Endogenous mortality, human capital and economic growth

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Abstract

We consider growth and welfare effects of lifetime-uncertainty in an economy with human capital-led endogenous growth. We argue that lifetime uncertainty reduces private incentives to invest in both physical and human capital. Using an overlapping generations framework with finite-lived households we analyze the relevance of government expenditure on health and education to counter such growth-reducing forces. We focus on three different models that differ with respect to the mode of financing of education: (i) both private and public spending, (ii) only public spending, and (iii) only private spending. Results show that models (i) and (iii) outperform model (ii) with respect to long-term growth rates of per capita income, welfare levels and other important macroeconomic indicators. Theoretical predictions of model rankings for these macroeconomic indicators are also supported by observed stylized facts.
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1. Introduction

Countries differ dramatically in the way they finance their educational systems. Education can be provided through public funds, through private funds, or a combination of the two. As reported in the Education at a Glance (OECD, 2003), a number of countries in 2000 had public education shares close to 100%, such as Norway (98.7%), Turkey and Portugal (98.6% each), Finland (98%) and Sweden (97%). Of the 36 OECD and non-OECD countries covered in this study, 19 countries (53%) financed at least 90% of their overall educational expenditures through public spending in 2000. In contrast, a number of countries put a larger responsibility on the private provision of education. Among those, Chile has gone the furthest with a private education share of more than 46%. Other countries in the above dataset with large private education shares include South Korea (41%), Indonesia (36%), Jamaica (35%), and the United States (32%).

Overall investment in education, through its effect on the stock of human capital, has long been identified as a source for long-run growth in an economy (see, for example, Rebelo, 1991). In this broad category, a number of studies have specifically stressed the importance of public investment in education in further enhancing the growth performance of the economy (Glomm and Ravikmar, 1992, 1997; Boldrin, 1992; Benabou, 1996). Given the trade-off between public and private financing in education, the question arises how different degrees of public (versus private) involvement in the production of human capital affect welfare, long-term growth, and other indicators of economic performance.

Furthermore, budget components like public education spending must compete with other budget items such as internal and external security, infrastructure expenditures, and debt servicing, to name a few. One of the fastest growing budget components of many countries is public health expenditure. World Bank (2001) data show that between 1972 and 1999 the share of public health expenditure in total public spending has increased two-fold or more in many countries. Over the 1972–1999 period, the average annual growth rate of the public health expenditure share was 1.7%.[2] Just like the market for education, the provision of health services can also be linked to a number of positive externalities, which in turn explain the large public involvement in the health sector. One such externality that has not been sufficiently recognized in the literature is the impact of public health spending on longevity: Increased levels of public health expenditures are most likely to be positively associated with higher life expectancy (Lichtenberg, 2002). In addition to the individual benefits of living longer lives, increased life expectancy may confer important growth effects. These effects arise since increased longevity produces stronger incentives to invest in physical and human capital as these long-term assets yield high returns only later in life (Chakraborty, 2004). Importantly, since public spending on education competes with public spending on health, a second trade-off exists that, like the first one between public and private education shares, may matter for the long-term growth rate of the economy as well as its level of welfare.

In this paper, we study the trade-off between public and private spending on education in a model with uncertain lifetimes and endogenous growth. To this end, we construct a

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[2] This average growth rate is based on data from 26 countries. 10 countries have been excluded due to missing or unreliable time series data on public health care expenditures. Countries with the fastest growing public health expenditure share are Uruguay (from 1% to 6%), Israel (from 4% to 13.7%), the US (from 8.6% to 20.5%), Paraguay (from 3.5% to 7.3%), Australia (from 7% to 14.8%), and Thailand (from 3.7% to 7.4%).

three-period overlapping generations model in which survival of an individual in the third period is uncertain. Thus, like Chakraborty (2004), we model adult mortality rather than infant or child mortality. The probability of survival depends on her own health as well public health spending. To the extent that good (poor) health and consequent higher (lower) longevity generates (dis)incentive for private accumulation of human capital, public health expenditure plays an important role in generation of human capital, thereby affecting long-run growth. However, the more the government spends on health, the less it can spend on public education, adversely affecting future human capital. Since human capital accumulation is the engine of growth in this model, differences between public spending on health and education on one side and public and private spending on education on the other constitute the two fundamental trade-offs in our model that generate important growth and welfare consequences.

This paper connects two different strands of the growth literature. One of them focuses on government spending on education assuming certainty with regard to the length of the life of each individual (Glomm and Ravikmar, 1992, 1997; Boldrin, 1992; Benabou, 1996; de la Croix and Doepke, 2004). Glomm and Ravikmar (1992) and de la Croix and Doepke (2004) focus on the effect of private and public provision of education on long-term economic growth and income inequality, while Glomm and Ravikmar (1997) discuss different forms of productive public expenditures and their effects on long-run growth.[3] Boldrin (1992) and Benabou (1996), on the other hand, focus on endogenous determination of public policies in a political economy setting. The other, more recent strand of the literature, deals with uncertain lifetimes in models with endogenous growth mechanism but abstracts from government spending on education. Chakraborty (2004) treats mortality as endogenous and argues that a decline in adult mortality has a multiplier effect on growth, generating either a 'poverty trap' or a 'stagnation to growth' dynamics with endogenous growth. Blackburn and Cipriani (2002) use a discontinuous (step-function) endogenous survival function to explain the existence of multiple development regimes in which the survival of an agent into old age depends upon her inherited level of human capital. In Birchenall (2004) higher consumption growth and increased public health expenditure enable an economy to escape a high child-mortality Malthusian equilibrium by reducing child mortality from infectious diseases. In Kalemli-Ozcan et al. (2000) a lower mortality raises human capital investment and strengthens long-run growth because the return from such investment are typically earned over a longer time horizon. The effect of a change in adult mortality, however, is analyzed as a comparative statics exercise.

The objective of the paper is to examine the relative macroeconomic performance of alternative educational funding strategies when public spending on education and health gives rise to budgetary tensions. For this reason, we analyze three distinct model scenarios. We begin with the analysis of a model with both public and private education expenditures (the benchmark model) followed by two alternative specifications: One with only public investment in education (the public education model) and one with exclusive private provision of education (the private education model). Except for the differences in funding education, all models share identical taste, technology, and policy (tax) parameters.

Our main results are as follows. First, longevity is highest in the private education model, followed by the benchmark model. Second, with regard to long-run growth, interest rate,

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[3] Interestingly, public expenditure on health is *not* part of their analysis.

and human-to-physical capital ratio, the benchmark and private education models generate similar values, all of which are higher than the corresponding values in the public education model. Third, with respect to welfare, the private education model ranks highest, followed by the benchmark and the public education models. Fourth, we compare the benchmark to the public education model for an optimally chosen tax rate (in a second-best sense). We show that the welfare ranking of two regimes depends on the (exogenous) relative size of government spending on education and health. For high levels of public spending on education relative to health, the public education model is welfare superior, while the reverse is true for low public education spending. Finally, we show that the observed stylized facts support the theoretical model rankings for several macroeconomic indicators.

The conclusions drawn from our analysis add to our understanding of the link between longevity, growth, and welfare. The better performance of the benchmark and private education *vis-à-vis* the pure public education model has an intuitive explanation. In a world with limited government resources, a country is better off if the government can concentrate its scarce resources on fewer budget items (here: full health coverage, but limited role in education) instead of spreading itself thin on too many budgetary needs (here: full provision of both health and public education). Note that our simulation results reveal that the macroeconomic performance ranking of the public education model can be reversed, but only for very high levels of taxation that exceed those found in most countries in our sample.

