.15330	0.10456	1.47	0.1461	2

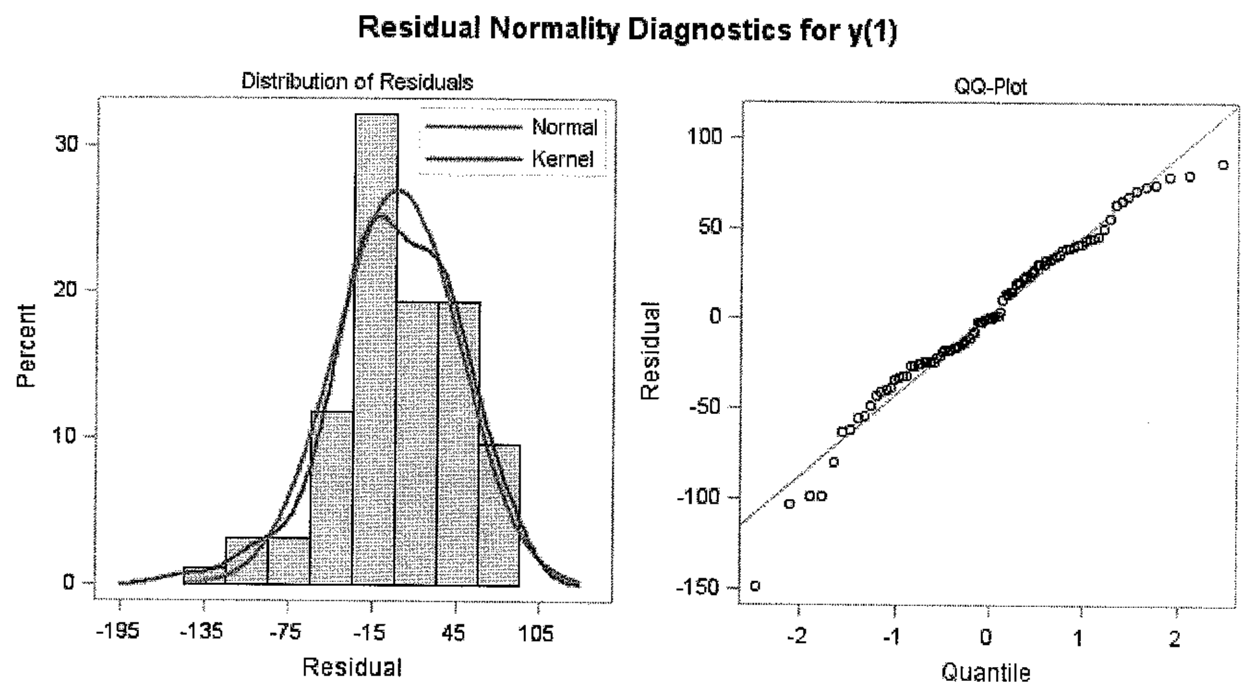
Constant Estimate	25.71258
Variance Estimate	2003.35
Std Error Estimate	44.7588
AIC	973.9127
SBC	981.5105
Number of Residuals	93

*** AIC and SBC do not include log determinant.**

Correlations of Parameter Estimates			
Parameter	**MU**	**AR1,1**	**AR1,2**
MU	1.000	-0.018	-0.036
AR1,1	-0.018	1.000	-0.365
AR1,2	-0.036	-0.365	1.000

Autocorrelation Check of Residuals									
To Lag	**Chi-Square**	**DF**	**Pr > ChiSq**	**Autocorrelations**					
6	2.14	4	0.7095	-0.003	-0.003	0.048	-0.002	-0.128	0.052
12	9.96	10	0.4438	0.045	-0.160	0.169	0.072	-0.075	-0.085
18	11.81	16	0.7570	-0.025	-0.003	-0.065	0.037	-0.041	0.089
24	15.05	22	0.8602	0.057	0.028	-0.128	0.008	-0.016	0.075





Model for variable y

Estimated Mean	47.76943
Period(s) of Differencing	1

Autoregressive Factors	
Factor 1:	1 - 0.30844 B**(1) - 0.1533 B**(2)

Conditional Least Squares Estimation					
Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**	**Lag**
MU	49.71299	6.04443	8.22	<.0001	0
MA1,1	-0.27289	0.10104	-2.70	0.0082	1

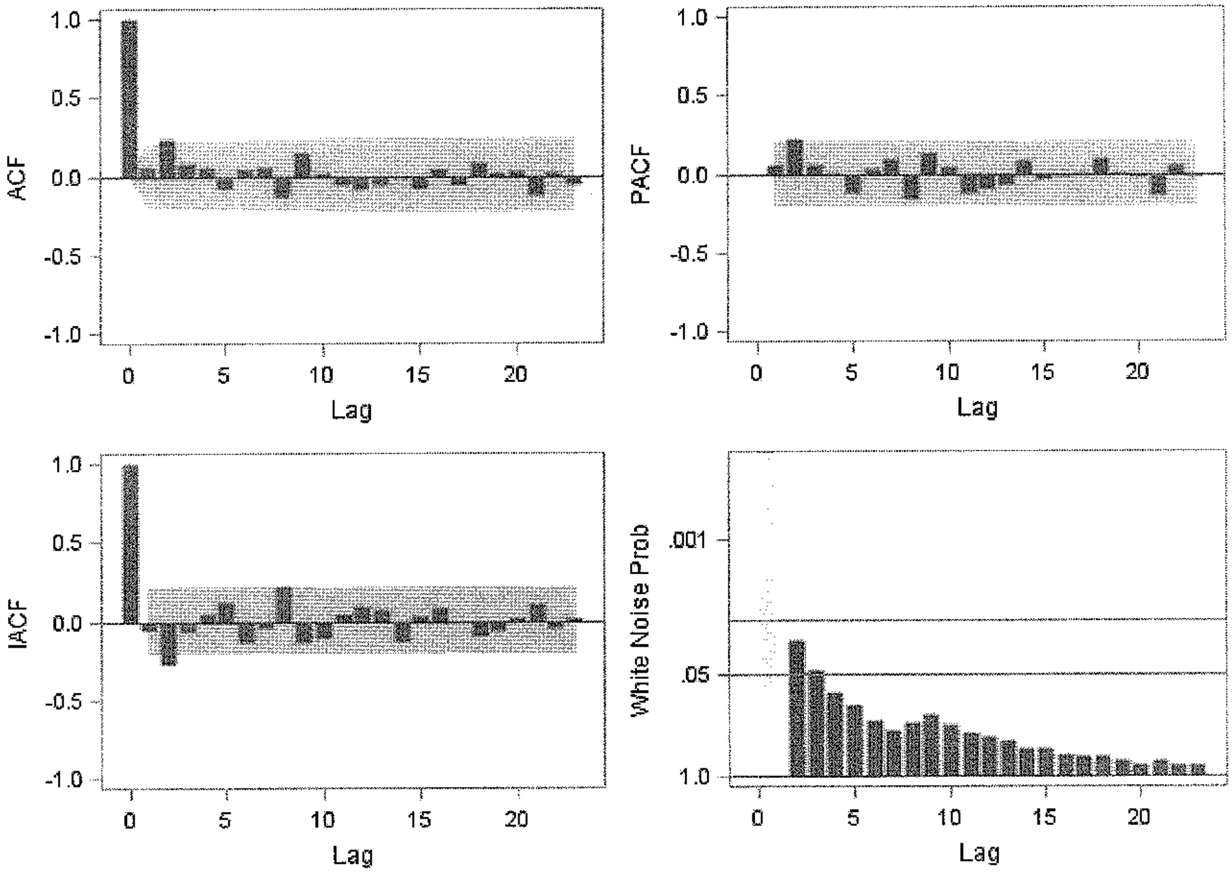
Constant Estimate	49.71299
Variance Estimate	2108.473
Std Error Estimate	45.91811
AIC	977.6966
SBC	982.7618
Number of Residuals	93

*** AIC and SBC do not include log determinant.**

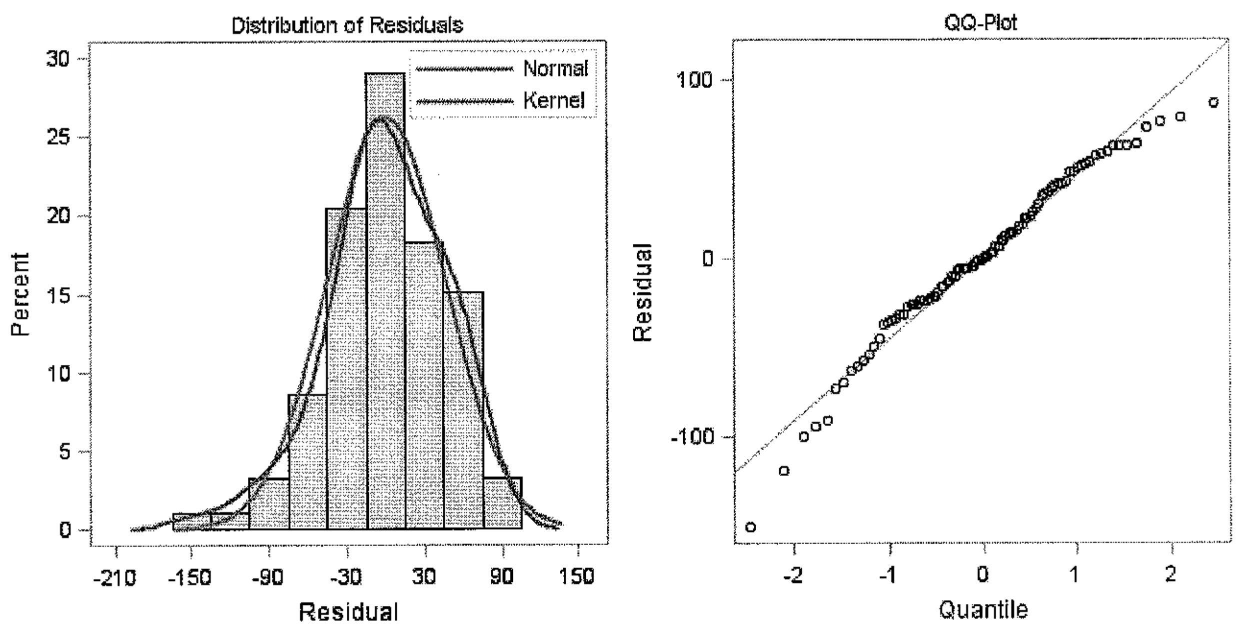
Correlations of Parameter Estimates		
Parameter	**MU**	**MA1,1**
MU	1.000	0.005
MA1,1	0.005	1.000

Autocorrelation Check of Residuals									
To Lag	**Chi-Square**	**DF**	**Pr > ChiSq**	**Autocorrelations**					
6	7.50	5	0.1859	0.059	0.233	0.082	0.064	-0.081	0.040
12	12.91	11	0.2991	0.053	-0.135	0.144	0.016	-0.055	-0.079
18	15.40	17	0.5665	-0.052	-0.021	-0.081	0.041	-0.052	0.087
24	18.01	23	0.7568	0.017	0.040	-0.120	0.019	-0.059	0.034

Residual Correlation Diagnostics for y(1)



Residual Normality Diagnostics for y(1)



Model for variable y	

Estimated Mean	49.71299
Period(s) of Differencing	1

Moving Average Factors	
Factor 1:	1 + 0.27289 B**(1)

Conditional Least Squares Estimation					
Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**	**Lag**
MU	49.09344	6.83021	7.19	<.0001	0
MA1,1	-0.28720	0.10419	-2.76	0.0071	1
MA1,2	-0.17595	0.10429	-1.69	0.0950	2

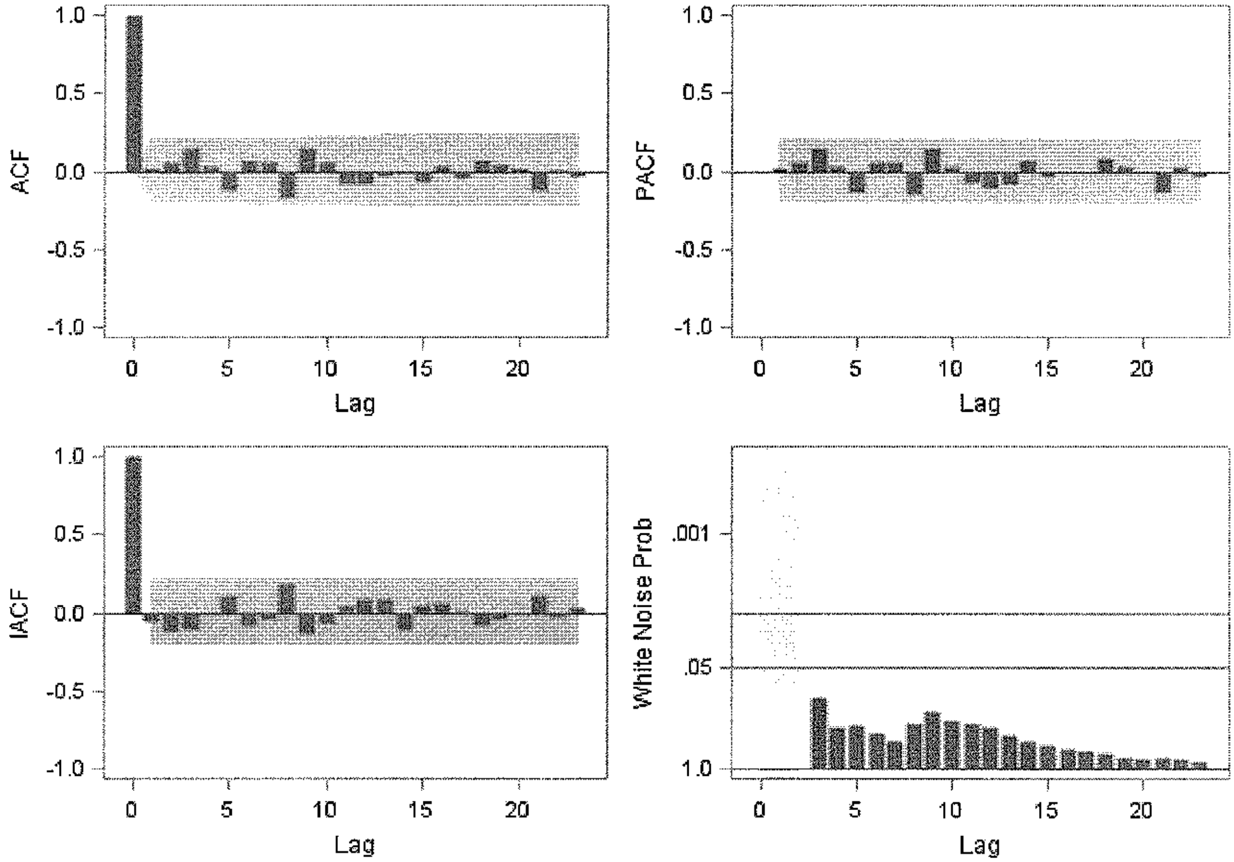
Constant Estimate	49.09344
Variance Estimate	2050.408
Std Error Estimate	45.28143
AIC	976.0719
SBC	983.6697
Number of Residuals	93

