

**Buys Ballot Plots:  
Graphical Methods for Detecting  
Seasonality in Time Series**

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**I. Introduction**

Christophorus Henricus Diedericus Buys Ballot (Oct. 10, 1817, Kloetinge – Feb. 3, 1890, Utrecht) was a Dutch chemist and meteorologist after whom Buys-Ballot's law of atmospheric pressure and the Buys Ballot table are named. Buys Ballot devised his tabular method for investigating periodicity in time series data. In this discussion we will be presenting various graphs suggested by his table for inspecting time series data for the presence of seasonal effects.

C.H.D. Buys Ballot  
(1817 – 1890)



Source: [http://en.wikipedia.org/wiki/C.H.D.\\_Buys\\_Ballot](http://en.wikipedia.org/wiki/C.H.D._Buys_Ballot)

**The Buys Ballot table** displays time series data in the following way:

### Buys-Ballot Table for a Seasonal Time Series

	<b>1</b>	<b>2</b>	<b>3</b>	<b>...</b>	<b>S</b>	<b>Total</b>	<b>Average</b>
	$y_1$	$y_2$	$y_3$	$\dots$	$y_s$	$T_{1\bullet}$	$\bar{y}_{1\bullet}$
	$y_{s+1}$	$y_{s+2}$	$y_{s+3}$	$\dots$	$y_{2s}$	$T_{2\bullet}$	$\bar{y}_{2\bullet}$
	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
	$y_{(T-1)s+1}$	$y_{(T-1)s+2}$	$y_{(T-1)s+3}$	$\dots$	$y_{Ts}$	$T_{T\bullet}$	$\bar{y}_{T\bullet}$
<b>Total</b>	$T_{\bullet 1}$	$T_{\bullet 2}$	$T_{\bullet 3}$	$\dots$	$T_{\bullet s}$	$T_{sum}$	$T_{sum}/s$
<b>Average</b>	$\bar{y}_{\bullet 1}$	$\bar{y}_{\bullet 2}$	$\bar{y}_{\bullet 3}$	$\dots$	$\bar{y}_{\bullet s}$	$T_{sum}/T$	$T_{sum}/(T \cdot s)$

$T_{j\bullet}$  = Total for j-th year

$T$  = Number of years of data

$T_{\bullet j}$  = Total for j-th season

$s$  = Number of seasons per year

$\bar{y}_{j\bullet}$  = Average for j-th year

$T_{sum}$  = Sum of all observations

$\bar{y}_{\bullet j}$  = Average for j-th season

In this table the main entries in the center of the table are the time series observations we wish to inspect for possible seasonality. Each row represents a year's worth of data observed over the  $s$  seasons of the year ( $s = 12$  for monthly data,  $s = 4$  for quarterly data, and  $s = 2$  for bi-annual data). The  $j$ -th year total is denoted by  $T_{j\bullet}$  while the year's average is denoted by  $\bar{y}_{j\bullet}$ . Each column represents a particular season's data observed over the  $T$  year span of the data. (Here we assume we have  $T$  complete year's worth of data.) The  $j$ -th season's total is denoted by  $T_{\bullet j}$  while the season's average is denoted by  $\bar{y}_{\bullet j}$ . Three additional averages are calculated.  $T_{sum}$  denotes the sum of all of the row totals (or equivalently the sum of all of the column totals). Therefore, the "overall season average" is  $T_{sum} / s$ , the "overall yearly average" is  $T_{sum} / T$ , and the "overall observation average" is  $T_{sum} / (T \cdot s)$ .

Several plots based on the Buys Ballot table suggest themselves. First, a plot of the yearly averages  $\bar{y}_{1\bullet}, \bar{y}_{2\bullet}, \dots, \bar{y}_{T\bullet}$  as a function of the years  $t = 1, 2, \dots, T$  will reveal the basic trend in the data but will have nothing to say about seasonality because the yearly

averages have been formed by aggregating the observations of the seasons. In contrast, a plot of the seasonal averages  $\bar{y}_{\bullet 1}, \bar{y}_{\bullet 2}, \dots, \bar{y}_{\bullet s}$ , could reveal an interesting seasonal pattern. If the seasonal averages fall along a fairly straight line as a function of the seasons  $j = 1, 2, \dots, s$ , then the data is dominated by trend and there is not much seasonal variation in it. On the other hand, if the seasonal averages do not fall along a straight line then there is a hint of seasonal variation in the data. Such a plot we will call the **Buys Ballot Seasonal Averages plot**.