The paper unfolds as follows: Section 2 lays down the basic model framework and the characterization of the competitive equilibrium. The three models are analyzed in Sections 3–5, respectively. Section 6 provides a numerical comparison of the three models including welfare analysis when tax policy is endogenous. Section 7 concludes.

2. The common framework

2.1. Individuals

We consider an overlapping generations economy populated by individuals who potentially live for three periods, "childhood", "youth" and "old age". As children, individuals attend school and accumulate human capital while remaining economically inactive until they reach youth. Lifetime of an individual is uncertain. Survival during the first two periods of life is a certain event, but whether or not a youth will survive into old age depends on a positive probability ϕ, which depends on her own accumulated health stock. An individual gives birth to a single offspring and takes decisions regarding saving, expenditure on child's education before the mortality shock is realized at the end of her youth. If she survives into old age, she derives 'satisfaction' from her now-young offspring's educational attainment, which can be thought of as the pleasure derived from improved social status that her educated descendant bestows. It may also represent the satisfaction from mental security in her old age of having an educated caregiver and companion.

Assuming logarithmic felicity functions for simplicity, a young parent's expected lifetime utility at time t is given by

$$U_t = \log(c_t) + \phi_{t+1}[\log(c_{t+1}) + \beta \log(h_{t+1})] \tag{1}$$

where c_t and c_{t+1} are levels of current and future consumption, respectively, ϕ_{t+1} is the old age survival probability, h_{t+1} is child's human capital stock and parameter $\beta > 0$ represents

the weight the parent attaches to the utility derived from her child's youthful human capital. Child's consumption is subsumed in parental consumption.[4]

At time $t = 0$, there is an initial generation of old agents with health stock x_0, human capital stock h_0, and physical capital stock k_0. At any period in t, the health and human capital stocks of a young adult are given by x_t and h_t, respectively. Individuals in each generation are identical, and the size of each generation is normalized to unity. Each of them is endowed with a unit of labor when young. During the first period of their lifetimes, individuals accumulate human capital, in the second period they supply their labor inelastically, and in the third period, they retire. The post-tax earning of a young individual is $(1 - \tau)w_t h_t$, which is spent on first period consumption, c_t, saving, s_t and child's education, e_t. Since there is no wage income in old age, all individuals save in order to finance old age consumption, c_{t+1}. Thus, today's savings by the young constitute tomorrows stock of physical capital, i.e. $k_{t+1} = s_t$. We assume that each agent deposits her savings with a mutual fund that guarantees a gross return of $\widehat{R}_{t+1}$ to the surviving old.[5] If a fund earns a gross return R_{t+1} on its investment, then perfect competition would ensure that $\widehat{R}_{t+1} = R_{t+1}/\phi_{t+1}$. Therefore, for a young individual, the budget constraints at time t are given by

$$c_t \leqslant (1 - \tau)w_t h_t - s_t - e_t \tag{2a}$$

$$c_{t+1} = \widehat{R}_{t+1}s_t, \quad \text{where } \widehat{R}_{t+1} = R_{t+1}/\phi_{t+1} \tag{2b}$$

where $w_t h_t$ is the gross wage earning and $0 \leqslant \tau \leqslant 1$ is the income tax rate.

2.2. Government

The government finances both the provision of public health and public education. To generate funds, wage income is taxed at a uniform rate. A balanced budget condition is assumed to be fulfilled in each period, so that the budget constraint of the government is given by

$$\text{Total Revenue} = \tau w_t h_t = \underbrace{\gamma_h \tau w_t h_t}_{g_t^h} + \underbrace{(1 - \gamma_h)\tau w_t h_t}_{g_t^x} = \text{Total Expenditure} \tag{3}$$

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[4] Note that Eq. (1) incorporates the warm glow formalization of imperfect altruism instead of the more familiar Barro–Becker type of perfect altruism. We do so for several reasons. First, there is no settled model of the family, as far as the assumption about altruism is concerned (Banerjee, 2002). Second, the Barro–Becker preference reflects a dynastic model that reduces to a single-agent infinite horizon model. Whether an extended family can be modeled as a single representative consumer with an infinite horizon seems highly questionable, especially when mortality of agents is introduced explicitly. Third, detailed micro econometric studies by Altonji et al. (1992, 1997) and many others do not support the Barro–Becker type altruism. However, there exists evidence of impure altruism in Cox (1987) and Cox and Rank (1992). Furthermore, Altonji et al. (1992) suggest that altruistically motivated transfers observed in the US (and many other countries) may materialize in the form of, among other things, less than fully efficient educational support from parents to the liquidity-constrained children – a situation that forms the essence of our analysis.

[5] An alternative assumption is one where the government takes over the assets of generation-t agents who do not survive and transfers them lump-sum to those alive. This gives qualitatively similar results as long as the transfers are made to the surviving members of the same cohort. See Chakraborty (2004) for a justification of this assumption.

where γ_h and $(1 - \gamma_h)$ are the fixed shares of tax revenue spent on human capital and health, respectively. Public investment in health (g_t^x) and education (g_t^h) are perfect substitutes in this model from a budgetary point of view.

2.3. Production of output

As in the standard neoclassical model, the production process utilizes a constant returns-to-scale, time invariant technology utilizing physical capital and effective labor units. The output produced at time t, y_t, is governed by

$$y_t = Ak_t^\alpha (h_t)^{1-\alpha}, \quad \alpha \in (0,1) \tag{4}$$

Perfect competition in the factor markets ensures that each factor is paid its marginal product so that at time t (assuming full depreciation of physical capital during the length of one period), the wage rate and the rental rate, respectively, are given by

$$w_t h_t = A(1 - \alpha)k_t^\alpha (h_t)^{1-\alpha} \tag{5a}$$

$$R_t k_t = A\alpha k_t^\alpha (h_t)^{1-\alpha} \tag{5b}$$

2.4. Health

The probability of survival at the beginning of the third period, ϕ_{t+1} depends on an old person's health stock, x_{t+1} at that time. In particular, it is given by a non-decreasing concave function

$$\phi_{t+1} = \phi(x_{t+1}), \quad \phi' > 0, \quad \phi'' < 0, \quad \phi(0) = 0, \quad \lim_{x \to \infty} \phi(x) = 1 \tag{6}$$

To focus on the longevity effect on human capital-led growth, health stock of an individual is assumed to affect only her survival in old age, but not her youthful labor productivity. We further assume that her health stock depends not on her direct private health spending, but on the level of public spending on health. Specifically, health stock of a young individual at the end of her youth, x_{t+1}, depends positively on her own young age health stock, x_t, and the government investment in health, g_t^x, as a proportion of the total capital stock k_t. Note that government investment in health improves health of individuals only with a lag of one period. Health is also assumed to depreciate at a constant rate $\delta_x \in (0,1)$. The health stock of an old is given by

$$x_{t+1} = M(x_t)^{1-\psi}(g_t^x/k_t)^\psi - \delta_x x_t \tag{7}$$

where M represents the productivity of health production, $1 - \psi$ measures the degree to which an individual's own intrinsic state of health x_t matters in determining her final health status and ψ measures the strength or effectiveness of the public health system on the future health of the young.[6]

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[6] Note that the normalization of g_t^x by k_t, and the resulting finiteness of the steady state value of health (x^*) is necessary for the existence of a balanced (rather than an unbalanced) growth path equilibrium. In addition, a finite x^* also guarantees that the steady state value of ϕ is different from unity for all three education regimes, a distinction that is crucial for meaningful differences in welfare and macroeconomic performances across regimes. The choice of normalizing g_t^x by k_t was also used in Blackburn and Cipriani (1998) to capture the idea that greater public spending on health is an essential means of mitigating the potential adverse effects on longevity of greater economic activity as proxied by the capital stock.