*** AIC and SBC do not include log determinant.**

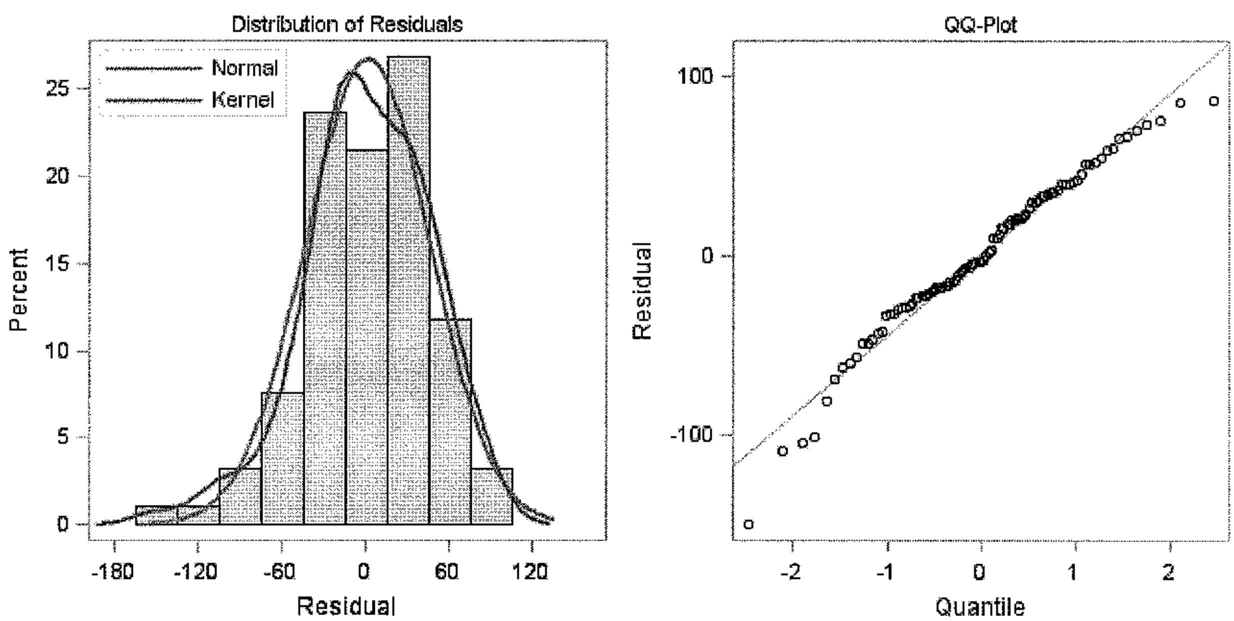
Correlations of Parameter Estimates			
Parameter	**MU**	**MA1,1**	**MA1,2**
MU	1.000	0.007	0.014
MA1,1	0.007	1.000	0.240
MA1,2	0.014	0.240	1.000

Autocorrelation Check of Residuals									
To Lag	**Chi-Square**	**DF**	**Pr > ChiSq**	**Autocorrelations**					
6	4.47	4	0.3459	0.027	0.056	0.145	0.036	-0.117	0.073
12	11.90	10	0.2920	0.055	-0.167	0.153	0.057	-0.084	-0.078
18	13.51	16	0.6351	-0.024	-0.017	-0.070	0.038	-0.045	0.069
24	15.89	22	0.8214	0.048	0.025	-0.116	0.008	-0.034	0.043

Residual Correlation Diagnostics for y(1)



Residual Normality Diagnostics for y(1)



Model for variable y	

Estimated Mean	49.09344
Period(s) of Differencing	1

Moving Average Factors	
Factor 1:	1 + 0.2872 B**(1) + 0.17595 B**(2)

Conditional Least Squares Estimation					
Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**	**Lag**
MU	47.70871	8.73031	5.46	<.0001	0
MA1,1	0.32538	0.24581	1.32	0.1890	1
AR1,1	0.64697	0.19839	3.26	0.0016	1

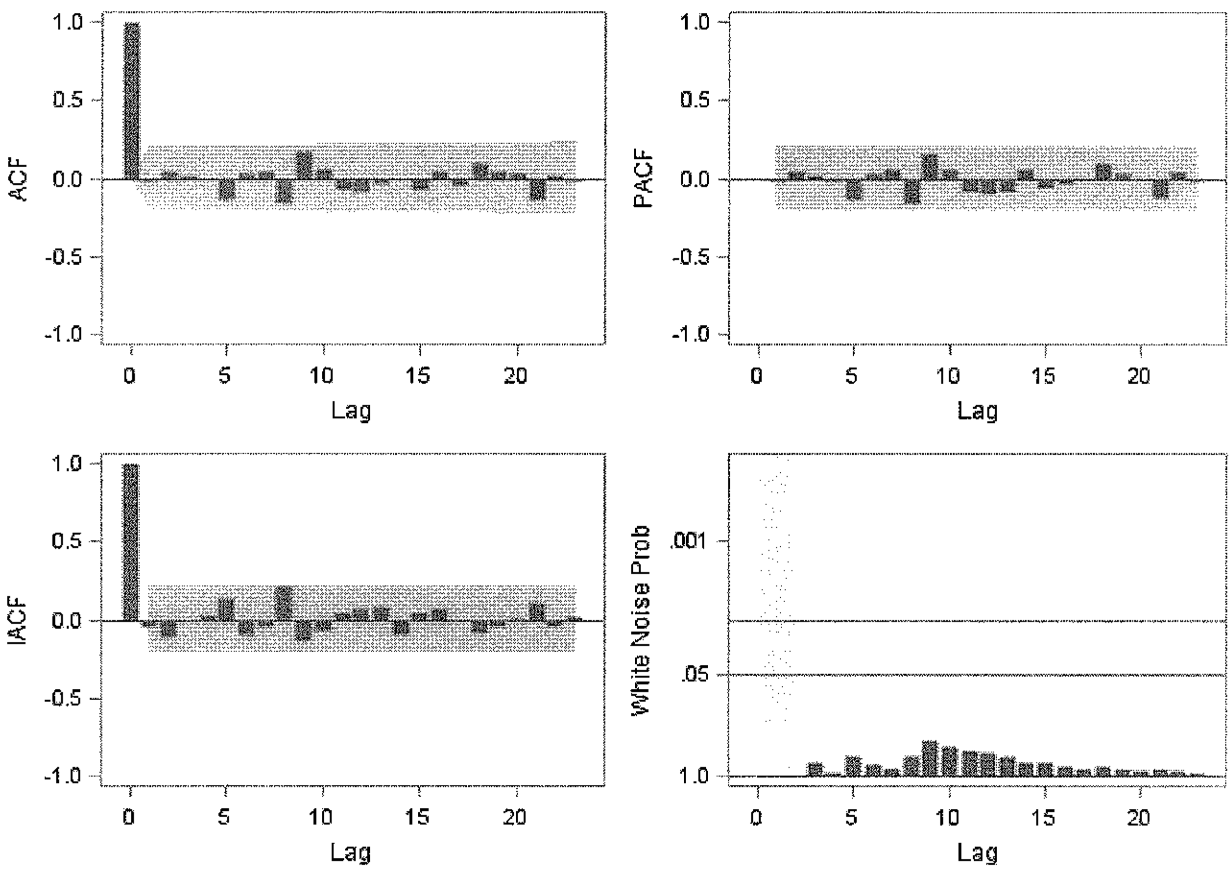
Constant Estimate	16.84271
Variance Estimate	2008.722
Std Error Estimate	44.81877
AIC	974.1618
SBC	981.7596
Number of Residuals	93

*** AIC and SBC do not include log determinant.**

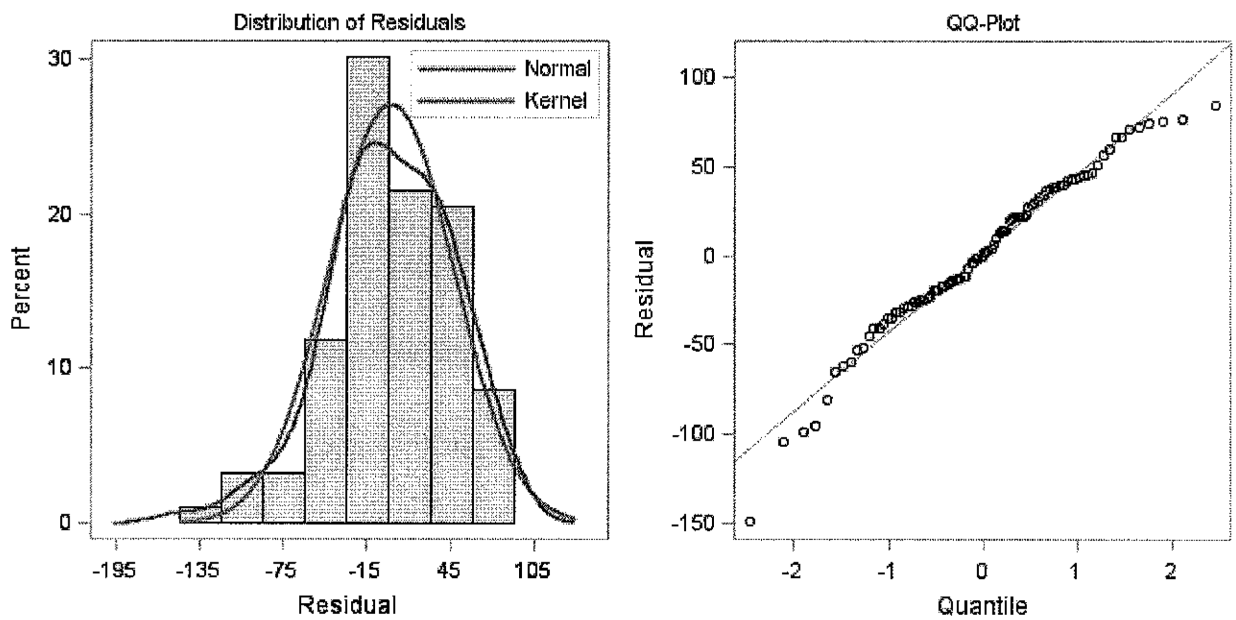
Correlations of Parameter Estimates			
Parameter	**MU**	**MA1,1**	**AR1,1**
MU	1.000	-0.043	-0.059
MA1,1	-0.043	1.000	0.914
AR1,1	-0.059	0.914	1.000

Autocorrelation Check of Residuals									
To Lag	**Chi-Square**	**DF**	**Pr > ChiSq**	**Autocorrelations**					
6	2.17	4	0.7036	-0.014	0.043	0.019	-0.010	-0.134	0.037
12	9.49	10	0.4866	0.044	-0.155	0.171	0.060	-0.065	-0.082
18	11.68	16	0.7657	-0.033	-0.005	-0.068	0.043	-0.042	0.098
24	14.96	22	0.8641	0.049	0.035	-0.127	0.016	-0.022	0.077

Residual Correlation Diagnostics for y(1)



Residual Normality Diagnostics for y(1)



Model for variable y

Estimated Mean	47.70871
Period(s) of Differencing	1

Autoregressive Factors	
Factor 1:	1 - 0.64697 B**(1)

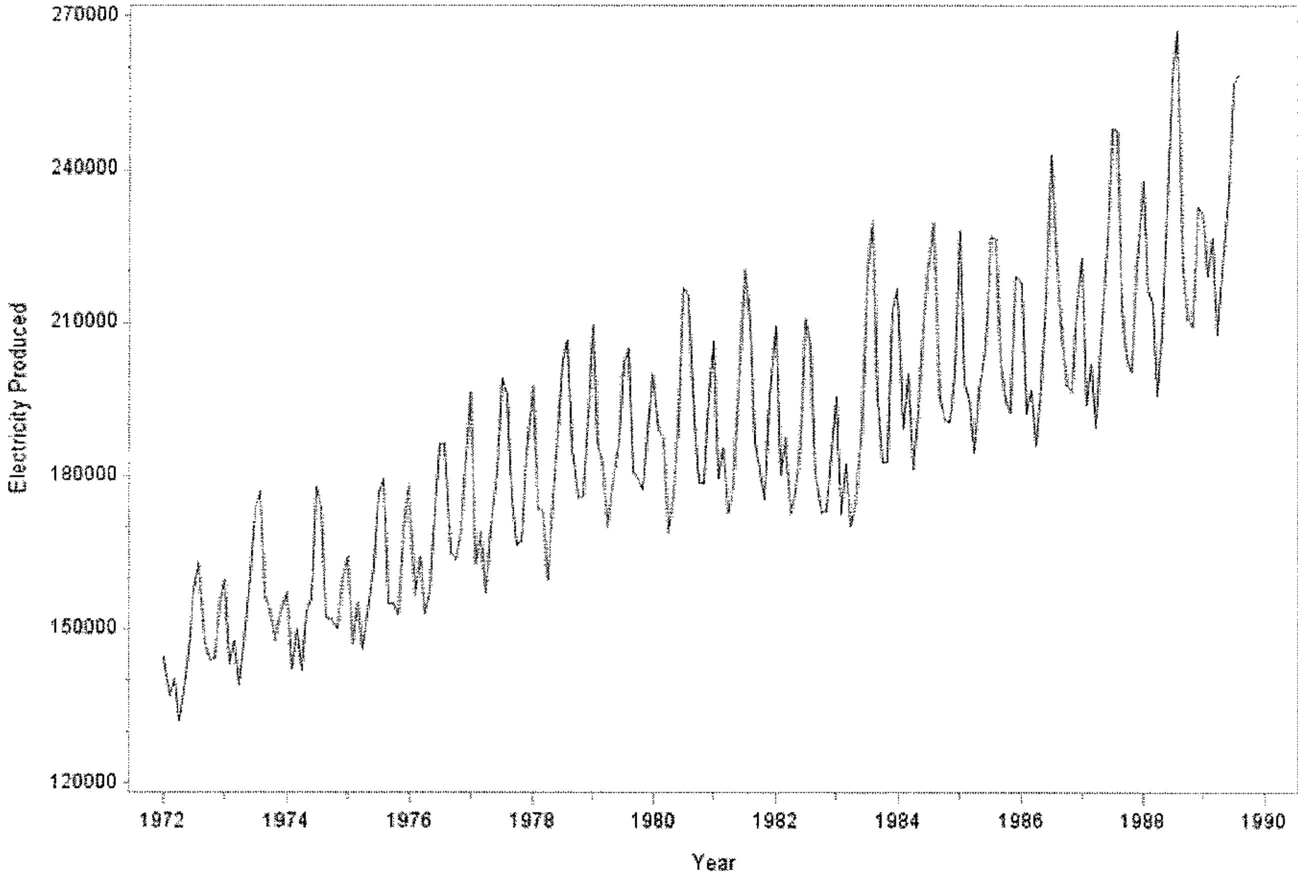
Moving Average Factors	
Factor 1:	1 - 0.32538 B**(1)