The Buys Ballot table suggests two other plots that can be useful for diagnosing seasonal variation in time series data. The first one we will call the **Buys Ballot Seasons-by-Year plot** and the second we will call the **Buys Ballot Seasonal Trends plot**. In the Buys Ballot Seasons by Year plot we simply produce overlay plots the seasons by year (i.e. row by row plotting of the data). In the Buys Ballot Seasonal Trends plot we produce an overlay plot of the interpolated trend graphs of each season (the columns).

The best way to gain intuition concerning the use of these graphs to diagnose the presence of seasonality in our data is to demonstrate their use on some data.

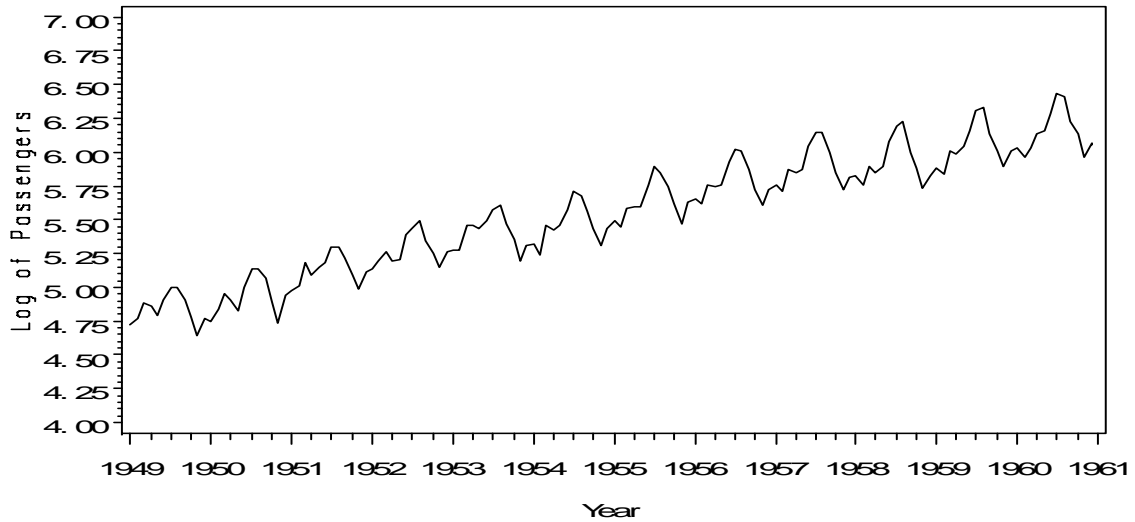
## II. Some Applications of the Buys Ballot Plots

Here we are going to compare the above mentioned Buys Ballot plots over two contrasting time series data sets, one that is distinctly seasonal and the other distinctly non-seasonal. We analyze these data sets using the SAS program Nonseasonal Airline data.sas. The seasonal data set is the original Airline Passenger data set where we analyze the logarithm of the passengers traveling by month. The variable is labeled lpass. The Non-seasonal Airline data.sas program also produces a non-seasonal version of the Airline Passenger data by generating a Monte Carlo data set that has the same trend and same standard error as that of the Airline Passenger data but without the seasonal variation in it. The variable is labeled lpassns. The data series is labeled “Non-seasonal Airline Passenger data”. These two data sets are reproduced below:

