Lecture 7

Two-Step Estimation Methods that are Asymptotically Equivalent to MLE

(Linearized Maximum Likelihood Procedures)
See A.C. Harvey EATS, Chapter 4 Section 5

For proof that linearized ML estimators are asymptotically equivalent to MLE see A.C. Harvey EATS, pp 140-141.

The iterative procedures of the previous discussion are of the form:

$$\underset{\sim}{\theta}_{(i+1)} = \underset{\sim}{\theta}_{(i)} - H_{(i)}^{-1}\; g_{(i)}$$

with $-H_{(i)}^{-1}$ being replaced by $I(\underset{\sim}{\theta}_{(i)})^{-1}$ in the case of the method of scoring, and the outer product matrix $(\hat{G}\hat{G}^{'})^{-1}$ in the case of BHHH.

Now the linearized ML estimator consists of "starting" the iteration at a *consistent* estimate of $\underset{\sim}{\theta}$, say $\tilde{\underset{\sim}{\theta}}$, and iterating once to obtain:

$$\tilde{\tilde{\underset{\sim}{\theta}}} = \tilde{\underset{\sim}{\theta}} - H^{-1}(\tilde{\underset{\sim}{\theta}})\, g(\tilde{\underset{\sim}{\theta}})$$

(or replacing $-H^{-1}$ with $I(\underset{\sim}{\theta})^{-1}$ or the outer product matrix with $\underset{\sim}{\theta} = \tilde{\underset{\sim}{\theta}}$).

Given that a consistent estimator "starts" all of these two-step methods, under the standard regularity conditions, they are asymptotically equivalent to MLE and thus are asymptotically efficient.