Table 1
Life expectancy and health shares

	Obs.	Mean	SD	Min.	Max.
Life expectancy index	175	0.67	0.21	0.14	0.94
Share of pub. exp. on health (% of GDP)	173	3.37	1.78	0.30	8.10
Share of pvt. exp. on health (% of GDP)	173	2.17	1.41	0.10	8.50

Source: Human Development Report (2003); UNDP.

The assumption that health stock does not depend on direct private spending on health is supported by available empirical evidence. In a study of sources of US longevity during 1960–1997, Lichtenberg (2002) finds strong evidence that public, rather than private health expenditure positively affects longevity. Blackburn and Cipriani (2002) argue that the probability of life-extension could be driven by public spending on health. Our own empirical analysis supports this conclusion. Table 1 reports the summary statistics for the Life Expectancy Index (LEI) as constructed by UNDP (2003) and shares of private and public expenditures on health as a percentage of GDP. Figs. 1 and 2 show the relationship between LEI and private and public spending on health as a percentage of GDP, respectively, for 173 countries. The share of public spending on health has a significant positive effect, whereas the share of private spending on health has no significant effect on LEI.[7] The results are very similar if, instead of LEI, we use "Probability of survival beyond age 40" as the dependent variable. Also, if the health of an economy is measured by the number of doctors per 100,000 population, empirical evidence shows that such a measure of health is positively (and significantly) associated with share of public health expenditure, while its association with the share of private health expenditure is statistically not different from zero.[8] The specific channels through which public health expenditure improves health and raises longevity include the provision of better health infrastructure such as clean air, water and sanitation, as well as immunization campaigns and dissemination of knowledge regarding protection against fatal diseases (see also Lichtenberg, 2002; Blackburn and Cipriani, 2002).

2.5. Human capital

The stock of human capital of a young adult of generation t, in period $t + 1$, h_{t+1}, depends on three factors. First, it depends on the stock of human capital of her parent, h_t. This assumption captures the intergenerational transmission of knowledge (e.g. education or skill) between parents and their children. Next, h_{t+1} depends on the level of edu-

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[7] Incidentally, these results do not contradict the findings of the World Bank's (1993) World Development Report that total investment in health is unrelated to longevity (p. 53). If we were to used total health expenditure as the 1993 report does, we too would not find a strong positive relationship between spending on health and life expectancy.

[8] Regression results using the HDR dataset (UNDP, 2003) show that, while controlling for population size and an index of education, a one percent increase in share of public health expenditure (in GDP) raises the number of physicians (per 100,000 population) by 0.73%, while the share of private health expenditure (in GDP) does not seem to have any effect on the number of physicians. The coefficient of the share of public health expenditure is statistically significant (t-stat = 3.86), while that of the share of private health expenditure is negative and statistically insignificant (t-stat = −1.33). Detailed results are available upon request.
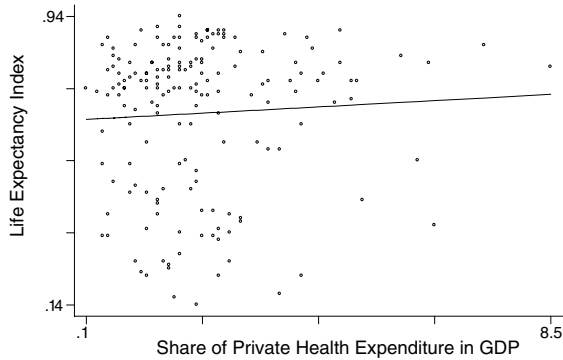
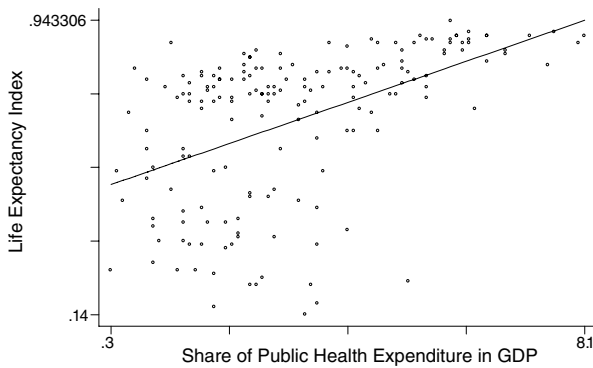Fig. 1. Life expectancy index and private health expenditure in GDP.

Fig. 2. Life expectancy in index and public health expenditure in GDP.

cational expenditure undertaken by her parent at time t, e_t. Note that since individuals receive direct utility from the level of human capital (not health) of their descendants, parents consciously spend on their children's schooling whenever they see the need to raise the level of future human capital. Finally, h_{t+1} depends on the level of government investment in public education in the previous period g_t^h (e.g. education infrastructure). Unlike health, education is financed both by private individuals as well as the government, and the two are substitutes (Buiter and Keltzer, 1995). The evolution process of human capital is thus as follows:

$$h_{t+1} = B(h_t)^{1-\theta}(\mu e_t + v g_t^h)^{\theta}, \quad B > 0, \ \theta \in (0,1), \ \mu, v > 0 \tag{8}$$

where B is a parameter measuring the productivity of the human capital accumulation process. Private and public spending carries weights μ and v, respectively, characterizing the relative strength of private versus public education systems. The parameter θ denotes the elasticity of real investment in education (private and public), while $1 - \theta$ captures the degree of intergenerational transmission of human capital. Since human capital is not needed in old age (the old agents do not work), individuals do not care about their own human capital and instead invest in their child's education.

A young individual maximizes (1) subject to the constraints (2a), (2b) and equations of motions (7)–(8) and the non-negativity constraints $s_t > 0$, and $e_t \geqslant 0$, taking as given the vector of prices $(w_t, \widehat{R}_{t+1})$ and the survival probability ϕ_{t+1}.

2.6. Market equilibrium

Given the initial k_0, x_0 and h_0, a competitive equilibrium for the model described in the previous section is a sequence of allocations for aggregate production $\{k_t, h_t, x_t\}_{t=0}^{\infty}$, a set of prices $\{w_t, \widehat{R}_{t+1}\}_{t=0}^{\infty}$ and a set of choices for the representative agent $\{c_t, c_{t+1}, e_t\}_{t=0}^{\infty}$ that solve the young agent's problem (1), the representative firm's profits such that conditions (5a), (5b), and conditions (3), (7) and (8) are always satisfied, and product market clears, i.e. $s_t = k_{t+1}$.

A balanced growth path $\{k_t, h_t, x_t, c_t, c_{t+1}\}_{t=0}^{\infty}$ is a competitive equilibrium such that $\{k_t, h_t, c_t, c_{t+1}\}_{t=0}^{\infty}$ grow at a constant rate η and $\{x_t\}_{t=0}^{\infty}$ stays constant. Throughout our analysis, we assume that the balanced growth path is interior ($\eta > 0$), a rather mild requirement.