SAS Program for Output #3

```
data electric;
   format date monyy5.;
   input date:monyy5. elec @@;
   datalines;
jan72 144549 feb72 137310 mar72 140151 apr72 132112
may72 138168 jun72 145583 jul72 157878 aug72 162901
sep72 147584 oct72 144062 nov72 144366 dec72 154966
jan73 159913 feb73 143257 mar73 147846 apr73 139292
may73 147088 jun73 160945 jul73 173467 aug73 177109
sep73 156385 oct73 153951 nov73 147881 dec73 153305
jan74 157254 feb74 142472 mar74 150043 apr74 142021
may74 153513 jun74 156161 jul74 177992 aug74 173871
sep74 152222 oct74 151978 nov74 149841 dec74 159736
jan75 164325 feb75 147080 mar75 155481 apr75 146217
may75 153231 jun75 162442 jul75 176816 aug75 179714
sep75 155223 oct75 154944 nov75 152794 dec75 169372
jan76 178329 feb76 156684 mar76 164174 apr76 153167
.
.
.
.
sep87 213008 oct87 203009 nov87 200258 dec87 220500
jan88 237600 feb88 216702 mar88 213838 apr88 195809
may88 208180 jun88 232507 jul88 257235 aug88 267408
sep88 220023 oct88 210377 nov88 209394 dec88 232550
jan89 231343 feb89 219066 mar89 226436 apr89 207749
may89 219803 jun89 235397 jul89 256744 aug89 258335
;

title 'Electricity Production Data (1972-89)';
title2 '(in thousands of kilowatt-hours)';

axis1 order=('01jan72'd to '05jan90'd by 731)
      label=('Year') minor=(number=1) offset=(3);
axis2 order=(120000 to 270000 by 30000) minor=(number=2)
      label=(angle=90 'Electricity Produced');

proc gplot data=electric;
   plot elec*date / haxis=axis1 vaxis=axis2;
   symbol1 i=join;
   format date year4.;

RUN;

Proc Arima data = electric;
   identify var = elec(1) nlag = 48;
   run;
```

computer Output #3

Electricity Production Data (1972-89)
(in thousands of kilowatt-hours)



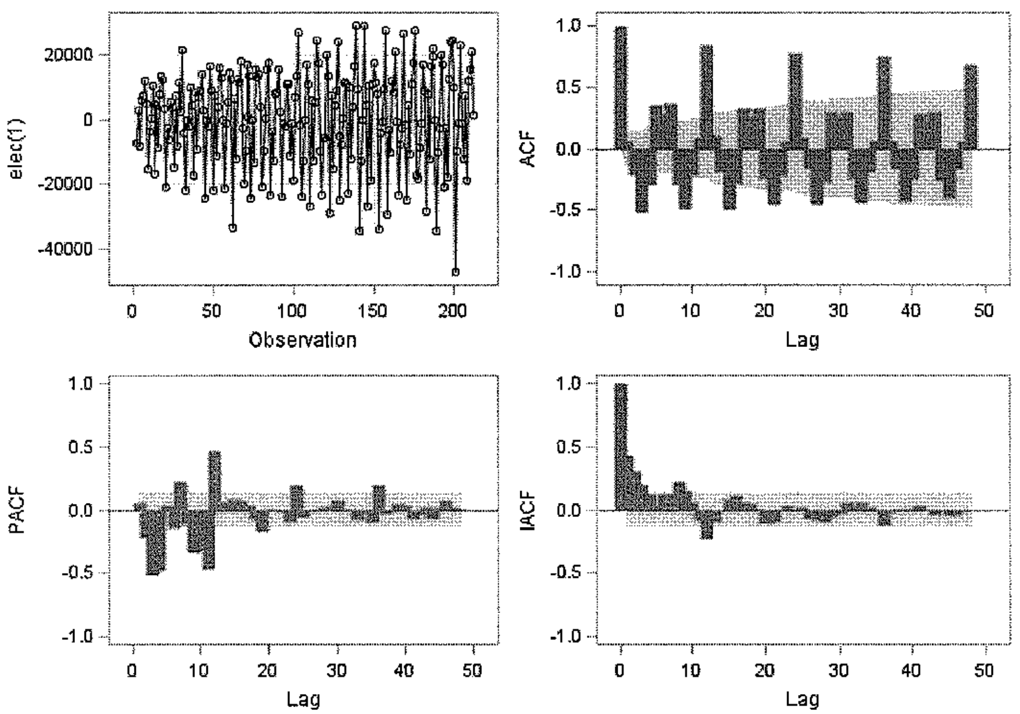
Electricity Production Data (1972-89)
(in thousands of kilowatt-hours)

The ARIMA Procedure

Name of Variable = elec	
Period(s) of Differencing	1
Mean of Working Series	539.2701
Standard Deviation	15112.15
Number of Observations	211
Observation(s) eliminated by differencing	1

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	132.98	6	<.0001	0.056	-0.221	-0.519	-0.298	0.348	0.285
12	408.74	12	<.0001	0.362	-0.300	-0.500	-0.216	0.081	0.840
18	533.89	18	<.0001	0.096	-0.186	-0.494	-0.288	0.320	0.265
24	778.26	24	<.0001	0.329	-0.249	-0.459	-0.211	0.066	0.771
30	892.10	30	<.0001	0.084	-0.158	-0.455	-0.268	0.298	0.253
36	1129.21	36	<.0001	0.304	-0.239	-0.437	-0.188	0.059	0.742
42	1235.89	42	<.0001	0.062	-0.162	-0.424	-0.246	0.285	0.233
48	1459.11	48	<.0001	0.305	-0.255	-0.406	-0.159	0.066	0.681

Trend and Correlation Analysis for elec(1)



SAS Program for Output #4

```
data electric;
   format date monyy5.;
   input date:monyy5. elec @@;
   datalines;
jan72 144549 feb72 137310 mar72 140151 apr72 132112
may72 138168 jun72 145583 jul72 157878 aug72 162901
sep72 147584 oct72 144062 nov72 144366 dec72 154966
jan73 159913 feb73 143257 mar73 147846 apr73 139292
may73 147088 jun73 160945 jul73 173467 aug73 177109
sep73 156385 oct73 153951 nov73 147881 dec73 153305
jan74 157254 feb74 142472 mar74 150043 apr74 142021
may74 153513 jun74 156161 jul74 177992 aug74 173871
sep74 152222 oct74 151978 nov74 149841 dec74 159736
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sep88 220023 oct88 210377 nov88 209394 dec88 232550
jan89 231343 feb89 219066 mar89 226436 apr89 207749
may89 219803 jun89 235397 jul89 256744 aug89 258335
;

data electric;
  set electric;
  lelec = log(elec);
  lelec_1 = lag(lelec);
  lelec12 = dif12(lelec);
  lelec12_1 = lag(lelec12);
  lelec112 = dif(lelec12);
  lelec1 = dif(lelec);
  lelec1_12 = lag12(lelec1);
  lelec112_1 = lag(lelec112);
  lelec112_2 = lag2(lelec112);
  lelec112_3 = lag3(lelec112);
  lelec112_4 = lag4(lelec112);
  lelec112_5 = lag5(lelec112);
  lelec112_6 = lag6(lelec112);
  lelec112_7 = lag7(lelec112);
  lelec112_8 = lag8(lelec112);
  lelec112_9 = lag9(lelec112);
  lelec112_10 = lag10(lelec112);
  lelec112_11 = lag11(lelec112);
  lelec112_12 = lag12(lelec112);

  proc reg data = electric;
     model lelec = lelec_1 lelec12_1 lelec1_12 lelec112_1 lelec112_2
                         lelec112_3 lelec112_4 lelec112_5 lelec112_6
                         lelec112_7 lelec112_8 lelec112_9 lelec112_10
                         lelec112_11 lelec112_12/noint;
     test lelec_1 = 1, lelec12_1 = 0, lelec1_12 = 1;

run;
```

The SAS System

The REG Procedure
Model: MODEL1
Dependent Variable: lelec

Number of Observations Read	212
Number of Observations Used	187
Number of Observations with Missing Values	25

Note: No intercept in model. R-Square is redefined.

Analysis of Variance					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	15	27637	1842.44530	2153695	<.0001
Error	172	0.14714	0.00085548		
Uncorrected Total	187	27637			

Root MSE	0.02925	**R-Square**	1.0000
Dependent Mean	12.15623	**Adj R-Sq**	1.0000
Coeff Var	0.24061		

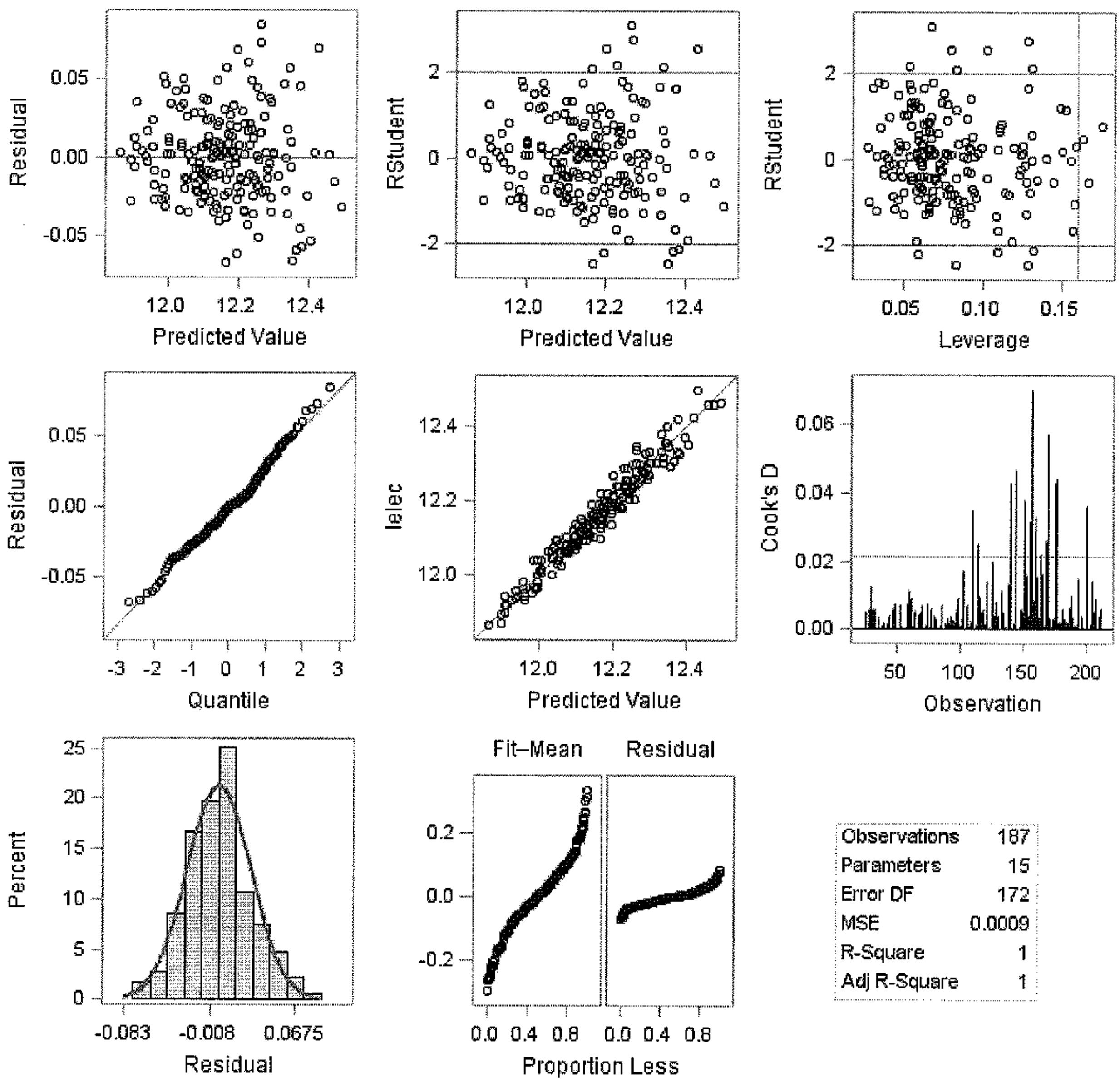
Parameter Estimates					
Variable	**DF**	**Parameter Estimate**	**Standard Error**	**t Value**	**Pr > |t|**
lelec_1	1	1.00079	0.00027496	3639.80	<.0001
lelec12_1	1	-0.38222	0.10064	-3.80	0.0002
lelec1_12	1	0.96301	0.02843	33.87	<.0001
lelec112_1	1	-0.16317	0.10085	-1.62	0.1075
lelec112_2	1	-0.08226	0.09993	-0.82	0.4115
lelec112_3	1	0.01760	0.09858	0.18	0.8586
lelec112_4	1	0.05412	0.09714	0.56	0.5781
lelec112_5	1	0.08034	0.09489	0.85	0.3984
lelec112_6	1	0.15981	0.09337	1.71	0.0888
lelec112_7	1	0.15197	0.09279	1.64	0.1033
lelec112_8	1	0.13112	0.09173	1.43	0.1547
lelec112_9	1	0.19596	0.08952	2.19	0.0300
lelec112_10	1	0.23190	0.08734	2.66	0.0087

lelec112_11	1	0.21802	0.08303	2.63	0.0094
lelec112_12	1	-0.19558	0.07625	-2.57	0.0112