**Figure 1**

**Original Seasonal Time Series – Airline Passenger Data**

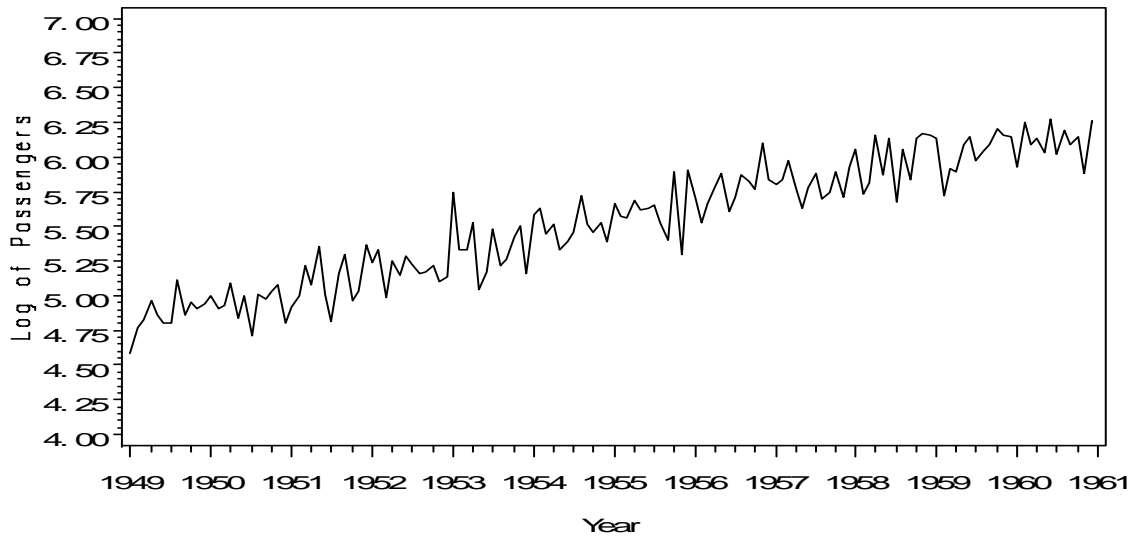
Log of Airline Passengers Jan. 1949 – Dec. 1960  
(Log of thousands of passengers)



**Figure 2**

**Non-seasonal Monte Carlo Version of the Airline Passenger Data**

Log of Non-Seasonal Airline Passengers Jan. 1949 – Dec. 1960  
(Log of thousands of passengers)



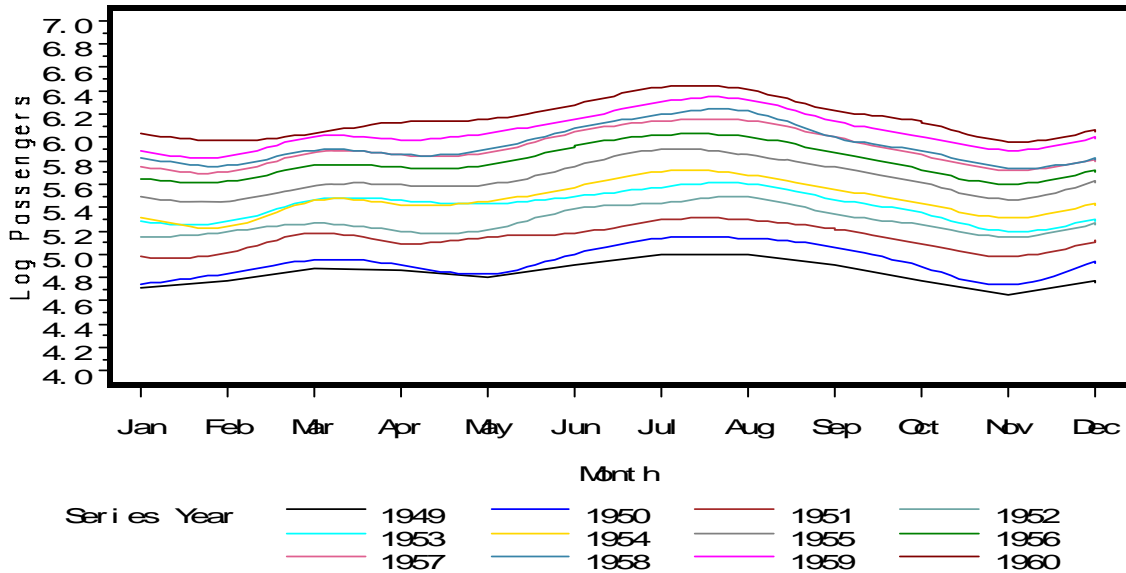
Below the competing Buys Ballot plots for these two time series are compared with each other.

