Augmented Dickey-Fuller Unit Root Tests

How do we know when to difference time series data to make it stationary? You use the Augmented Dickey-Fuller t-statistic.

Here are the **various cases** of the test equation:

a. When the time series is **flat** (i.e. doesn't have a trend) and **potentially slow-turning around zero**, use the following test equation:

$$\Delta z_t = \theta z_{t-1} + \alpha_1 \Delta z_{t-1} + \alpha_2 \Delta z_{t-2} + \cdots + \alpha_p \Delta z_{t-p} + a_t$$

where the number of augmenting lags (p) is determined by minimizing the Schwartz Bayesian information criterion or minimizing the Akaike information criterion or lags are dropped until the last lag is statistically significant. EVIEWS allows all of these options for you to choose from. Notice that this test equation does **not** have an intercept term or a time trend. What you want to use for your test is the t-statistic associated with the Ordinary least squares estimate of θ. This is called the **Dickey-Fuller t-statistic**. Unfortunately, the Dickey-Fuller t-statistic does not follow a standard t-distribution as the sampling distribution of this test statistic is skewed to the left with a long, left-hand-tail. EVIEWS will give you the correct critical values for the test, however. Notice that the test is **left-tailed**. The null hypothesis of the Augmented Dickey-Fuller t-test is

$H_0 : \theta = 0$ (i.e. **the data needs to be differenced to make it stationary**)

versus the alternative hypothesis of

$H_1 : \theta < 0$ (i.e. **the data is stationary and doesn't need to be differenced**)

b. When the time series is **flat and potentially slow-turning around a non-zero value**, use the following test equation:

$$\Delta z_t = \alpha_0 + \theta z_{t-1} + \alpha_1 \Delta z_{t-1} + \alpha_2 \Delta z_{t-2} + \cdots + \alpha_p \Delta z_{t-p} + a_t .$$

Notice that this equation has an intercept term in it but no time trend. Again, the number of augmenting lags (p) is determined by minimizing the Schwartz Bayesian information criterion or minimizing the Akaike information criterion or lags are dropped until the last lag is statistically significant. EVIEWS allows all of these options for you to choose from. You then use the t-statistic on the θ coefficient to test whether you need to difference the data to make it stationary or not. Notice the test is **left-tailed**.

The null hypothesis of the Augmented Dickey-Fuller t-test is

$H_0 : \theta = 0$ (i.e. **the data needs to be differenced to make it stationary**)

versus the alternative hypothesis of

$H_1 : \theta < 0$ (i.e. **the data is stationary and doesn't need to be differenced**)

c. When the time series **has a trend in it (either up or down) and is potentially slow-turning around a trend line you would draw through the data**, use the following test equation:

$$\Delta z_t = \alpha_0 + \theta z_{t-1} + \gamma t + \alpha_1 \Delta z_{t-1} + \alpha_2 \Delta z_{t-2} + \cdots + \alpha_p \Delta z_{t-p} + a_t \, .$$

Notice that this equation has an intercept term **and** a time trend. Again, the number of augmenting lags (p) is determined by minimizing the Schwartz Bayesian information criterion or minimizing the Akaike information criterion or lags are dropped until the last lag is statistically significant. EVIEWS allows all of these options for you to choose from. You then use the t-statistic on the θ coefficient to test whether you need to difference the data to make it stationary or you need to put a time trend in your regression model to correct for the variables deterministic trend. Notice the test is left-tailed.

The null hypothesis of the Augmented Dickey-Fuller t-test is

$H_0 : \theta = 0$ (i.e. **the data needs to be differenced to make it stationary**)

versus the alternative hypothesis of

$H_1 : \theta < 0$ (i.e. **the data is trend stationary and needs to be analyzed by means of using a time trend in the regression model instead of differencing the data**)

Just a note of caution: Sometimes if you have data that is **exponentially trending** then you might need to take the log of the data first before differencing it. In this case in your Dickey-Fuller unit root tests you will need to take the differences of the log of the series rather than just the differences of the series.