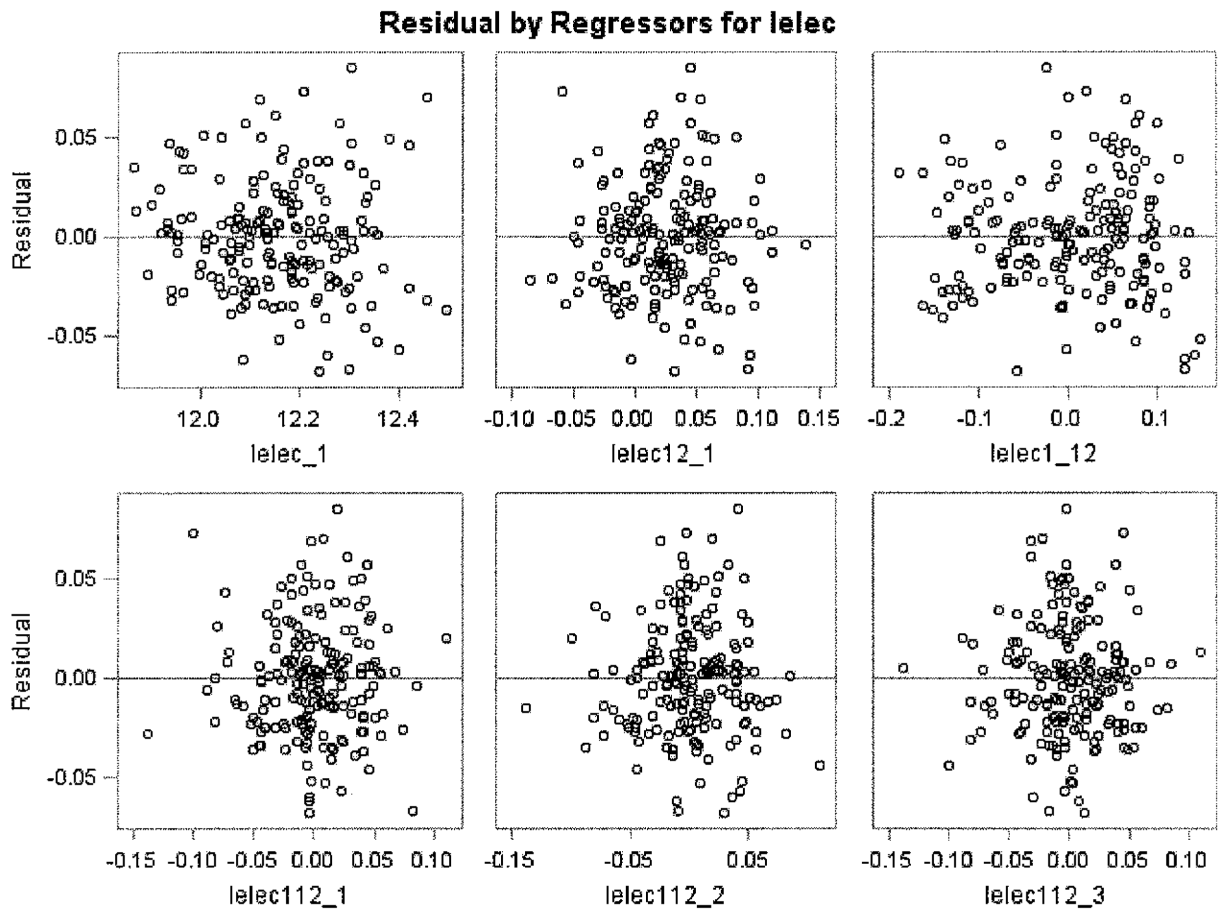
The SAS System

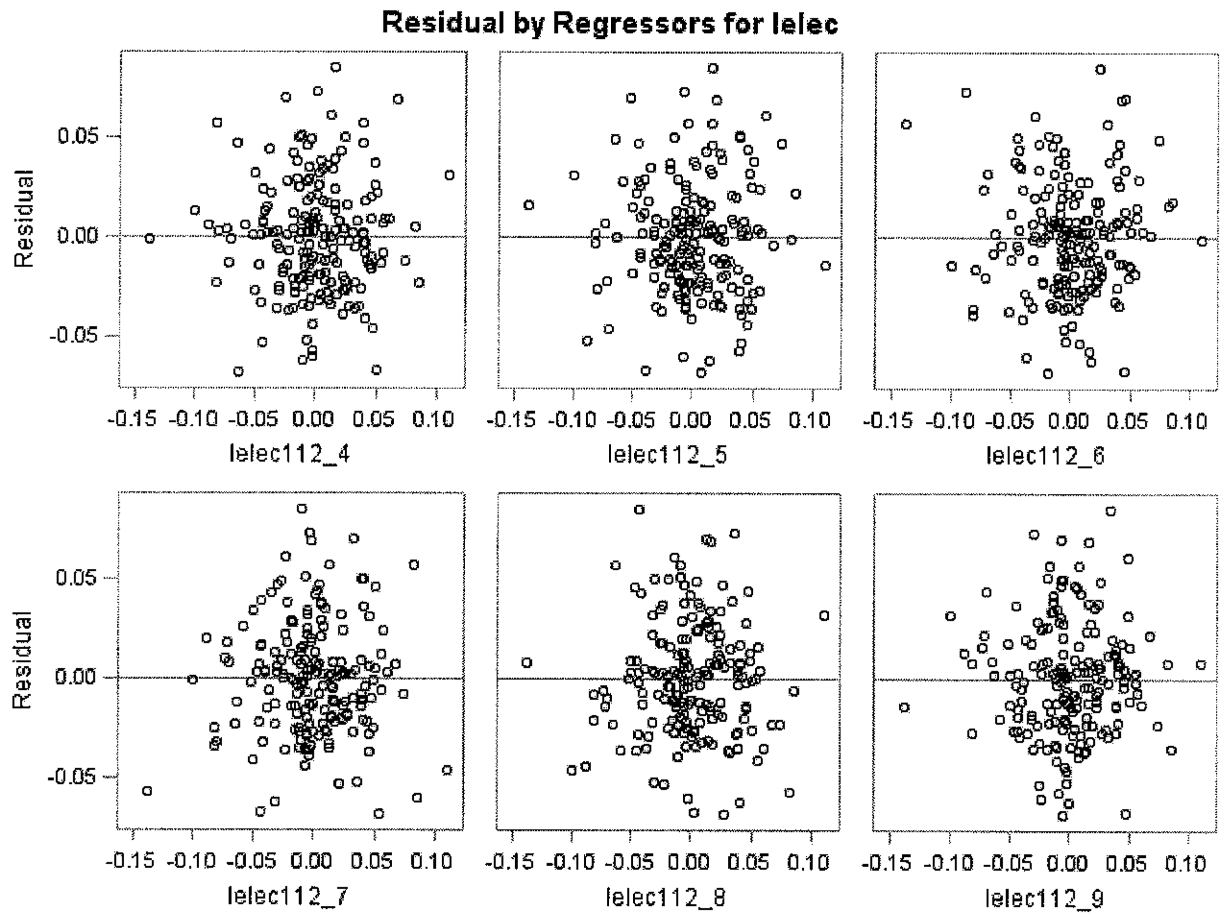
The REG Procedure
Model: MODEL1
Dependent Variable: lelec

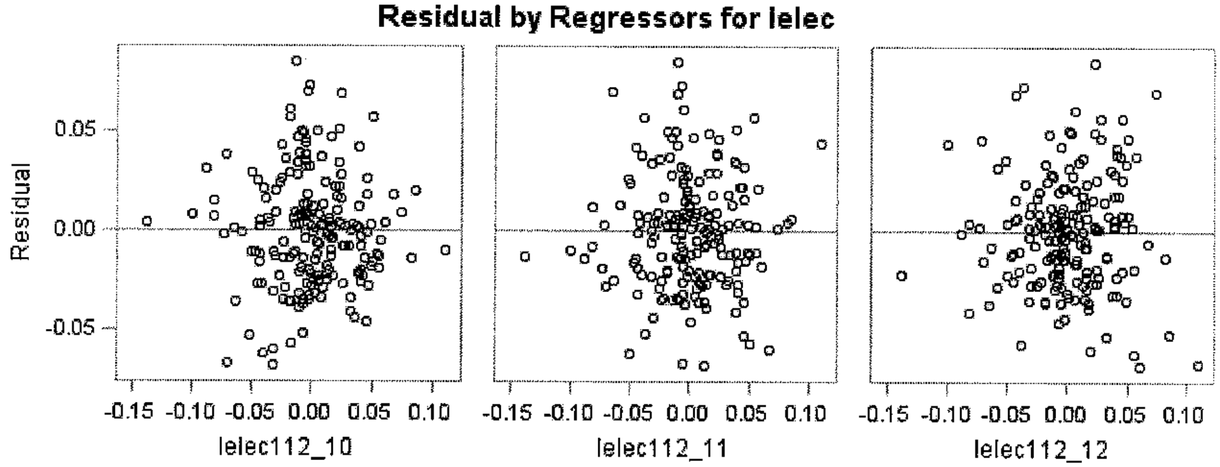
Fit Diagnostics for lelec



Observations	187
Parameters	15
Error DF	172
MSE	0.0009
R-Square	1
Adj R-Square	1


Residual by Regressors for lelec
Residual
0.05
0.00
-0.05
12.0 12.2 12.4
lelec_1
-0.10 -0.05 0.00 0.05 0.10 0.15
lelec12_1
-0.2 -0.1 0.0 0.1
lelec1_12
-0.15 -0.10 -0.05 0.00 0.05 0.10
lelec112_1
-0.15 -0.05 0.05
lelec112_2
-0.15 -0.10 -0.05 0.00 0.05 0.10
lelec112_3


Residual by Regressors for lelec
Residual
0.05
0.00
-0.05
-0.15 -0.10 -0.05 0.00 0.05 0.10
lelec112_4
lelec112_5
lelec112_6
lelec112_7
lelec112_8
lelec112_9


Residual by Regressors for lelec
Residual
0.05
0.00
-0.05
-0.15 -0.10 -0.05 0.00 0.05 0.10
lelec112_10
lelec112_11
lelec112_12

The SAS System

The REG Procedure
Model: MODEL1

Test 1 Results for Dependent Variable lelec				
Source	DF	Mean Square	F Value	Pr > F
Numerator	3	0.00470	5.49	0.0013
Denominator	172	0.00085548		

Hasza & Fuller Test $H_0: \Delta_1 \Delta_s$
(1982), Annals of Statistics H_1: not $\Delta_1 \Delta_s$

TABLE 5.1
Empirical percentiles for test statistics

	m	$d = 2$			$d = 4$			$d = 6$			$d = 12$		
		0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
$\Phi^{(3)}_{n-3}$	10	2.52	3.24	5.09	2.46	3.06	4.49	2.42	3.00	4.35	2.37	2.93	4.23
	20	2.48	3.09	4.51	2.44	3.01	4.30	2.41	2.97	4.24	2.37	2.92	4.19
	50	2.45	3.02	4.29	2.44	2.99	4.22	2.41	2.96	4.19	2.37	2.92	4.17
	∞	2.44	2.98	4.20	2.44	2.98	4.19	2.41	2.96	4.17	2.37	2.92	4.16
$\Phi^{(3)}_{n-d-4}$	10	8.33	10.08	14.86	8.95	10.42	13.72	10.09	11.63	14.58	13.49	15.18	18.47
	20	7.36	8.58	11.47	8.53	9.69	12.33	9.72	10.97	13.56	13.40	14.78	17.76
	50	6.97	7.93	9.92	8.26	9.28	11.36	9.54	10.68	12.86	13.24	14.55	17.46
	∞	6.67	7.50	9.17	8.04	8.96	10.90	9.36	10.39	12.43	13.16	14.41	16.93
$\Phi^{(d+4)}_{n-d-4}$	10	5.31	6.37	9.36	4.32	4.96	6.35	3.99	4.46	5.56	3.54	3.84	4.47
	20	4.44	5.09	6.69	3.94	4.44	5.42	3.71	4.08	4.87	3.40	3.65	4.18
	50	4.06	4.54	5.63	3.73	4.12	4.96	3.57	3.90	4.59	3.29	3.52	4.05
	∞	3.81	4.22	5.05	3.60	3.93	4.62	3.45	3.73	4.35	3.22	3.44	3.88
$\Phi^{(2)}_{n-2}$	10	2.61	3.46	5.76	2.58	3.36	5.20	2.53	3.28	5.04	2.46	3.19	4.90
	20	2.59	3.34	5.17	2.54	3.29	4.99	2.50	3.24	4.92	2.45	3.17	4.84
	50	2.55	3.27	4.99	2.53	3.26	4.92	2.49	3.22	4.88	2.45	3.16	4.83
	∞	2.52	3.24	4.93	2.52	3.24	4.90	2.49	3.20	4.87	2.45	3.15	4.82
$\Phi^{(2)}_{n-d-3}$	10	7.94	9.77	14.72	9.84	11.68	15.87	11.75	13.68	17.91	17.37	19.53	24.22
	20	7.43	8.89	12.03	9.63	11.12	14.39	11.57	13.24	16.81	17.29	19.24	23.65
	50	7.23	8.44	11.30	9.41	10.78	13.82	11.46	12.93	16.30	17.21	19.15	23.18
	∞	7.05	8.16	10.48	9.28	10.59	13.26	11.36	12.82	15.72	17.20	19.08	22.61
$\Phi^{(d+3)}_{n-d-3}$	10	4.39	5.30	7.64	3.87	4.47	5.84	3.64	4.11	5.15	3.37	3.68	4.31
	20	3.87	4.50	6.00	3.59	4.05	5.08	3.44	3.84	4.61	3.24	3.50	4.04
	50	3.61	4.14	5.26	3.45	3.86	4.74	3.34	3.68	4.38	3.16	3.40	3.90
	∞	3.45	3.92	4.88	3.35	3.73	4.48	3.26	3.58	4.23	3.11	3.34	3.80

THEOREM 4.1. *Let Y_t satisfy model* (4.1) *with e_t that are* iid$(0, \sigma^2)$. *Let* H, h *be as defined in Corollary* 3.1. *Then under $H_0: \beta' = (1, 0, 1)$*

(i) $$\{n^2(\hat\beta_1 - 1), n\hat\beta_2, n(\hat\beta_3 - 1)\}' \to_{\mathscr{L}} c\mathbf{H}^{-1}\mathbf{h},$$

(ii) $$n^{1/2}(\hat\theta - \theta) \to_{\mathscr{L}} N_p(0, \Gamma^{-1}\sigma^2),$$

where $(\Gamma)_{ij} = \lim_{t\to\infty} \mathrm{Cov}(X_t, X_{t+|i-j|})$.

PROOF. Define

$$W_t^\dagger = \sum_{j=1}^{t} e_j,\ Z_t^\dagger = \sum_{j=1}^{[t/d]} e_{t-dj},\ Y_t^\dagger = \sum_{j=1}^{t} Z_j^\dagger.$$

It is not difficult to show that the asymptotic properties of $(\sum_{t=1}^n \psi_t\psi_t')^{-1}(\sum_{t=1}^n \psi_t e_t)$ are not affected by replacing Y_{t-1}, W_{t-1}, and Z_{t-d} by $c^{-1}Y_{t-1}^\dagger$, $c^{-1}W_{t-1}^\dagger$, and $c^{-1}Z_{t-d}^\dagger$ respectively. Furthermore

$$\sum_{t=1}^n Y_{t-1}X_{t-i} = O_p(n^2), \quad \sum_{t=1}^n W_{t-1}X_{t-i} = O_p(n), \quad \sum_{t=1}^n Z_{t-d}X_{t-i} = O_p(n),$$
$$i = 1, 2, \cdots, p.$$

The result then follows from Theorem 3.1 and the well-known asymptotic distributional theory for stationary autoregressive processes. □

Now let $\Phi^{(3)}_{n-p-3}$ denote the test statistic for testing $H_0: \beta' = (1, 0, 1)$ analogous to the usual $F_{3,n-p-3}$ test statistic in a fixed normal regression model.