**Buys Ballot "Seasons-By-Year" Plots**

**Figure 3**

**Log of Airline Passenger Volume 1949 to 1960**

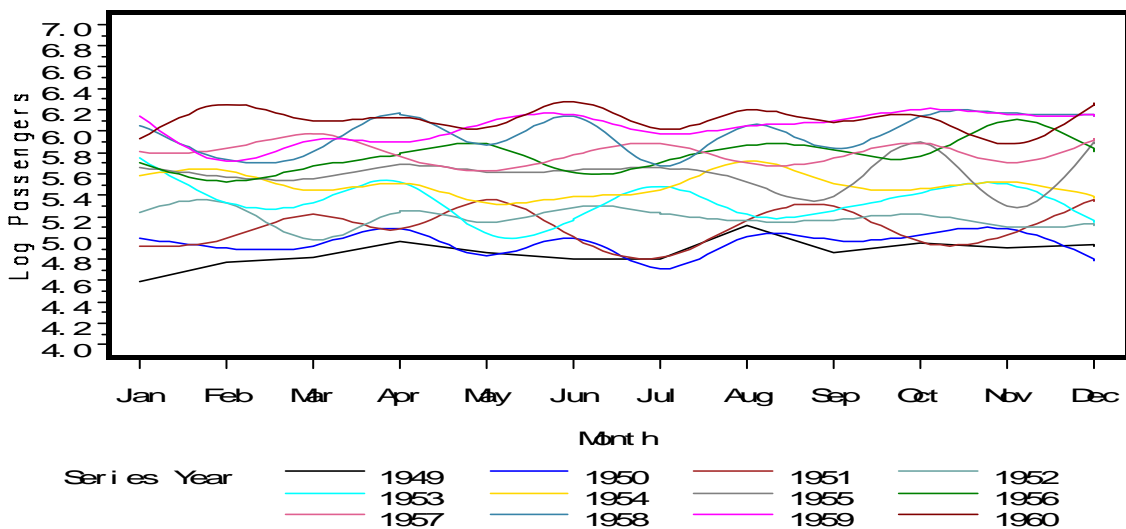
Buys Ballot "Seasons By Year" Plot



**Figure 4**

**Log of Non-seasonal Airline Passenger Volume 1949 to 1960**

Buys Ballot "Seasons By Year" Plot



In the seasonal series (Figure 3) there is a parallel appearance of the “seasons-by-year” graphs with repeating patterns (peaks, troughs) for given months over years emphasizing the distinctive differences in the data at the various seasons, apart from the means (levels) of the respective years (i.e.  $\bar{y}_{j\cdot}$ ). For example, July and August continue to be strong months, year over year, while May and November appear to be weak. Of course, given the trend in the data, each seasons-by-year graph tends to be higher year by year as time progresses. In the non-seasonal seasonal data (Figure 4), the seasons-by-year graphs are totally intertwined and seem to show no repeating patterns over years as in the seasonal data.