3. The benchmark model

3.1. Model specifications ($\mu > 0$, $v > 0$)

A young individual (parent) in the benchmark model chooses the optimal consumption and education expenditure levels given her budget constraints. Therefore the problem facing a young adult at time t is one of maximizing Eq. (1) by choosing s_t and e_t subject to Eqs. (2a) and (2b). Assuming interior solutions and omitting the generation superscripts, optimization on the part of the young agent of generation-t implies

$$s_t = \left[\frac{\phi_{t+1}[(1-\tau)\mu + v\gamma_h\tau]}{\mu[1 + (1+\theta\beta)\phi_{t+1}]}\right] w_t h_t \tag{9}$$

and

$$e_t = \left[\frac{\mu\theta\beta\phi_{t+1}(1-\tau) - v\gamma_h\tau(1+\phi_{t+1})}{\mu[1 + (1+\theta\beta)\phi_{t+1}]}\right] w_t h_t \tag{10}$$

where $\phi_{t+1} = \phi(x_{t+1})$. The equilibrium saving function (Eq. (9)) has the following properties: $s_\phi > 0$, $s_w > 0$, $s_h > 0$, $s_\mu < 0$, $s_\theta < 0$, $s_\beta < 0$, $s_\tau \lessgtr 0$. Equilibrium saving is increasing in the probability of survival (ϕ_{t+1}) since the higher expected length of life raises the return from savings and hence increases the incentive to save. Interestingly, the impact of tax rate on saving is ambiguous. The ambiguity arises from two opposing forces that an increase in tax rate generates. A rise in τ raises (i) health and longevity of parents as well as (ii) public spending on education, while lowering the disposable income for saving and private educational spending. If the positive effects (i) and (ii) dominate the negative income effect, an increase in tax rate would raise private saving.

Eq. (10) denotes the equilibrium private expenditure on education (for the next generation-$t + 1$) and is characterized by: $e_\phi > 0$, $e_w > 0$, $e_h > 0$, $e_\mu > 0$, $e_\theta > 0$, $e_\beta > 0$, $e_\tau \lessgtr 0$. As expected, a higher survival probability, wage income, productivity of educational spending and the degree of altruism, all raise parent's investment in child's education, while the tax rate on income has an ambiguous effect on e_t. The last result stems, once again, from the

dual impact that a change in tax rate has on parental decision on educating her child. First, an increase in tax rate reduces disposable income, hence e_t falls. On the other hand, as a result of the increased tax rate parental longevity will be higher inducing them to spend more on child's education. The final effect will depend on whether the income effect is stronger than the incentive effect.

It is evident from Eq. (10) that the model gives rise to possibility of a corner solution ($e_t = 0$), where low survival probability eliminates any incentive for the parents to invest in child's education. Such a situation arises if $\phi_{t+1} \leqslant \frac{v\gamma_h\tau}{\theta\beta(1-\tau)-v\gamma_h\tau} \equiv \phi$, i.e. when mortality is higher than a threshold level, ϕ, which is determined by, among other things, the tax revenue spent on public education. The higher this amount, the lower is public health spending and the higher is the mortality rate. An interior solution is ensured by ensuring that the tax revenue allotted to public health is not too low. In the subsequent analysis we assume that this condition is satisfied.[9]

Let us define a new variable z_t as a ratio of two stocks h_t and k_t:

$$z_t \equiv \frac{h_t}{k_t} \quad \forall t > 0 \tag{11}$$

The ratio z_t can be interpreted as the inverse of the capital-labor ratio with labor being measured in units of human capital. A steady state of the model is achieved when the ratio z_t becomes stationary, i.e. when $z_t = z^*$. Since there is no perpetual growth in health, the steady state health stock x^* is given by

$$x^* = \left(\frac{M}{1+\delta_x}\right)^{1/\psi} (1-\gamma_h)\tau A(1-\alpha)(z^*)^{(1-\alpha)} \tag{12}$$

Using the factor prices given in Eqs. (5a) and (5b), the market clearing condition, $s_t = k_{t+1}$, the allocation of government tax revenue (Eq. (3)), the saving function (Eq. (9)) and the steady state health stock (Eq. (12)), the BGP growth rates for physical and human capital stocks are given by, respectively,

$$\eta_k = \frac{k_{t+1}}{k_t} = \left[\frac{\phi(x^*)[(1-\tau)\mu - v\gamma_h\tau]}{\mu[1+(1+\theta\beta)\phi(x^*)]}\right] A(1-\alpha)(z^*)^{1-\alpha} \tag{13}$$

and

$$\eta_h = \frac{h_{t+1}}{h_t} = B \left[\frac{\phi(x^*)[\mu(1-\tau)+v\gamma_h\tau]}{1+(1+\theta\beta)\phi(x^*)}\right]^\theta (\theta\beta A(1-\alpha))^\theta (z^*)^{-\alpha\theta} \tag{14}$$

In turn, Eqs. (13) and (14) implicitly determine z^*

$$[B\mu(\theta\beta)^\theta]^{\frac{1}{1-\theta}} - \left[\frac{\phi(x^*)[(1-\tau)\mu+v\gamma_h\tau]}{1+(1+\theta\beta)\phi(x^*)}\right] A(1-\alpha)(z^*)^{\frac{1}{1-\theta}-\alpha} = 0 \tag{15}$$

where x^* is given by Eq. (12). Using Eq. (15), the balanced growth rate of physical and human capital can be written as

$$\eta^* = Q(z^*)^{\frac{-\theta}{1-\theta}}, \quad \text{where } Q = B^{\frac{1}{1-\theta}}(\mu\theta\beta)^{\frac{\theta}{1-\theta}} \tag{16}$$

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[9] In the numerical simulations, the parameters are chosen so as to satisfy $e_t > 0$.

3.2. Analysis of the balanced growth path

The difference equations governing the evolution of z_t are given by

$$z_{t+1} - z_t = f(z_t, x_{t+1}) = [B\mu(\theta\beta)^\theta]^{\frac{1}{1-\theta}} - \left[\frac{\phi(x_{t+1})[(1-\tau)\mu + v\gamma_h\tau]}{1 + (1+\theta\beta)\phi(x_{t+1})}\right] A(1-\alpha)z_t^{\frac{1}{1-\theta}-\alpha} \quad (17)$$

and

$$x_{t+1} = M(x_t)^{1-\psi}[(1-\gamma_h)\tau A(1-\alpha)z_t^{1-\alpha}]^\psi - \delta_x x_t \quad (18)$$

The last equation is the general equilibrium version of Eq. (7). Suppose a solution to the system of difference equations given by Eqs. (17) and (18) exists and is given by $z_t = z^*$, and $x_t = x^*$. While it is not possible to demonstrate local stability or uniqueness of the steady state equilibrium in general, numerical simulations based on a parameterized version of the model (to be discussed below) reveal that the parameterized BGP equilibrium is indeed locally stable as well as unique.

Next, we investigate the BGP relationship between the balanced growth rate η^* (as given by Eq. (16)) and z^*. Differentiating Eq. (16) with respect to z^* yields

$$\frac{\partial\eta^*}{\partial z^*} = Q\left(\frac{-\theta}{1-\theta}\right)(z^*)^{\frac{-1}{1-\theta}} < 0$$

The negative sign of the above partial implies that z^* unambiguously lowers balanced growth, a result that reflects the stronger diminishing returns to human capital accumulation relative to physical capital accumulation in our model.

3.3. Comparative statics for the benchmark model

Having analyzed some of the properties of the BGP equilibrium, we now address the issue of comparative statics along the BGP. While most of the comparative-static result are analytically ascertained, some are derived numerically (see Table 2).[10] We start with changes in z^*, from which we derive changes in η^* and x^*. In particular, we find that $dz/dj > 0$, for $j = \psi, \beta, B, \delta_x, \theta, \mu, \tau$ and $dz/dj < 0$ for $j = M, A, v, \gamma_h$. $dz/d\alpha > 0$ for $z^* > 1$ and ambiguous otherwise.[11]

Eq. (12) shows that certain parameters such as γ_h, A, α, ψ, δ_x and M have a direct impact on x^*, in addition to having an indirect effect through z^*. Given that, along the BGP, health and human capital formation are 'competing' with each other at the margin, changes in these parameters will cause opposing changes in x^* and z^*. In contrast, changes in the remaining parameters ($B, \beta, \theta, \tau, \mu, v$) cause z^* and x^* to change in the same direction. Eq. (16) reveals an inverse equilibrium relationship between z^* and η^*. Therefore, we would expect that most parameter changes that change z^* in one direction would change η^* in the other. This is indeed the case, except for changes in B, θ, and β which, in addition to their indirect impact through changes in z^*, affect η^* directly (see Eq. (16)). The results are presented in Table 2.