(1984) Journal of American Statistical Association. Dickey, Hasza, and Fuller Test $H_0: \Delta_s$ $H_1:$ not Δ_s

Table 5. Percentiles for $\tau_{\mu d}^*$, the Studentized Test for the Single Mean Model

		Probability of a Smaller Value								
	n = md	*.01*	*.025*	*.05*	*.10*	*.50*	*.90*	*.95*	*.975*	*.99*
	20	−3.54	−3.08	−2.72	−2.32	−.97	.49	.92	1.31	1.76
	30	−3.44	−3.02	−2.69	−2.31	−1.01	.46	.88	1.26	1.68
	40	−3.40	−3.00	−2.68	−2.31	−1.02	.44	.86	1.22	1.65
d = 2	100	−3.31	−2.95	−2.65	−2.31	−1.05	.41	.83	1.19	1.61
	200	−3.28	−2.93	−2.64	−2.31	−1.05	.40	.83	1.19	1.60
	400	−3.27	−2.93	−2.64	−2.31	−1.06	.40	.82	1.18	1.59
	∞	−3.25	−2.92	−2.63	−2.31	−1.06	.40	.82	1.18	1.59
	40	−3.14	−2.73	−2.38	−2.00	−.60	.80	1.19	1.54	1.94
	60	−3.11	−2.71	−2.38	−2.00	−.63	.76	1.15	1.49	1.89
	80	−3.09	−2.71	−2.38	−2.01	−.64	.74	1.13	1.47	1.87
d = 4	200	−3.07	−2.70	−2.38	−2.02	−.66	.72	1.11	1.46	1.86
	400	−3.06	−2.70	−2.38	−2.02	−.67	.72	1.11	1.46	1.85
	800	−3.06	−2.70	−2.38	−2.02	−.67	.72	1.10	1.45	1.85
	∞	−3.05	−2.70	−2.38	−2.03	−.67	.72	1.10	1.45	1.85
	120	−2.73	−2.33	−2.01	−1.65	−.31	1.02	1.38	1.71	2.08
	180	−2.73	−2.35	−2.02	−1.66	−.33	.99	1.35	1.68	2.06
	240	−2.73	−2.36	−2.02	−1.66	−.34	.98	1.34	1.67	2.05
d = 12	600	−2.73	−2.37	−2.04	−1.66	−.35	.97	1.34	1.66	2.03
	1,200	−2.73	−2.37	−2.05	−1.66	−.35	.96	1.33	1.65	2.02
	2,400	−2.74	−2.37	−2.06	−1.66	−.36	.96	1.33	1.65	2.02
	∞	−2.74	−2.37	−2.06	−1.66	−.36	.96	1.33	1.65	2.01

would obtain by regressing $Z_t - Z_{t-d} = \dot{Z}_t$ on Z_{t-d}, where $Z_t = Y_t - \theta_1 Y_{t-1} - \cdots - \theta_p Y_{t-p}$. The estimators $\hat{\theta}_i$, obtained by adding the estimates of $\theta_i - \hat{\theta}_i$ to $\hat{\theta}_i$, have the same asymptotic distribution as the coefficients in a regression of $\dot{Y}_t$ on $\dot{Y}_{t-1}, \dot{Y}_{t-2}, \ldots, \dot{Y}_{t-p}$.

The proof of Theorem 5 is given in Appendix B. Theorem 5 implies that the tabulated limit percentiles for estimators in model (1.1) are applicable in the multiplicative model for large sample sizes.

The extension of Theorem 5 to estimators with seasonal means or a single mean is immediate. Let

$$y_t = Y_t - \sum_{i=1}^{d} \delta_{it} \bar{\mu}_i. \tag{6.3}$$

Replacing Y_t by y_t in the two-step estimation procedure results in the regression of $e_t(1, \hat{\theta})$ on

$$(1 - \hat{\theta}_1 B - \hat{\theta}_2 B^2 - \cdots - \hat{\theta}_p B^p) y_{t-d},$$
$$\dot{Y}_{t-1}, \dot{Y}_{t-2}, \ldots, \dot{Y}_{t-d}.$$

Table 6. Percentiles for $n(\hat{\alpha}_{\mu d} - 1)$, the Ordinary Regression Coefficient for the Seasonal Means Model

		Probability of a Smaller Value								
	n = md	*.01*	*.025*	*.05*	*.10*	*.50*	*.90*	*.95*	*.975*	*.99*
	20	−18.98	−16.75	−14.96	−12.78	−6.48	−1.94	−.83	.10	1.16
	30	−20.89	−18.19	−16.01	−13.62	−6.76	−2.11	−1.01	−.11	.92
	40	−22.02	−19.03	−16.64	−14.09	−6.91	−2.18	−1.10	−.21	.79
d = 2	100	−24.32	−20.76	−17.95	−15.05	−7.20	−2.31	−1.25	−.40	.55
	200	−25.17	−21.39	−18.44	−15.40	−7.30	−2.35	−1.30	−.46	.46
	400	−25.62	−21.72	−18.69	−15.58	−7.35	−2.37	−1.33	−.49	.42
	∞	−26.07	−22.06	−18.95	−15.76	−7.41	−2.39	−1.35	−.52	.38
	40	−28.78	−25.59	−23.10	−20.40	−11.89	−5.40	−3.90	−2.51	−1.05
	60	−30.63	−27.18	−24.49	−21.42	−12.33	−5.73	−4.15	−2.81	−1.33
	80	−31.79	−28.12	−25.27	−22.00	−12.58	−5.87	−4.28	−2.96	−1.47
d = 4	200	−34.26	−30.03	−26.78	−23.15	−13.06	−6.09	−4.51	−3.20	−1.74
	400	−35.20	−30.73	−27.32	−23.56	−13.23	−6.15	−4.59	−3.28	−1.83
	800	−35.69	−31.10	−27.60	−23.78	−13.32	−6.18	−4.63	−3.32	−1.87
	∞	−36.19	−31.47	−27.88	−24.00	−13.41	−6.21	−4.67	−3.35	−1.92
	120	−60.72	−55.63	−51.22	−46.98	−33.65	−22.13	−19.46	−17.09	−14.25
	180	−63.40	−57.83	−53.57	−49.14	−34.95	−23.06	−20.00	−17.44	−14.57
	240	−64.90	−59.21	−54.89	−50.28	−35.57	−23.48	−20.31	−17.70	−14.86
d = 12	600	−67.87	−62.16	−57.52	−52.44	−36.65	−24.16	−20.92	−18.34	−15.57
	1,200	−68.93	−63.29	−58.47	−53.19	−36.99	−24.37	−21.14	−18.60	−15.87
	2,400	−69.48	−63.87	−58.95	−53.57	−37.16	−24.47	−21.26	−18.74	−16.03
	∞	−70.04	−64.48	−59.45	−53.95	−37.33	−24.56	−21.37	−18.88	−16.20

SAS Program for Output #5

```
data electric;
   format date monyy5.;
   input date:monyy5. elec @@;
   datalines;
jan72 144549 feb72 137310 mar72 140151 apr72 132112
may72 138168 jun72 145583 jul72 157878 aug72 162901
sep72 147584 oct72 144062 nov72 144366 dec72 154966
jan73 159913 feb73 143257 mar73 147846 apr73 139292
may73 147088 jun73 160945 jul73 173467 aug73 177109
.
.
.
sep87 213008 oct87 203009 nov87 200258 dec87 220500
jan88 237600 feb88 216702 mar88 213838 apr88 195809
may88 208180 jun88 232507 jul88 257235 aug88 267408
sep88 220023 oct88 210377 nov88 209394 dec88 232550
jan89 231343 feb89 219066 mar89 226436 apr89 207749
may89 219803 jun89 235397 jul89 256744 aug89 258335
;

data electric;
  set electric;
  lelec = log(elec);

title 'Model 1';
proc arima data=electric;
  identify var = lelec(1,12);
  estimate q=(1)(12); /* model 1 */
  run;

title 'Model 2';
proc arima data=electric;
  identify var = lelec(1,12);
  estimate q=(1,2)(12); /* model 2 */
  run;

title 'Model 3';
proc arima data=electric;
  identify var = lelec(1,12);
  estimate p=1 q=(1,2)(12); /* model 3 */
  run;

title 'Model 4';
proc arima data=electric;
  identify var = lelec(1,12);
  estimate p=(12) q=(1,2)(12); /* model 4 */
  run;
```

Computer Output #5

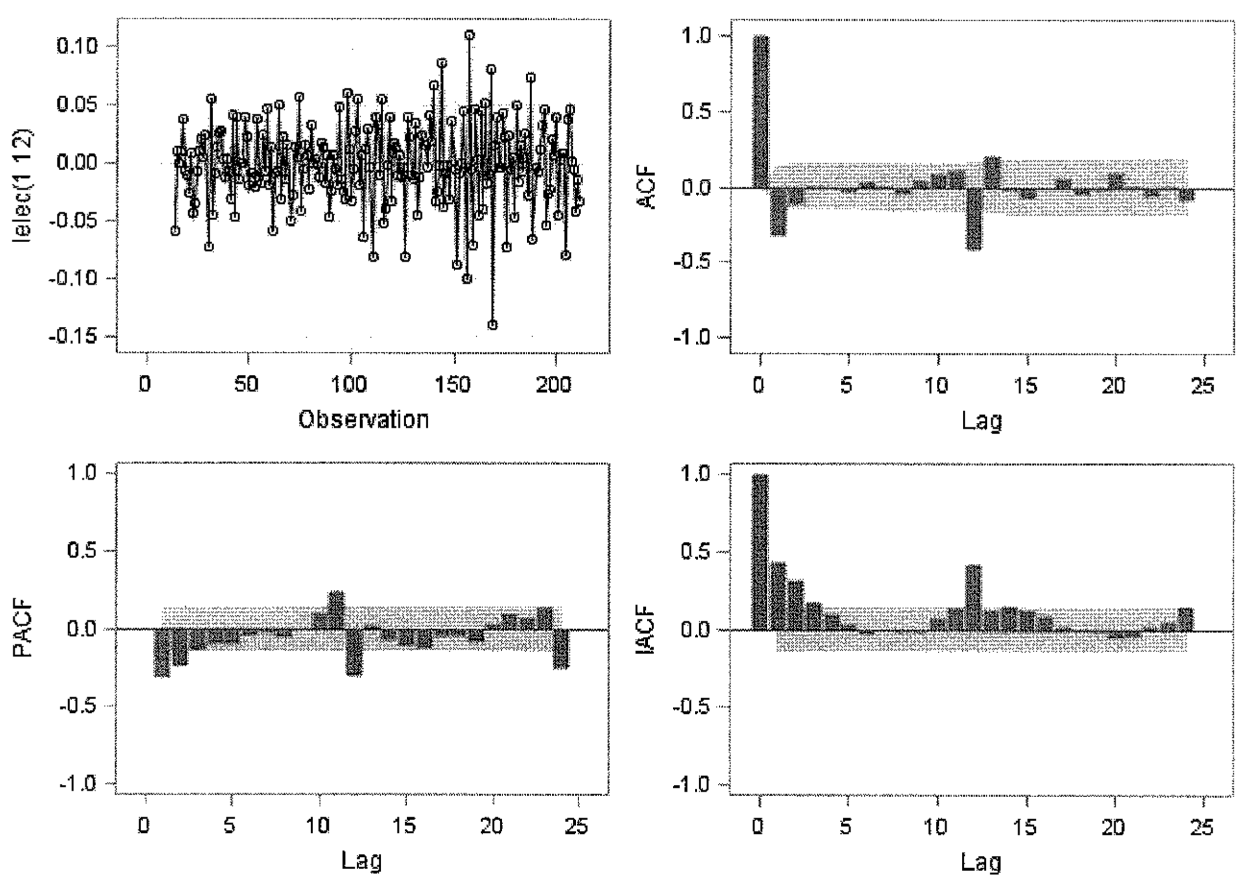
Model 1

The ARIMA Procedure

Name of Variable = lelec	
Period(s) of Differencing	1,12
Mean of Working Series	-0.00068
Standard Deviation	0.03578
Number of Observations	199
Observation(s) eliminated by differencing	13

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	23.59	6	0.0006	-0.319	-0.113	0.009	-0.005	-0.036	0.030
12	64.30	12	<.0001	0.002	-0.039	0.042	0.082	0.111	-0.410
18	76.29	18	<.0001	0.213	-0.024	-0.068	0.000	0.060	-0.039
24	81.12	24	<.0001	-0.018	0.099	0.004	-0.055	0.010	-0.089

Trend and Correlation Analysis for lelec(1 12)



Conditional Least Squares Estimation					
Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**	**Lag**
MU	-0.0002942	0.0002940	-1.00	0.3182	0
MA1,1	0.51083	0.06169	8.28	<.0001	1
MA2,1	0.73804	0.05085	14.51	<.0001	12