**Buys Ballot “Seasonal Trends” Plots**

**Figure 5**

**Log of Airline Passenger Volume 1949 to 1960**

Buys Ballot “Seasonal Trends” Plot

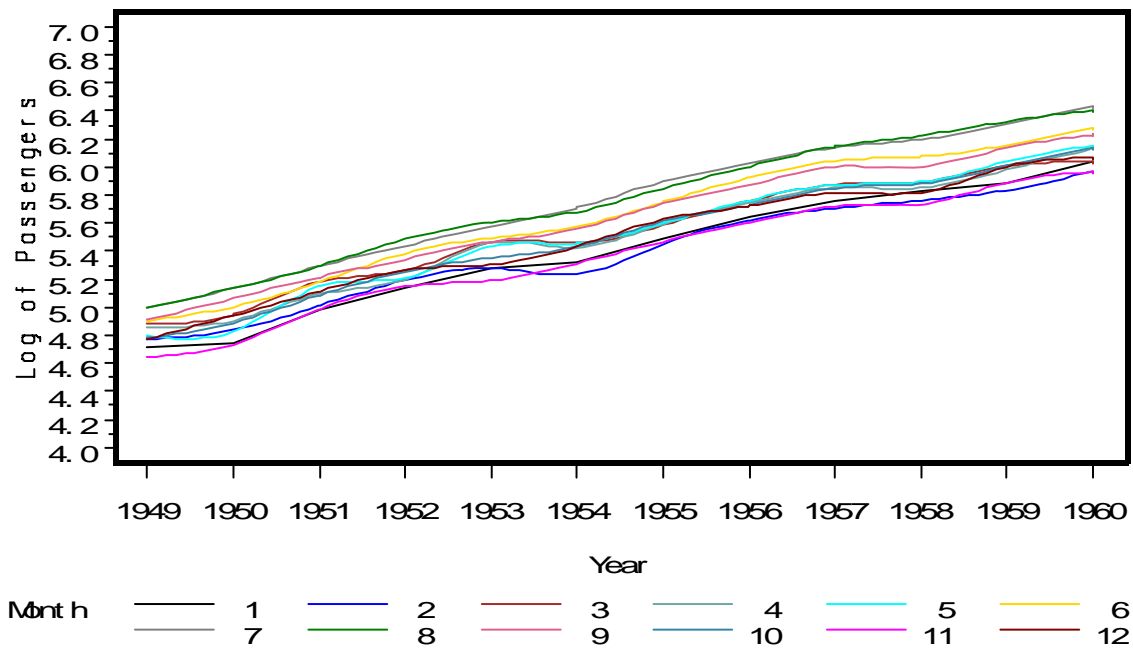
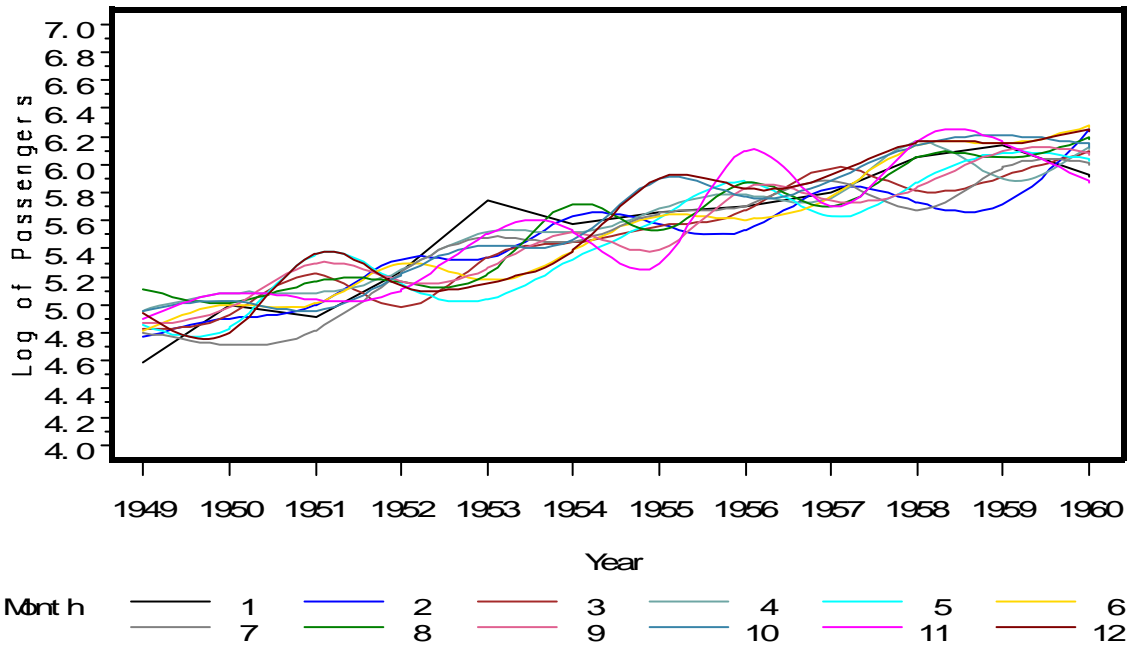


Figure 6

### Log of Non-Seasonal Airline Passenger Volume 1949 to 1960

Buy's Ball of "Seasonal Trends" Plot



In the seasonal series (Figure 5) there is a rough parallelism of the trends for each month of the year, the “seasonal trends”. That is, when one season’s trend graph is above another season’s trend graph it tends to remain so over the years with only a few, if any, instances of crossing. On the other hand, in the non-seasonal series there is no such dominance between the trends. They tend to cross each other back and forth a lot, indicating that no one season tends to dominate another in the non-seasonal data.