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[10] In our numerical exercise we use the functional specification $\phi(x) = x/(1 + x)$, This specific form has been chosen so that the 'survival probability' satisfies the properties described in Eq. (6). For a discussion of the parameter values used to derive the numerical results, see Section 6.1.

[11] The analytical derivatives are available upon request.

Table 2
Comparative statics for the benchmark model

Parameters related to	Health			Human capital					Output		Utility	Taxation
Parameters	M	ψ	δ_x	B	γ_h	θ	μ	v	A	α	β	τ
Effects on x^* (or ϕ)	+	−	−	+	−	−	+	−	+	−	+	+
Effects on z^*	−	+	+	+	$+^\S$	$-^\S$	+	−	−	+	+	$+^\S$
Effects on η^*	+	−	−	+	$-^\S$	$-^\S$	+	+	+	−	+	$-^\S$

Notes: The signs with the superscript § are obtained using numerical simulations (see Section 6.1 for calibration details) and reflect local responses in the neighborhood of the benchmark equilibrium.

Of special interest are the comparative-static signs of two policy parameters: the tax rate τ and the education share of government expenditures γ_h. Table 2 reveals that an increase in either has negative effects on growth and health. However, that result is derived numerically and is only valid in the neighborhood of the benchmark equilibrium. As we discuss in Section 6.2, the global properties of changes in τ and γ_h on growth and health are actually non-monotonic.

4. The public education model

4.1. Model specifications ($\mu = 0$, $v > 0$)

In this model the human capital accumulation is driven only by public spending on education and therefore there is no role for private spending on education in this model. Young individuals (parents) maximize their intertemporal utility only by choosing the optimal levels of consumption. In particular, a representative young agent maximizes (1) with respect to s_t, subject to the following budget constraints:

$$c_t \leqslant (1 - \tau)w_t h_t - s_t \tag{7a'}$$

$$c_{t+1} = \widehat{R}_{t+1} s_t, \quad \text{where } \widehat{R}_{t+1} = R_{t+1}/\phi_{t+1} \tag{7b'}$$

In this model, the only choice variable is saving, which, given the survival probability, ϕ_{t+1}, and interiority of solution, is chosen optimally as

$$s_t = \left(\frac{\phi_{t+1}}{1 + \phi_{t+1}} \right)(1 - \tau)w_t h_t, \quad \text{where } \phi_{t+1} = \phi(x_{t+1}) \tag{19}$$

The equilibrium saving function given by Eq. (19) has the following properties: $s_\phi > 0$, $s_w > 0$, $s_h > 0$, $s_\tau < 0$. Thus saving increases as individuals' perceived probability of survival rises. As expected, saving increases with wages and decreases with income tax. Note that in this model, the saving function is identical to the one used in Chakraborty (2004). As in the previous model, the steady state health stock is given by Eq. (12).

Using the factor prices, the market clearing condition, the saving function (Eq. (19)) the steady state health stock (Eq. (12)), and noting that $g_t^h = \gamma_h \tau w_t h_t = \gamma_h \tau A(1 - \alpha)k_t^\alpha h_t^{(1-\alpha)}$, the BGP growth rates for physical and human capital stocks are given by, respectively,

Table 3
Comparative statics for the public education model

Parameters related to	Health			Human capital				Output		Utility	Taxation
Parameters	M	ψ	δ_x	B	γ_h	θ	v	α	A	β	τ
Effects on x^* (or ϕ)	+	−	−	+	$-^{\S}$	−	+	−	+	0	+
Effects on z^*	−	+	+	+	+	$-^{\S}$	+	+	−	0	$+^{\S}$
Effects on η^*	+	−	−	$+^{\S}$	$+^{\S}$	$-^{\S}$	+	−	+	0	$+^{\S}$

Notes: The signs with the superscript § are obtained using numerical simulations (see Section 6.1 for calibration details) and reflect local responses in the neighborhood of the benchmark equilibrium.

$$\eta_k = \frac{k_{t+1}}{k_t} = \left[\frac{\phi(x^*)}{1 + \phi(x^*)}\right](1 - \tau)A(1 - \alpha)z_t^{1-\alpha} \qquad (20)$$

$$\eta_h = \frac{h_{t+1}}{h_t} = B[v\gamma_h\tau(1 - \alpha)A]^{\theta}z_t^{-\alpha\theta} \qquad (21)$$

Along the BGP stationarity of z_t leads to equality of Eqs. (20) and (21), yielding the following BGP condition:

$$B(v\gamma_h\tau)^{\theta} = \left[\frac{\phi(x^*)}{1 + \phi(x^*)}\right](1 - \tau)[A(1 - \alpha)]^{1-\theta}(z^*)^{1-\alpha(1-\theta)} \qquad (22)$$

where x^* is given by Eq. (12). The balanced growth rate of physical and human capital can be written as

$$\eta^* = B[v\gamma_h\tau A(1 - \alpha)]^{\theta}(z^*)^{-\alpha\theta} \qquad (23)$$

4.2. Comparative statics for public education model

Following the same method as in the benchmark model, we derive comparative-static results for the public education model (see Table 3). Following a change in the parameter value, we first obtain the responses of z^*, from which, as in the case of benchmark model, we derive changes in η^* and x^*, we can find that for parameters $j = \psi, \gamma_h, B, \delta_x, v, \tau$ and θ $dz/dj > 0$, and for $j = M, A$, $dz/dj < 0$. For $z^* > 1$, $dz/d\alpha > 0$ (and ambiguous otherwise).[12]

The comparative-static results for this model are presented in Table 3. A comparison with the corresponding results from the benchmark model (see Table 2) reveals that most signs are identical. The notable exception is the sign of v, the parameter representing the weight of public education spending in total human capital formation. Not surprisingly, in the case where there is no private involvement in funding education, it has a positive impact on human capital formation (therefore on z^*) and x^*. For the same reason, the effect of τ on z^* is now positive, compared to a negative impact in the benchmark model (see Table 2). Since the balanced growth rate η^* and the capital stock ratio z^* are inversely related, one would expect parameter changes to affect them in opposite directions. Exceptions are changes in B, v, γ_h, and τ, all of which affect η^* and z^* in the same direction. These parameters have a direct positive impact on η^* (see Eq. (23)) that dominates the negative indirect effect via z^*.

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[12] The derivations of the comparative-static results are available upon request.