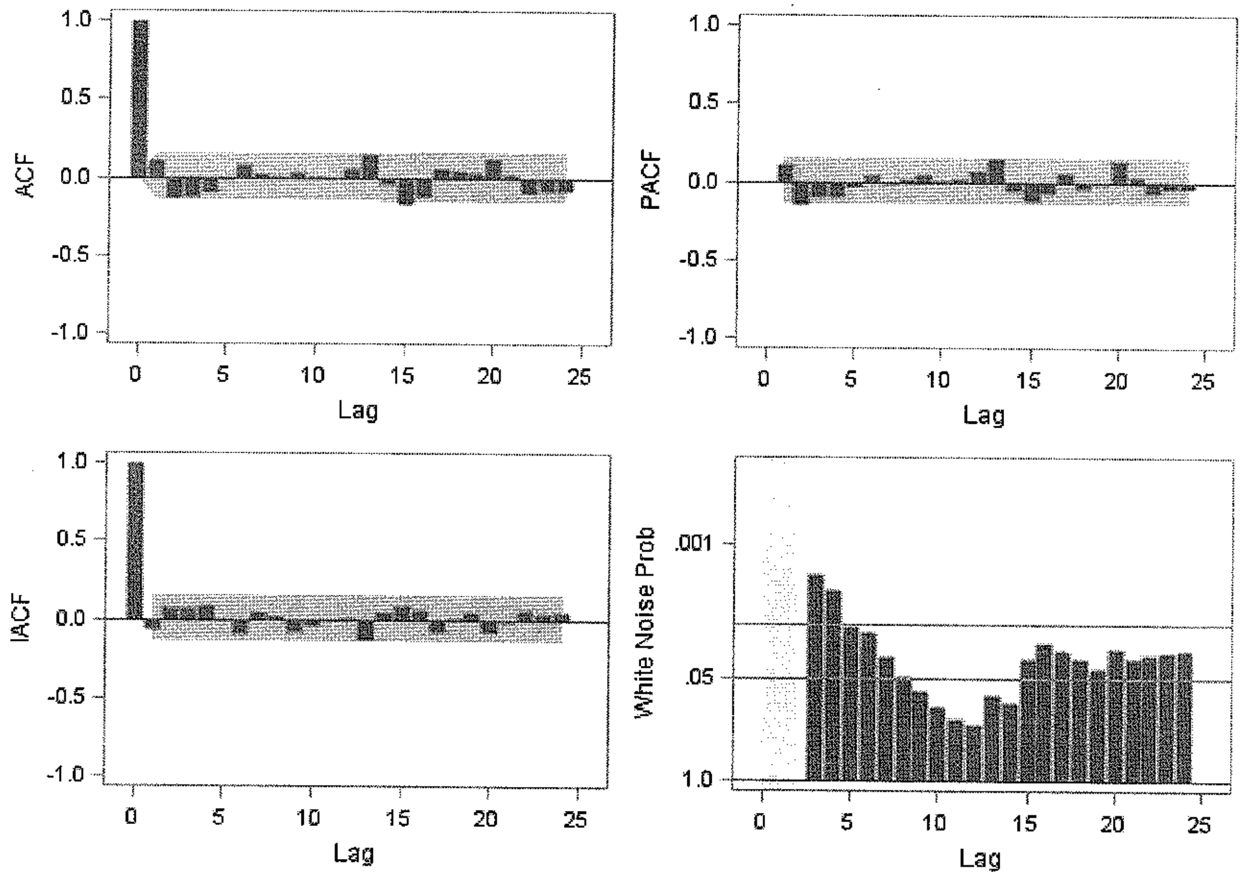
Constant Estimate	-0.00029
Variance Estimate	0.000753
Std Error Estimate	0.027445
AIC	-863.323
SBC	-853.443
Number of Residuals	199

*** AIC and SBC do not include log determinant.**

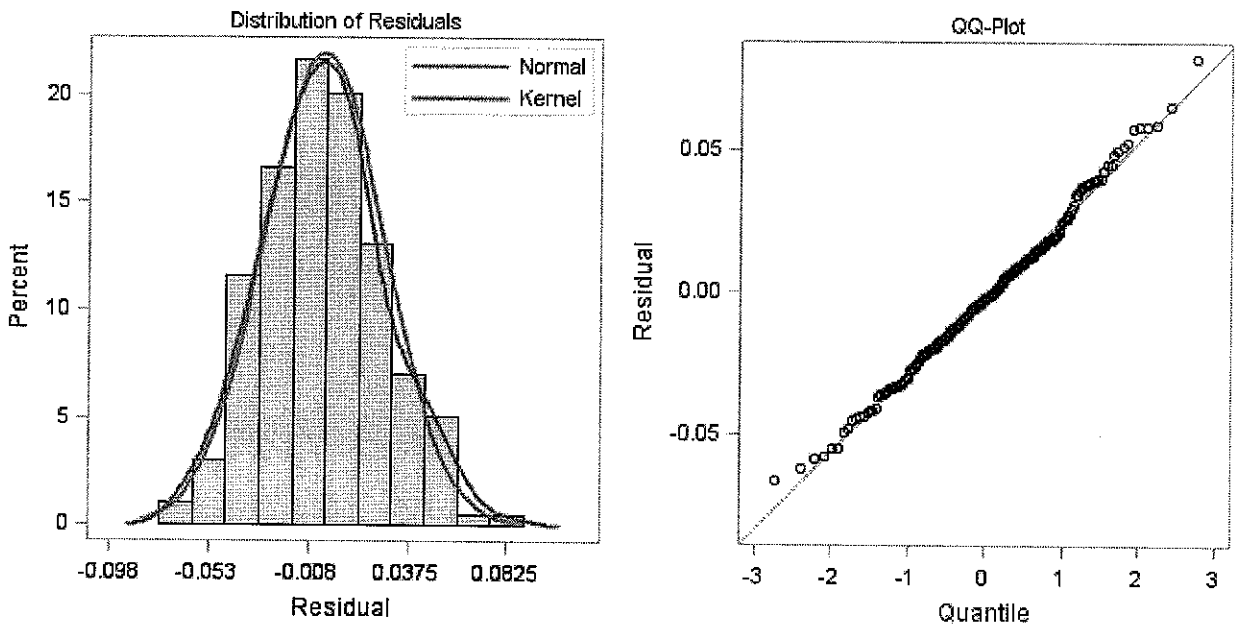
Correlations of Parameter Estimates			
Parameter	**MU**	**MA1,1**	**MA2,1**
MU	1.000	-0.007	-0.109
MA1,1	-0.007	1.000	0.082
MA2,1	-0.109	0.082	1.000

Autocorrelation Check of Residuals									
To Lag	**Chi-Square**	**DF**	**Pr > ChiSq**	**Autocorrelations**					
6	12.57	4	0.0136	0.110	-0.132	-0.125	-0.096	-0.012	0.084
12	13.70	10	0.1873	0.028	0.011	0.037	0.003	-0.003	0.055
18	28.28	16	0.0293	0.143	-0.040	-0.164	-0.116	0.056	0.040
24	37.10	22	0.0231	0.037	0.127	0.026	-0.088	-0.083	-0.080
30	46.20	28	0.0166	0.104	0.111	-0.053	-0.092	0.049	0.050
36	48.59	34	0.0501	-0.009	0.021	-0.031	-0.054	-0.071	0.020

Residual Correlation Diagnostics for lelec(1 12)



Residual Normality Diagnostics for lelec(1 12)



Model for variable lelec

Estimated Mean	-0.00029
Period(s) of Differencing	1,12

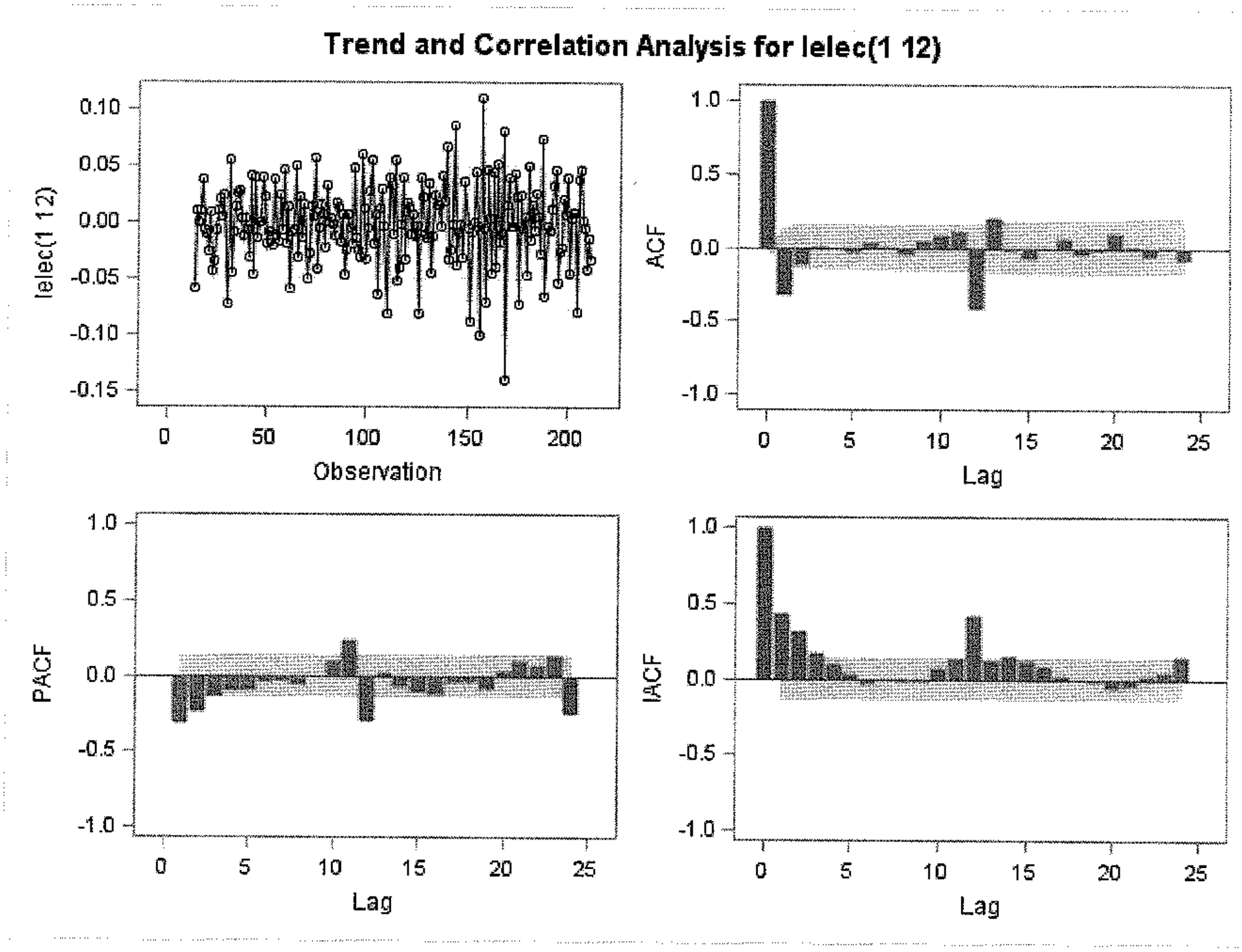
Moving Average Factors	
Factor 1:	1 - 0.51083 B**(1)
Factor 2:	1 - 0.73804 B**(12)

Model 2

The ARIMA Procedure

Name of Variable = lelec	
Period(s) of Differencing	1,12
Mean of Working Series	-0.00068
Standard Deviation	0.03578
Number of Observations	199
Observation(s) eliminated by differencing	13

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	23.59	6	0.0006	-0.319	-0.113	0.009	-0.005	-0.036	0.030
12	64.30	12	<.0001	0.002	-0.039	0.042	0.082	0.111	-0.410
18	76.29	18	<.0001	0.213	-0.024	-0.068	0.000	0.060	-0.039
24	81.12	24	<.0001	-0.018	0.099	0.004	-0.055	0.010	-0.089



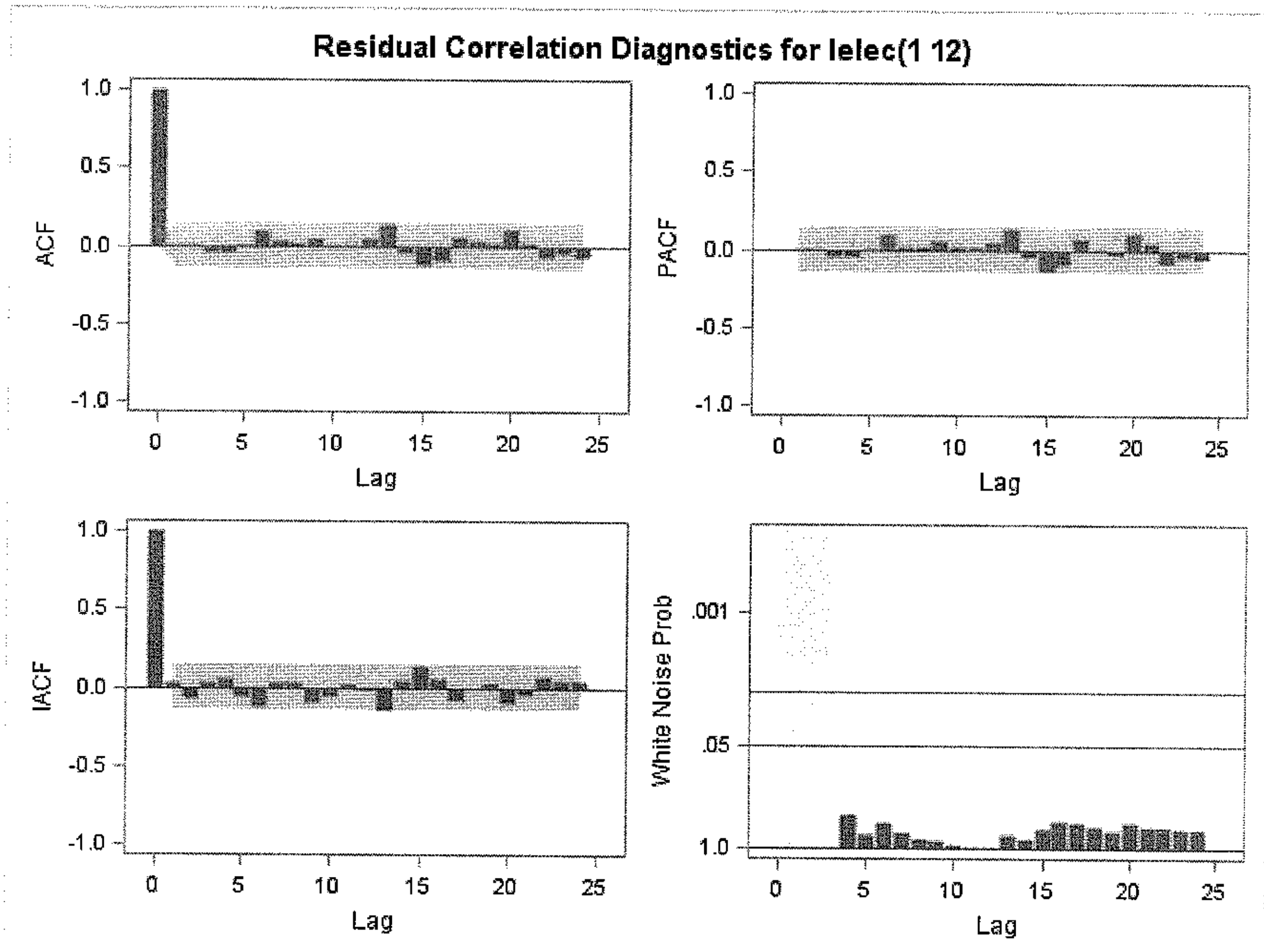
Conditional Least Squares Estimation					
Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**	**Lag**
MU	-0.0002926	0.0002294	-1.28	0.2036	0
MA1,1	0.40629	0.07024	5.78	<.0001	1
MA1,2	0.19550	0.07031	2.78	0.0060	2
MA2,1	0.74930	0.05042	14.86	<.0001	12