## Buys Ballot Seasonal Averages Plots

Figure 7

### Log of Airline Passenger Volume 1949 to 1960

Buy's Ballot "Seasonal Averages" Plot

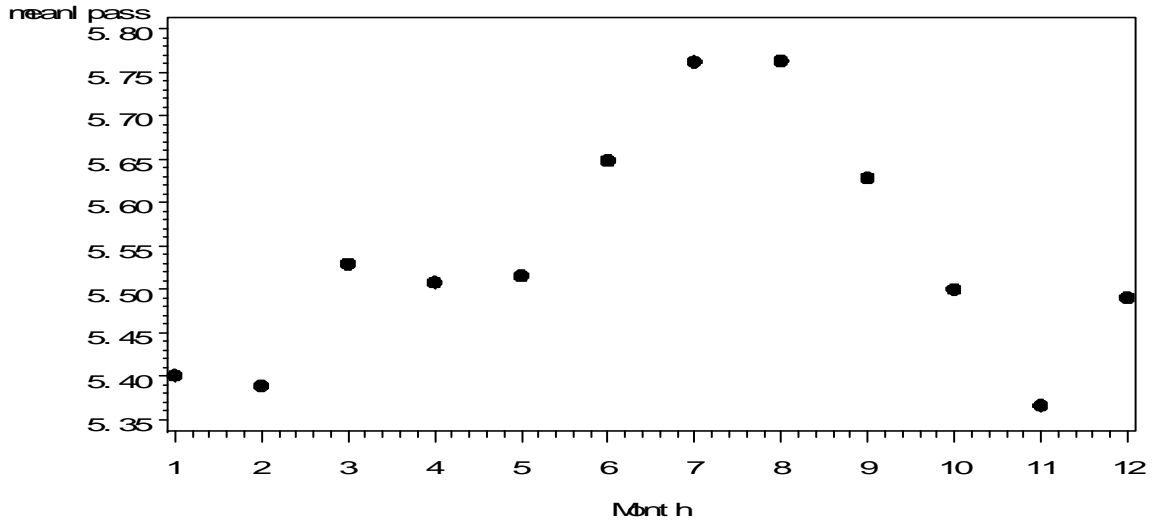
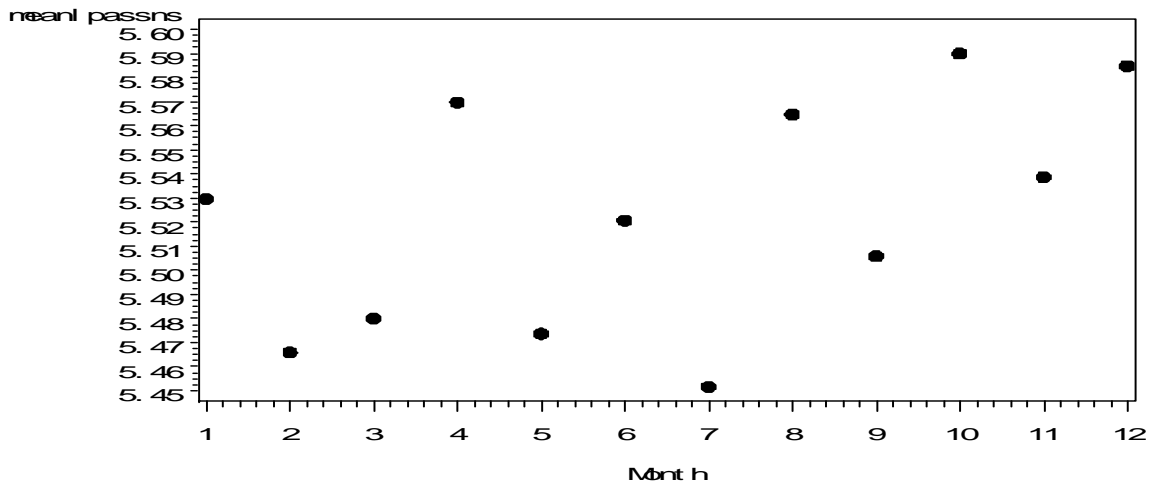


Figure 8

### Log of Non-Seasonal Airline Passenger Volume 1949 to 1960

Buy's Ballot "Seasonal Averages" Plot



In the non-seasonal "seasonal averages" plot (Figure 8) we can see that there is a definite trend in the seasonal means reflecting, in general, the overall trend in the data but nothing distinctive with respect to the seasons. In contrast, there is no such trend in the



seasonal data. The impact of the seasons on the locations of the averages is quite distinct and the seasonal effects are not being dominated by the overall trend in the data.

### **III. Conclusion**

Buy's Ballot plots are very useful for diagnosing the presence or absence of seasonal effects in time series. The three major Buy's Ballot plots are the "Seasons-By-Year" plot, "Seasonal Trends" plot, and "Seasonal Averages" plot. We have examined each of these plots for a seasonal time series and then for a companion non-seasonal time series and have found the plots to be very distinctive across the data sets. These distinctive patterns help us identify the presence seasonality in our time series so that it can be properly accounted for in our time series modeling process. Of course, these plots should be viewed as a **supplement to**, rather than a substitute for, more formal statistical tests such as the F-tests for trend and season in the DTDS model and the tests for seasonal components in the UC model.