5. Private education model

5.1. Model specifications ($\mu > 0$, $v = 0$)

In this model, we consider the scenario when there is no public expenditure on education. Thus education is entirely financed privately while the entire tax revenue is spent on health. This case is the opposite extreme of the public education model where education is financed entirely by the government. Another, and perhaps a better interpretation of this model is that it is a special case of the benchmark model in which the all public revenue is spent on health, $1 - \gamma_h = 1$ (or, $\gamma_h = 0$). With this interpretation we can use all the result from the benchmark model with the modification $\gamma_h = 0$. Thus, we have the following expressions for private saving and educational spending that are obtained from individual optimization:

$$s_t = \left(\frac{\phi_{t+1}(1 - \tau)}{1 + (1 + \theta\beta)\phi_{t+1}} \right) w_t h_t \tag{24}$$

and

$$e_t = \left(\frac{\theta\beta\phi_{t+1}(1 - \tau)}{1 + (1 + \theta\beta)\phi_{t+1}} \right) w_t h_t \tag{25}$$

Steady state health stock is given by

$$x^* = \left(\frac{M}{1 + \delta_x} \right)^{\frac{1}{\psi}} \tau A(1 - \alpha)(z^*)^{(1-\alpha)} \tag{26}$$

Note that for a given z^*, the steady state health stock in this model is higher than that in the benchmark model. Given Eq. (26), the growth rates of physical and human capital are, respectively, given by

$$\eta_k = \frac{k_{t+1}}{k_t} = \left[\frac{\phi(x^*)(1 - \tau)}{1 + (1 + \theta\beta)\phi(x^*)} \right] A(1 - \alpha)(z^*)^{1-\alpha} \tag{27}$$

and

$$\eta_h = \frac{h_{t+1}}{h_t} = B \left[\frac{\phi(x^*)(1 - \tau)}{1 + (1 + \theta\beta)\phi(x^*)} \right]^{\theta} (\mu\theta\beta A(1 - \alpha))^{\theta} (z^*)^{1-\alpha\theta} \tag{28}$$

As before, the BGP condition is obtained by equating the above two growth rates:

$$[B(\mu\theta\beta)^{\theta}]^{\frac{1}{1-\theta}} = \left[\frac{\phi(x^*)}{1 + (1 + \theta\beta)\phi(x^*)} \right] (1 - \tau)A(1 - \alpha)(z^*)^{\frac{1}{1-\theta}-\alpha} \tag{29}$$

Apparently the BGP condition in Eq. (29) is same as the BGP condition in Eq. (15) in the benchmark model, but the solution, z^* in the two models are different due to differences in x^*. Here the balanced growth rate (η^*) is given by

$$\eta^* = Q(z^*)^{\frac{-\theta}{1-\theta}}, \quad \text{where } Q = B^{\frac{1}{1-\theta}}(\mu\theta\beta)^{\frac{\theta}{1-\theta}} \tag{30}$$

For the parameter values chosen for the benchmark model, the BGP in this model is stable and unique.

In this model, the only utility of raising tax rate comes from higher health stock that raises longevity. However, marginal increase in longevity falls very quickly as health stock
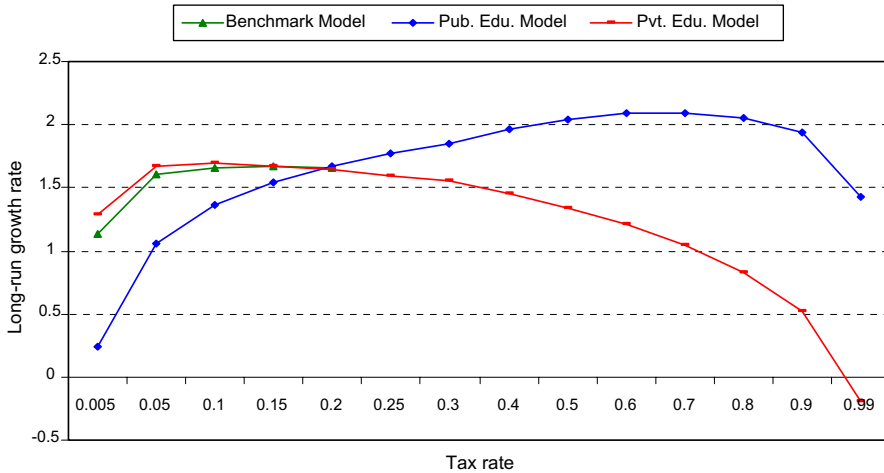
Fig. 3. Taxation and long-run growth.

increases. This is the reason why the peak growth rate is reached at a relatively low level of tax rate after which growth rate starts to decline as shown in Fig. 3.

To avoid repetition, we refrain from re-deriving the dynamic equilibrium conditions for the private education model. Furthermore, since the model is qualitatively similar to the benchmark model, the comparative-static results remain unchanged from those presented in Table 2.

6. Welfare and macroeconomic indicators

Government provision of education and health has important welfare effects through changes in consumption levels, survival probability and human capital. To compare welfare levels between the three models, we employ the standard social welfare index (W) that includes the utilities of individuals across generations and over time. More specifically, W is expressed as the discounted sum of utilities of young and old over an infinite time horizon:

$$W = \sum_{t=-1}^{\infty} \rho^t \{\log(c_t) + \phi(x_{t+1})[\log(c_{t+1}) + \beta \log(h_{t+1})]\} \quad \rho \in (0,1) \tag{31}$$

where ρ is the social discount factor and t denotes time periods. We impose $\rho < 1$ to ensure that the welfare index converges to a finite value as $t \to \infty$. We report the steady state values of the welfare indicators for the three models in Table 6.

6.1. Calibration

Our numerical analysis will be based on the parameters summarized in Table 4. With the exception of the education spending weights in the human accumulation function (μ and v), parameter values are identical across models.

The share of physical capital in output production, α, is set to 0.33, a value within the range of its empirical estimates. We chose the total productivity parameter A to be equal

Table 4
Benchmark model calibration

Parameters	M	ψ	δ_x	B	γ_h	θ	μ	v	β	α	A	τ	ρ
Values	4.5	0.55	0.5	4.5	0.46	0.20	0.65	0.35	0.65	0.33	5.0	0.16	0.40

to five so that the steady state net interest rate $(R - 1)$ yielded by the benchmark model comes close to the benchmark countries' average (1970–2001) real interest rate on savings deposits of 5.94% (World Bank, 2003). As in Soares (2005) we pick a value of $\theta = 0.2$ for the elasticity of total educational expenditure with regard to wage earnings (Eq. (3)). Our choice of $B = 4.5$ together with the values for the weights of private and public funding in the human capital production function guarantees that the share of private education in total education spending matches that of the benchmark countries of about 27% in 2000 (OECD, 2003). For the benchmark model, this requires setting $\mu = 0.65$ and $v = 0.35$. In contrast, μ (v) is set to zero in the public (private) education model.

No dependable empirical estimate for ψ, the elasticity of health spending in the survival function, exists. We thus choose an arbitrary value of $\psi = 0.55$. We assume that the health stock of an individual depreciates by about 50% from childhood to adulthood ($\delta_x = 0.5$). Our choice of $\tau = 0.16$ implies that the share of government spending on education and health in GDP approximates 10.6% – the corresponding value for the benchmark countries in 1995–2000 (UNDP, 2004). The parameter γ_h determines the allocation of government tax revenues between public health and education budget. The share of government spending on education as a percentage of total government spending on health and education equals 46% for the group of benchmark countries in 1995–2000 (UNDP, 2004). We thus choose $\gamma_h = 0.46$.

For the scaling parameter, M, in the health accumulation function we choose a value of 4.5 because it produces a probability of survival into old age that comes close to the observed average probability at birth of surviving to age 65 in the benchmark countries of roughly 0.82 (UNDP, 2004).[13] Finally, in the absence of appropriate guidance from the empirical literature, we choose the altruism parameter, $\beta = 0.65$, a value used in Raut (2003), who analyses a similar household problem in a similar framework as ours. This value of β, however, is not based on empirical estimation in Raut (2003). Furthermore, degree of altruism is generally conditioned by cultural norms, which differ across societies. Keeping in view of the heterogeneous nature of countries in our sample, we conduct a sensitivity analysis for varying degrees of altruism in Section 6.4. A social discount factor of $\rho = 0.4$ for the 30-year period is equivalent to a quarterly discount factor of 0.99, a widely used number in the real-business-cycle literature. Finally, since the rate of convergence of the welfare indicator is slow, we need to choose a long enough time horizon, such as $t = 150$, to guarantee convergence in our numerical simulations.