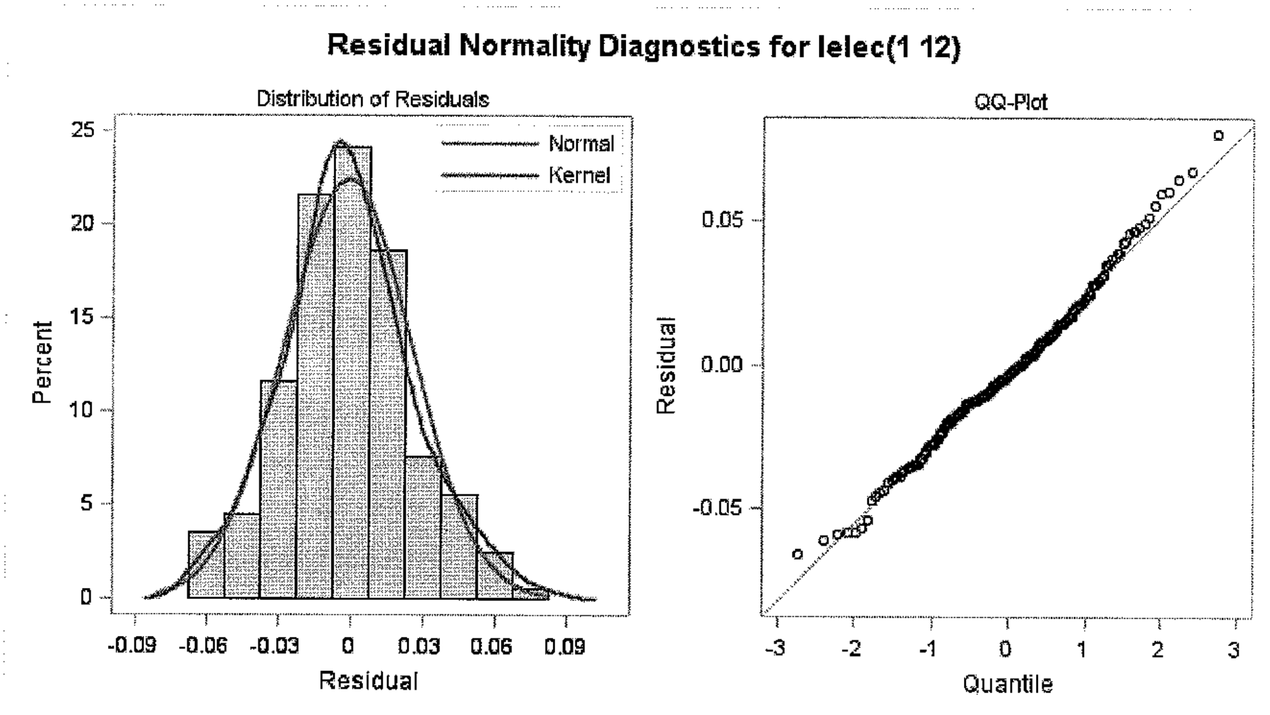
Constant Estimate	-0.00029
Variance Estimate	0.000725
Std Error Estimate	0.02693
AIC	-869.886
SBC	-856.713
Number of Residuals	199

*** AIC and SBC do not include log determinant.**

Correlations of Parameter Estimates				
Parameter	MU	MA1,1	MA1,2	MA2,1
MU	1.000	-0.005	0.002	-0.160
MA1,1	-0.005	1.000	-0.506	0.018
MA1,2	0.002	-0.506	1.000	-0.030
MA2,1	-0.160	0.018	-0.030	1.000

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	2.63	3	0.4527	0.005	0.007	-0.045	-0.041	0.005	0.094
12	3.88	9	0.9194	0.030	0.021	0.049	0.014	0.015	0.042
18	14.07	15	0.5204	0.133	-0.039	-0.117	-0.097	0.061	0.030
24	19.84	21	0.5314	0.020	0.112	0.018	-0.072	-0.039	-0.075
30	26.28	27	0.5032	0.090	0.086	-0.042	-0.076	0.054	0.042
36	28.04	33	0.7125	-0.019	0.027	-0.036	-0.032	-0.058	0.022





Model for variable lelec	
Estimated Mean	-0.00029
Period(s) of Differencing	1,12

Moving Average Factors	
Factor 1:	1 - 0.40629 B**(1) - 0.1955 B**(2)
Factor 2:	1 - 0.7493 B**(12)

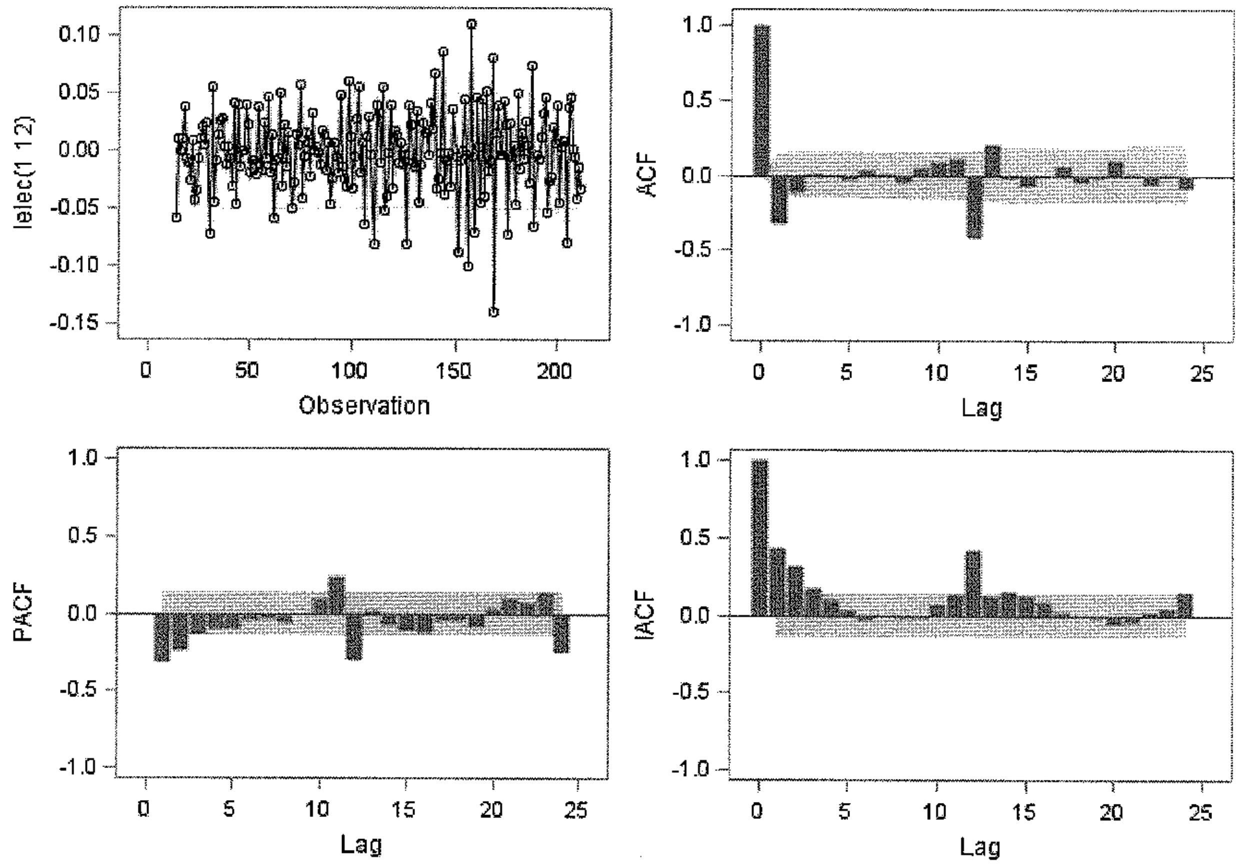
Model 3

The ARIMA Procedure

Name of Variable = lelec	
Period(s) of Differencing	1,12
Mean of Working Series	-0.00068
Standard Deviation	0.03578
Number of Observations	199
Observation(s) eliminated by differencing	13

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	23.59	6	0.0006	-0.319	-0.113	0.009	-0.005	-0.036	0.030
12	64.30	12	<.0001	0.002	-0.039	0.042	0.082	0.111	-0.410
18	76.29	18	<.0001	0.213	-0.024	-0.068	0.000	0.060	-0.039
24	81.12	24	<.0001	-0.018	0.099	0.004	-0.055	0.010	-0.089

Trend and Correlation Analysis for lelec(1 12)



Conditional Least Squares Estimation					
Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**	**Lag**
MU	-0.0002928	0.0002272	-1.29	0.1990	0
MA1,1	0.49212	0.36408	1.35	0.1781	1
MA1,2	0.14981	0.19741	0.76	0.4488	2
MA2,1	0.74985	0.05083	14.75	<.0001	12
AR1,1	0.09083	0.36805	0.25	0.8053	1

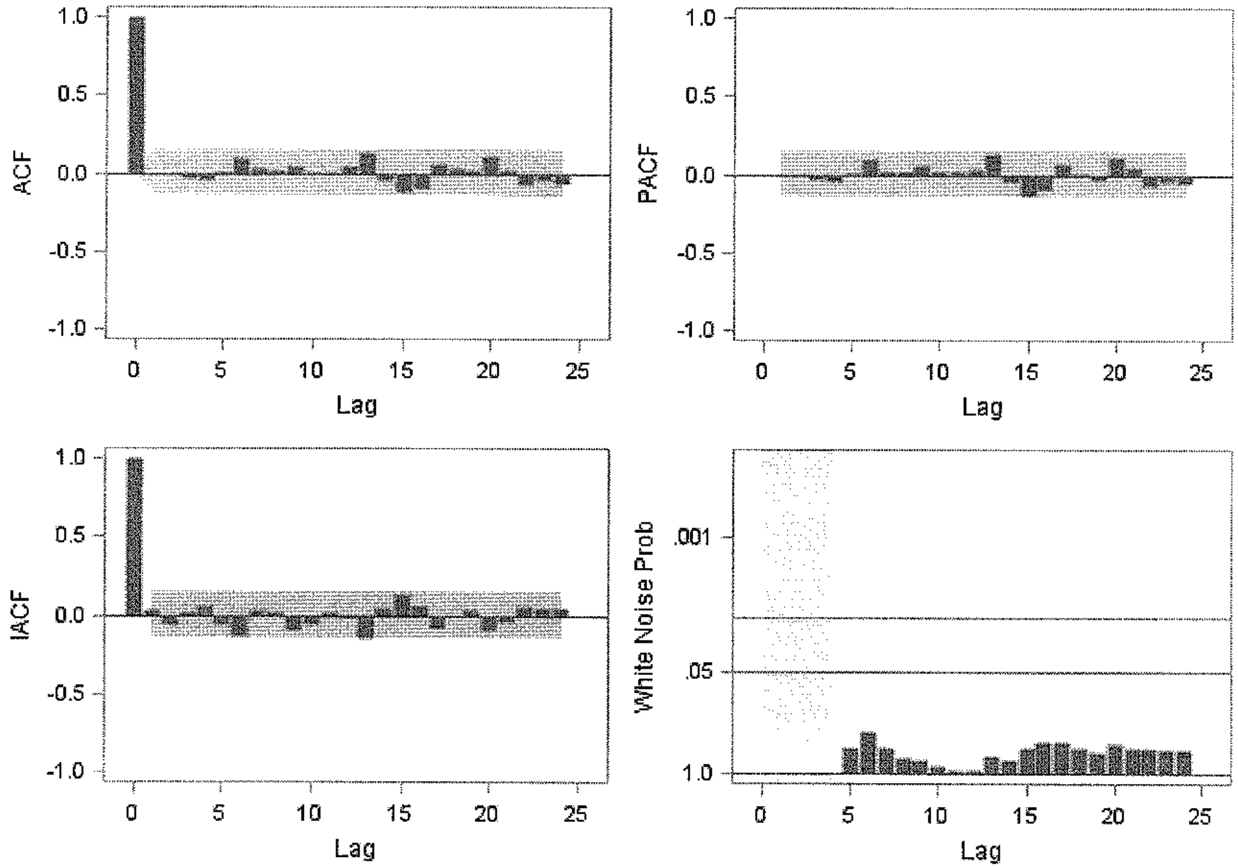
Constant Estimate	-0.00027
Variance Estimate	0.000729
Std Error Estimate	0.026993
AIC	-867.975
SBC	-851.508
Number of Residuals	199

*** AIC and SBC do not include log determinant.**

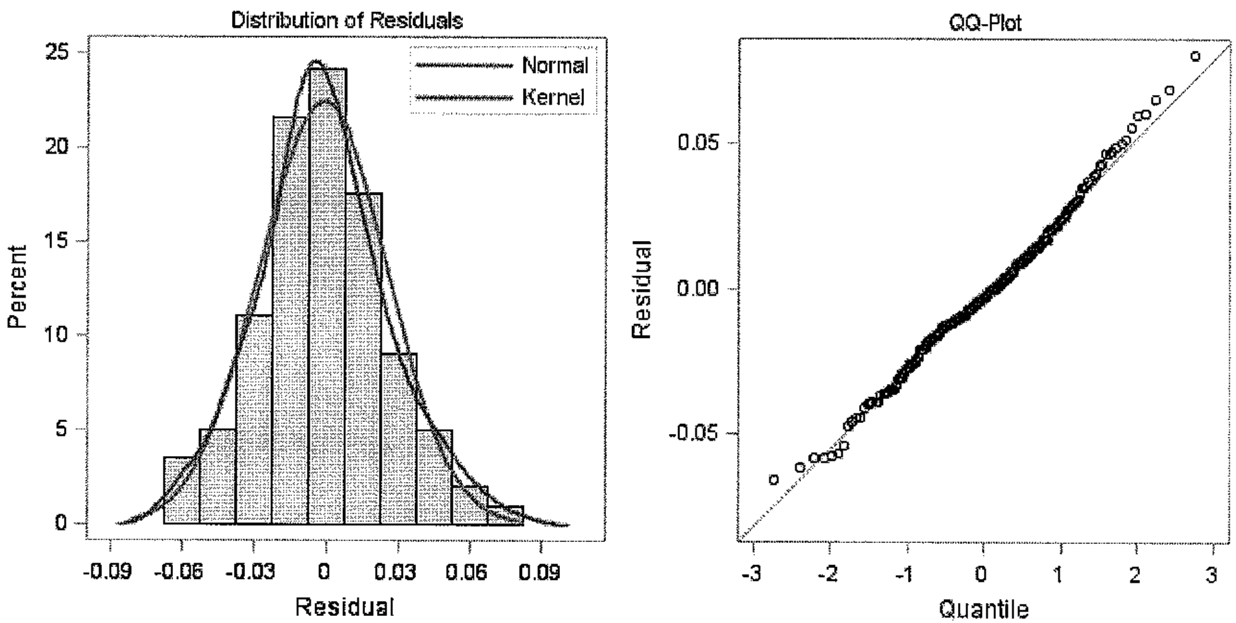
Correlations of Parameter Estimates					
Parameter	**MU**	**MA1,1**	**MA1,2**	**MA2,1**	**AR1,1**
MU	1.000	0.008	-0.008	-0.163	0.009
MA1,1	0.008	1.000	-0.956	-0.104	0.981
MA1,2	-0.008	-0.956	1.000	0.088	-0.933
MA2,1	-0.163	-0.104	0.088	1.000	-0.110
AR1,1	0.009	0.981	-0.933	-0.110	1.000

Autocorrelation Check of Residuals									
To Lag	**Chi-Square**	**DF**	**Pr > ChiSq**	**Autocorrelations**					
6	2.48	2	0.2888	0.001	-0.003	-0.032	-0.037	0.009	0.097
12	3.76	8	0.8779	0.031	0.021	0.050	0.015	0.013	0.042
18	13.95	14	0.4536	0.135	-0.037	-0.117	-0.095	0.064	0.028
24	19.63	20	0.4815	0.016	0.113	0.018	-0.071	-0.038	-0.074
30	26.30	26	0.4469	0.090	0.088	-0.044	-0.077	0.056	0.043
36	28.01	32	0.6688	-0.020	0.027	-0.034	-0.031	-0.058	0.022

Residual Correlation Diagnostics for lelec(1 12)



Residual Normality Diagnostics for lelec(1 12)



Model for variable lelec

Estimated Mean	-0.00029
Period(s) of Differencing	1,12

Autoregressive Factors	
Factor 1:	1 - 0.09083 B**(1)

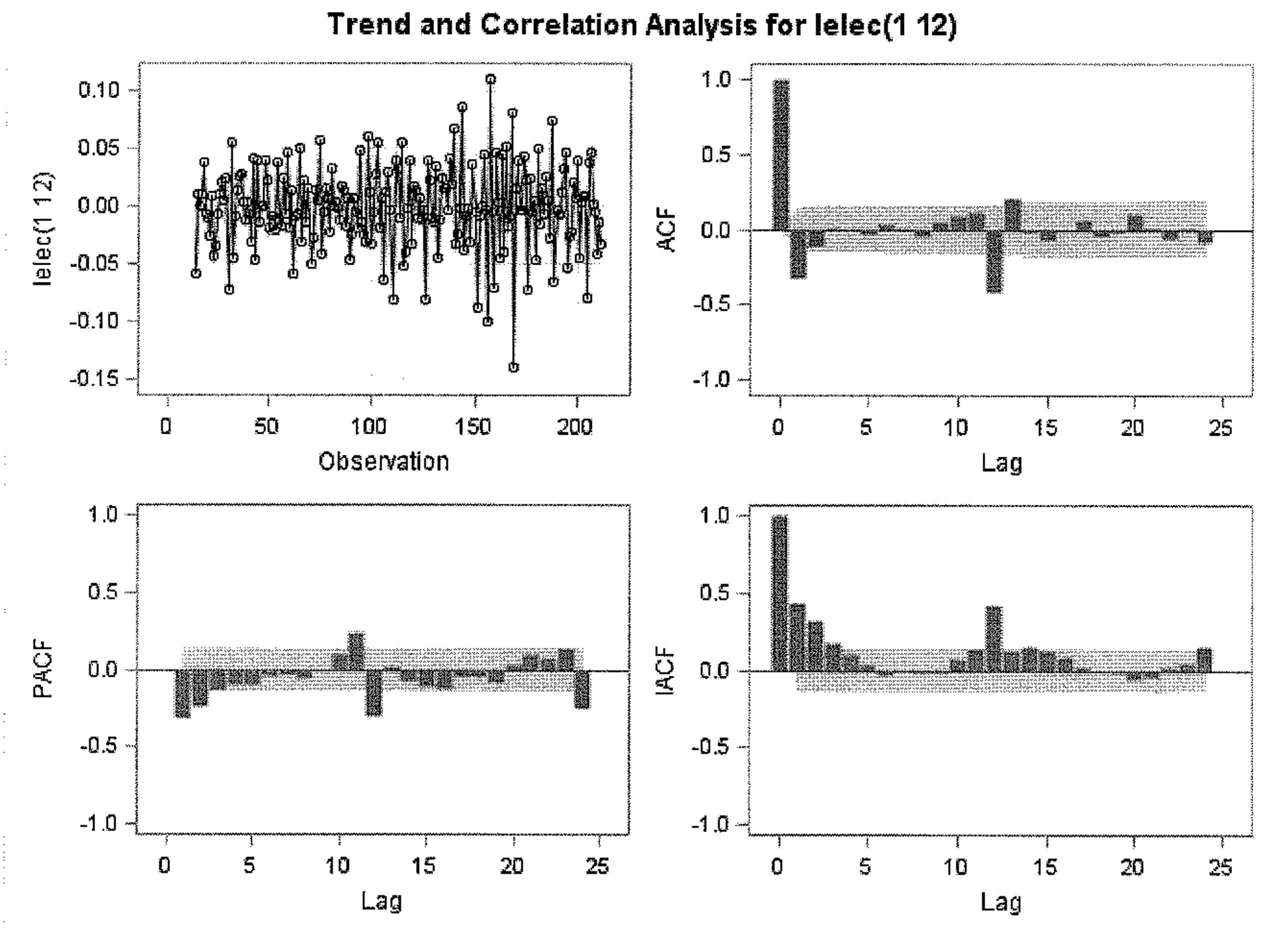
Moving Average Factors	
Factor 1:	1 - 0.49212 B**(1) - 0.14981 B**(2)
Factor 2:	1 - 0.74985 B**(12)

Model 4

The ARIMA Procedure

Name of Variable = lelec	
Period(s) of Differencing	1,12
Mean of Working Series	-0.00068
Standard Deviation	0.03578
Number of Observations	199
Observation(s) eliminated by differencing	13

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	23.59	6	0.0006	-0.319	-0.113	0.009	-0.005	-0.036	0.030
12	64.30	12	<.0001	0.002	-0.039	0.042	0.082	0.111	-0.410
18	76.29	18	<.0001	0.213	-0.024	-0.068	0.000	0.060	-0.039
24	81.12	24	<.0001	-0.018	0.099	0.004	-0.055	0.010	-0.089



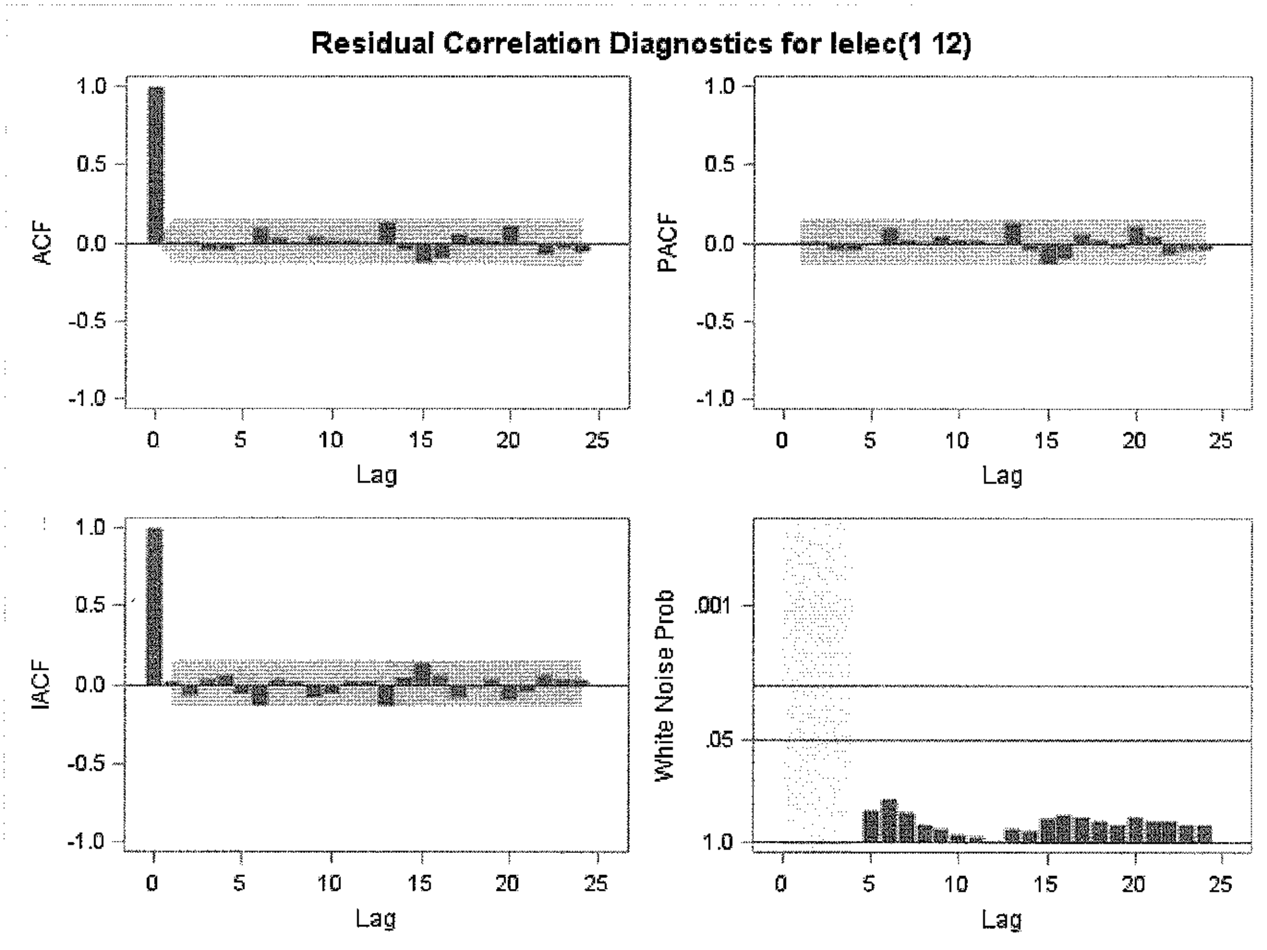
Conditional Least Squares Estimation					
Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**	**Lag**
MU	-0.0003089	0.0002271	-1.36	0.1753	0
MA1,1	0.41143	0.07056	5.83	<.0001	1
MA1,2	0.19126	0.07057	2.71	0.0073	2
MA2,1	0.78180	0.06615	11.82	<.0001	12
AR1,1	0.07811	0.10238	0.76	0.4464	12

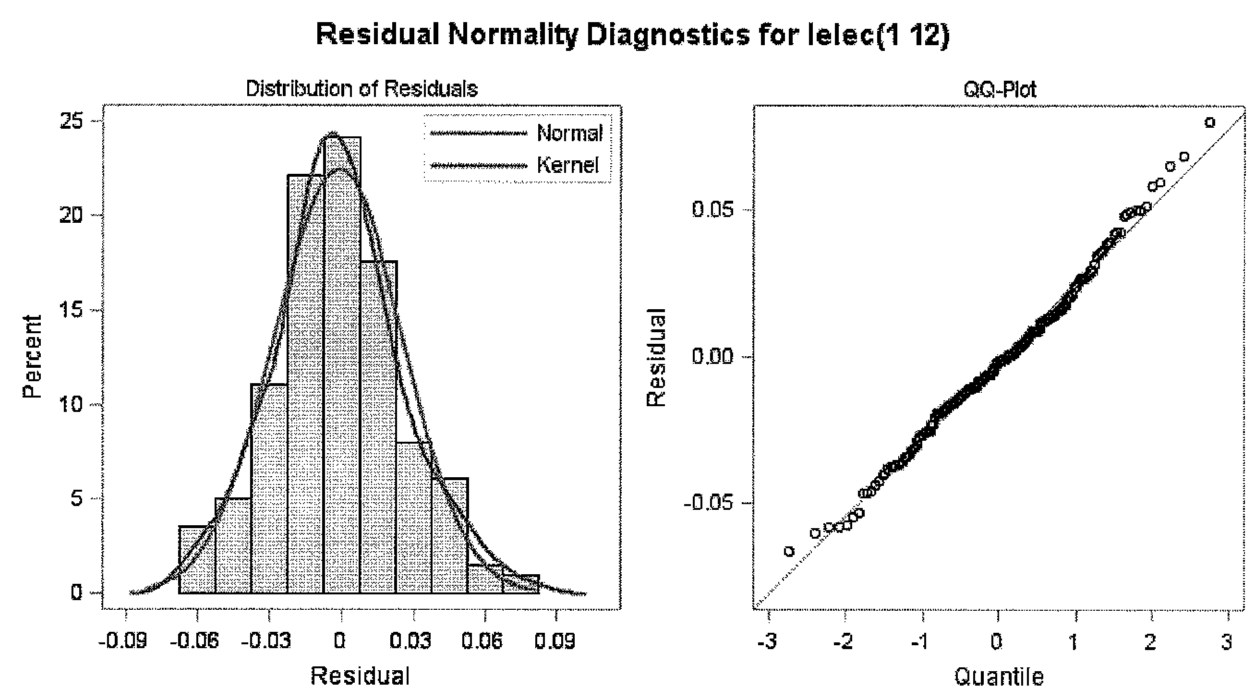
Constant Estimate	-0.00028
Variance Estimate	0.000727
Std Error Estimate	0.026955
AIC	-868.538
SBC	-852.071
Number of Residuals	199

*** AIC and SBC do not include log determinant.**

Correlations of Parameter Estimates					
Parameter	**MU**	**MA1,1**	**MA1,2**	**MA2,1**	**AR1,1**
MU	1.000	-0.010	-0.005	-0.242	-0.163
MA1,1	-0.010	1.000	-0.506	0.041	0.049
MA1,2	-0.005	-0.506	1.000	0.013	0.036
MA2,1	-0.242	0.041	0.013	1.000	0.692
AR1,1	-0.163	0.049	0.036	0.692	1.000

Autocorrelation Check of Residuals									
To Lag	**Chi-Square**	**DF**	**Pr > ChiSq**	**Autocorrelations**					
6	2.68	2	0.2618	0.005	0.007	-0.045	-0.039	0.003	0.097
12	3.51	8	0.8983	0.032	0.014	0.044	0.018	0.020	0.004
18	13.24	14	0.5074	0.129	-0.044	-0.116	-0.092	0.058	0.033
24	18.14	20	0.5779	0.025	0.107	0.013	-0.071	-0.035	-0.057
30	24.07	26	0.5718	0.080	0.088	-0.037	-0.073	0.049	0.047
36	25.98	32	0.7643	-0.018	0.021	-0.037	-0.036	-0.049	0.045





Model for variable lelec

Estimated Mean	-0.00031
Period(s) of Differencing	1,12

Autoregressive Factors	
Factor 1:	1 - 0.07811 B**(12)

Moving Average Factors	
Factor 1:	1 - 0.41143 B**(1) - 0.19126 B**(2)
Factor 2:	1 - 0.7818 B**(12)