Table 5 summarizes certain economic indicators for the benchmark model as well as actual data for a group of countries with levels of private spending on education that make up a sizable share of overall expenditures on education (hereafter called 'Benchmark' countries).

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[13] Alternatively, one could employ the probability of surviving into old age when middle-aged instead of at birth. However, for our purpose, it is safe to use at birth probabilities since we know from our simulation exercises that the relative performance of the three models is not particularly sensitive to changes in M.

Table 5
Select economic indicators: Comparison between benchmark model and benchmark countries (in %)

	Probability of survival to age 65	Real per capita growth rate[a]	Real interest rate[a]	Private share in total educational spending	Share of government health-spending in GDP	Share of government educational spending in GDP
Benchmark model	81.94	3.32	4.27	27.43	5.79	4.93
Benchmark countries	82.31	2.96	5.94	26.59	5.82	4.79

[a] Numbers reflect annualized rates.

6.2. Comparison of the three models

Table 6 summarizes select macroeconomic indicators along the BGP as well as welfare levels for each of the three models. In terms of longevity (ϕ), the private education model has by far the highest survival probability compared to both the benchmark and public education model. This result is a direct consequence of the differences in the level of health expenditures in the three models (see g^x/Y column). In the private education model, the government concentrates its entire revenues on health, while in the benchmark and public education models the government has a dual responsibility and diverts some its revenues toward education.

With regard to long-run growth (η^*), the net interest rate (r^*), and the human-to-physical capital ratio (h/K) the benchmark and private education model generate similar values. In addition, these values are higher than the corresponding values in the public education model. There are two forces affecting the human-to-capital ratio. In the public education model, the government is the sole provider of education spending leaving human capital at a comparatively lower level. At the same time, given the lower level of income in the public education model, the actual amount saved is less than what is being saved in the other models causing a lower stock of physical capital.[14] With both human and physical capital below the levels in the other models, the relative magnitude of the human-to-physical capital ratio in the public education model is ambiguous a priori. However, based on our numerical simulations, we find that the ratio is lower in the public education model than in the other models. Since the production function exhibits constant returns to scale and diminishing returns to each factor, the rate of return to physical capital is an increasing function of the human-to-physical-capital ratio. Given the positive impact of the rate of interest on long-run growth, the public education model must exhibit lower balanced growth than both the benchmark and the private education model.

Differences in health expenditures and thus longevity among the three models have direct implications for the model rankings with regard to welfare.[15] The private education model ranks highest, followed by the benchmark model, while the public education model generates the lowest level of welfare. While many factors influence the level of welfare, the

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[14] The reduced longevity in the public education model does not have a strong adverse impact on private saving since returns are annuitized.

[15] To ensure compatibility of the welfare indicators, we impose identical initial values for the three stock variables (k, h, and x) across all models in our numerical simulations.

Table 6
Main economic indicators: a comparison between the three models

	ϕ (%)	g^x/Y	η^{*a}	h/K	r^{*a}	Welfare
Benchmark model	81.94	5.79	3.32	3.08	4.27	14.21
Pub. Edu model	81.29	5.79	3.20	2.89	4.12	14.05
Pvt. Edu model	89.38	10.72	3.32	3.09	4.28	14.49

[a] Numbers reflect annualized rates.

most important single determinant of welfare is the survival probability. A high level of ϕ translates into a low discount rate which explains the superior welfare level of the private education regime. Similarly, the low level of welfare in the public education models stems to a large degree from its low survival probability and thus its high discount rate.

Lastly, we examine the relationship between the level of taxation and the corresponding balanced growth rate across models. As is typical for models with endogenous growth mechanisms driven by production externalities (see Barro, 1990; Barro and Sala-i-Martin, 1995), we find a hump-shaped relation between the two variables in all models (see Fig. 3).[16] Furthermore, while the growth rates in the benchmark and the private education models peak at relatively low tax rates (between 10% and 15%, approximately), the growth-maximizing tax rate in the public education model turns out to be around 75%.[17] The difference in the growth-maximizing tax rates between the public education model and the other two models arises mostly due to the expanded range of services (health and education) that the government must provide in the public education model.

6.3. Comparing models: Empirical evidence

In this section, we investigate whether the ordering of models by key economic indicators implicit in Table 6 is born out by the data. We therefore collect data for two sets of countries: the Benchmark countries defined in Section 6.1 above and countries with little private spending on education (public education countries).[18] Since no countries in our sample rely more or less exclusively on privately funded education, the relative performance of the private education model cannot be analyzed empirically. We define 'Public Ed' countries as those with a public share in total educational spending of more than 85%, while all other countries in our sample are considered Benchmark countries.[19] We use OECD data on education spending for a total of 36 countries (Table B3.1; OECD, 2003), while the economic and demographic indicators are taken from UNDP (2003),

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[16] The relevant curve for the benchmark model is truncated at around $\tau = 20\%$ because for higher tax rates a corner solution with no private spending on education occurs.

[17] A similar non-monotonic relationship can be derived for the effect of changes in γ_h on growth.

[18] These economic indicators include life expectancy at birth, income, real GDP per capita growth, human-to-physical capital ratio, rate of interest, and GDP-to-capital ratio.

[19] Our sample includes only 36 countries due to the unavailability of the share of private spending in total educational spending for many countries. The group of countries with mixed funding of their education systems include Chile, Republic of Korea, Indonesia, Jamaica, United States, Paraguay, Japan, Australia, Argentina, Canada, Israel and Germany. Countries that use mostly public funds to finance education include United Kingdom, Mexico, Spain, Hungary, Czech Republic, The Netherlands, Italy, Iceland, Ireland, Switzerland, Belgium, France, Greece, Austria, Uruguay, Thailand, India, Denmark, Slovakia, Sweden, Finland, Portugal, Turkey and Norway.

Table 7
Economic indicators by mode of education funding

	Life-expectancy at birth	Real GDP per capita	Growth rate of real per capita GDP	Human-physical capital ratio	Interest rate
Benchmark countries	74.99	21,987	2.96	0.0139	5.94
Public Ed countries	67.61	5442	2.50	0.0064	6.79
Mean	70.23	11,318.5	2.66	0.0106	6.48

Note: Human-physical capital ratio is defined as number of workers with secondary education divided by gross fixed capital formation.

World Bank (2003) and version 5.6 of Heston and Summers (1994). All indicators except for the growth rate have been calculated as averages over 1995–2000. The growth rate is calculated as the average annual growth rate of real GDP per capita over 1961–2001 to represent long-run growth rate.

The results, given in Table 7, show that – for four out of five indicators – the predicted relative model performance summarized in Table 6 is supported by the data. In particular, life expectancy at birth, the welfare indicator (as measured by real GDP per capita in international dollar), long-term growth and human-to-physical capital ratio are higher for Benchmark countries than for Public Ed countries. In terms of the interest rate – defined as the lending interest rate adjusted for inflation as measured by the GDP deflator – Benchmark countries tend to have lower values than Public Ed countries, contrary to the prediction from Table 5. However, if the 'rate of interest on deposits' is used instead of the interest rate, the relative group averages are again in line with their predicted magnitudes. Using alternative measures for longevity and the human-to-physical capital ratio leads to qualitatively similar results as the ones reported in Table 7.[20]

6.4. Sensitivity analysis

We investigate the sensitivity of our results with regard to changes in two key parameters. First, we examine the welfare effects of alternative values of the discount factor ρ. We find that changing ρ does not alter the welfare ranking of the three education regimes.[21] Second, we analyze the macro and welfare consequences of changes in the altruism parameter β (see Table 8). As shown before (see Table 3), changing β does not change the steady state values of any variables in the public education model except for the level of welfare. The BGP values in the other two models are, however, sensitive to changes in β. A lower β implies a lower weight attached to a child's human capital and thus slows down human capital accumulation, which in turn lowers z^*, the survival probability, growth and welfare (see Table 2). The adverse impact on longevity is stronger in the benchmark model where sub-optimality is associated with two aspects (public education and health spending), compared to the private education model where one source of sub-optimality (public spending on education) is absent. However, the adverse impact on welfare is stronger in the private education model where welfare declines by 34% when β is reduced from 0.8 to 0.15, compared to a decline of 30% in the benchmark model. The cause of this difference

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[20] The figures in Table 7 represent weighted averages of the two groups. The weight used is population. Using per capita GDP as weight yields qualitatively similar results.
[21] The results are available from the authors upon request.

Table 8
Sensitivity of steady states values with respect to the altruism parameter (β)

	ϕ (%)	η^{*a}	h/K	r^{*a}	Welfare
$\beta = 0.15$					
Benchmark model	77.49	2.42	2.04	3.32	10.28
Pub. Edu model	81.29	3.20	2.89	4.12	10.77
Pvt. Edu model	86.33	2.43	2.01	3.29	10.50
$\beta = 0.3$					
Benchmark model	80.00	2.85	2.46	3.75	10.46
Pub. Edu model	81.29	3.20	2.89	4.12	11.63
Pvt. Edu model	87.79	2.86	2.44	3.73	11.92
$\beta = 0.8$					
Benchmark model	82.58	3.45	3.29	4.43	14.68
Pub. Edu model	81.29	3.20	2.89	4.12	14.49
Pvt. Edu model	89.80	3.44	3.31	4.44	15.92

[a] Numbers reflect annualized rates.

is the absence of the mitigating factor of government spending on education in the private education model. When parents show very low degrees of altruism toward their children (for example, $\beta = 0.15$ in Table 8) both the benchmark and private education models are outperformed by the public education model in most macroeconomic aspects including welfare levels.

6.5. Endogenous fiscal policy

What would be the preferred tax rates in the three models? And which of the models would yield higher levels of welfare? To answer these questions we produce numerical values of the welfare index given in Eq. (31) for various levels of γ_h and the optimally chosen tax rate (in a second-best sense) for each of the models.[22] Table 9 reports these maximum welfare levels and the corresponding optimal second-best tax rates (in parentheses).[23]

We find that the welfare ranking of the education regimes depends on the (exogenous) split between government spending on education versus health. For high levels of public spending on education relative to health ($\gamma_h \geqslant 0.5$), the public education model is welfare superior, while the reverse is true for low public education spending (values of $\gamma_h < 0.5$). That is, in a world where the government is forced to spend most of its revenues on education, the country is better of if schooling is provided exclusively by the public sector. In contrast, if the government decides to allocate its resources mostly to finance public health, the country is better off with a mixed funding for education. In the extreme case that the government spends all its resources on health and nothing on education, welfare is the same as under the mixed spending regime. The intuition behind these results is the follow-

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[22] The private education regime is omitted from the table since it represents the special case of $\gamma_h = 0$. For the optimally chosen tax rate ($\tau = 0.15$), the level of welfare achieved under this regime is 14.49, a value that exceeds the maximum welfare levels achievable in either of the other two models (see Table 9).

[23] Another fiscal policy exercise would involve finding out the optimal allocation of public revenue (γ_h) between health and education in the benchmark and public education models. The analysis of optimal choice of γ_h for various levels of τ is very similar to that reported in Table 9 and omitted to avoid repetitiveness.

Table 9
Optimal welfare levels: Comparison between benchmark and public education model

γ_h	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Benchmark model	*14.47*	*14.46*	*14.42*	*14.38*	*14.33*	14.20	13.90	13.31	12.27	10.56
	(15.5)	*(16.1)*	*(17.5)*	*(19.2)*	*(21.2)*	(17.9)	(14.8)	(12.1)	(9.47)	(6.0)
Pub. Edu. model	12.38	12.99	13.57	13.90	14.10	*14.22*	*14.28*	*14.28*	*14.24*	*13.73*
	(31.3)	(31.9)	(32.7)	(33.4)	(34.3)	*(35.5)*	*(37.3)*	*(44.5)*	*(41.9)*	*(42.7)*

Note: (i) Welfare maximizing second-best tax rates (%) in parentheses.
(ii) For private education model, see Footnote 21.

ing: When a majority of the public resources is allocated toward financing education, it leaves little funds for health. As a result, longevity is reduced in equilibrium causing disincentives for households to invest in education. In fact, the lower the extent of public spending on health, the higher is the extent of sub-optimality associated with private spending on education and the lower is the associated welfare level. Conversely, when most government spending is targeted toward health, private spending on education becomes more and more desirable from society's point of view (because of higher longevity and consequent lower discount rates over time), thus making higher private educational spending optimal.

Table 9 raises the question why the public education model requires substantially higher welfare-maximizing tax rates (for any level of γ_h) than the benchmark (and private education) model. Again, the answer has to do with the fact that the scope of government services in the public education model is more comprehensive than in the other two models.

7. Concluding remarks

We incorporate dual public spending on education and health in a general equilibrium overlapping generations framework in which individuals have lifetime uncertainty. Private decisions regarding saving and expenditure on child's education depend crucially on the incentive effect generated by better health and therefore higher longevity. Health accumulation depends on public funding on health alone, while the accumulation of human capital depends on both private and public funding. We analyze three different models. In the benchmark model, the most general form of human capital accumulation function is used requiring both private and public spending. In the public education model, the government is the sole provider of education, while private individuals are the sole providers in the private education model. The implications of these three models shed important light on policy issues such as the welfare maximizing mix of public versus private spending on education.

By combining 'productive' government spending with endogenous length-of-life effects we can investigate not only the long-run (steady state) growth and welfare implications for three different education regimes, but also the short-term (transitional) growth impact of variable longevity.[24] Another novel assumption of this paper is the simultaneous inclusion

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[24] We do not report the short-term (transitional) dynamics in the paper for the purpose of brevity. It is, however, available from the authors on request.

of two productive government investments (education and health) and their specific roles in generating and sustaining long-term growth under lifetime uncertainty.

An important result of the paper is that the public education model produces results that are inferior to both the benchmark and the private education model with regard to a number of macroeconomic indicators. For a fixed income tax rate, long-run growth, interest rates, and the ratio of human-to-physical capital are all lower in the public education model than in the other two models, while longevity and welfare are highest in the private education model and lowest in the public education model. The poor performance of the public education model has its root in an income tax rate that is too low relative to its optimal level. In other words, the public education model cannot perform properly if the government cannot raise the funds needed to run both a public health and a public education system. This simple intuition is validated when we endogenize the income tax rate: Welfare-maximizing tax rates are on average more than three times higher in the public education model compared to the benchmark model. In addition, a comparison of welfare levels based on second-best tax rates reveals that the welfare ranking of benchmark versus public education model depends on the (exogenous) relative size of government spending on education and health. For high levels of public spending on education relative to health, the public education model is welfare superior, while the reverse is true for low public education spending.

Our results point to an interesting policy implication. Faced with a given technology and a level of taxation that is too low relative to its optimal size, a government that cares about longevity, welfare, and long-run growth should encourage private participation in funding of education, thereby freeing up public funds for the provision of health care and related services.

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