Revenue from the Saints, the Showoffs, and the Predators: Comparisons of Auctions with Price-Preference Values

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Abstract

Traditional auction theory assumes that bidders possess values defined solely on the auctioned object. There may, however, be cases in which bidders possess preferences over the revenue achieved by the auctioneer. We present here a comprehensive framework of price preference valuations, unifying several phenomenon ranging from preference for charitable giving to shill bidding. We compare expected efficiency and revenue of first and second price auctions for some specific cases of key interest. We also incorporate heterogeneous bidder preferences and examine the effects of mis-specified beliefs and show that both are crucial for understanding these situations.
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1 Introduction

While traditional auction literature has focused almost exclusively on situations in which bidders possess preferences only over the items being auctioned, bidders in real auctions will quite often have preferences over more than just what they win. One sort of these preferences might be preferences over the revenue achieved by the auctioneer. In previous literature such preferences have been called "price preference valuations". There are, however, a large number of different variations and applications of a model of this sort. Several such models have been explored in recent literature. We will show that these are all special cases of a more general model and we will extend this model to some additional applications.

In section 2 of this paper we will construct a general framework of price preferences in auctions. In doing so, we will demonstrate the necessity of considering complicating factors such as asymmetry and heterogeneity among the bidders as well as bidders possessing mis-specified beliefs about the population they are bidding against. Section 3 will present the formal model along with some analytical results. This will set-up section 4, which will use a computational approach to analyze multiple different cases and environments for which analytical results are unavailable.

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2 A Unified Model of Price Preference Valuations

There are a number of different reasons why a bidder in an auction might derive additional utility based on the revenue of the auctioneer. When specifying some form of these alternative preferences there are additions that must be made to the standard story of preferences in auctions. It is typically assumed in auction models that each bidder i possesses some value v_i of winning the object. The notion of price preferences supposes that bidders may also derive utility from the level of the final price. Engelbrecht-Wiggans (1994) (EW) introduces such a model in the context of bidding rings and using auctions to divide partnerships. He assumes that each individual bidder derives some amount of utility proportional to the final price with this parameter, β, constant across all bidders and symmetric in regard to whether or not a bidder wins or loses the auction.

There are, however, a number of alternative applications of a model of this sort beyond the examples in EW that require a more flexible specification of these preferences. The first addition to the specification is to allow asymmetric benefits to a bidder between winning and losing. This involves introducing a new parameter, α, such that when a bidder wins, his utility is increased by $\beta * p$, where p is the seller's revenue or price, but his utility is increased by $\alpha * p$ when the bidder loses. It is relatively straightforward to extend the results in EW to allow for this case and some of the details of doing so will be shown in the appendix[1].

In many of the cases in which price preference valuations are an issue it is inherent to the motivation of the scenario that bidders have asymmetric or heterogenous values for these parameters. In certain cases it is also crucial to the story that this asymmetry will not be known during the bidding process. In the descriptions of different cases below we will note these issues and most of our computational results in section 4 will be aimed at dealing with these situations since they are ones for which analytical results can not be found. Introducing asymmetries across bidders means different bidders will have different values for β and α, requiring the notation of β_i and α_i. Our analysis will focus on computing optimal bid functions in first and second price auctions in three different cases.

One detail to note is that we have not actually specified whether these preferences result from pecuniary (e.g. a payment from the seller) or non-pecuniary (e.g. a preference to see the seller collect more money) motivations. Either can be modeled in the same structure from the bidders' perspective but this will prove to be an important distinction for seller revenue, which will be net of any direct payments to the bidders.

While our basic model of auctions with price preferences can be used to study a wide range of phenomena, in this paper we will concentrate our focus on three cases of particular interest. The first involves charity auctions. In auctions that are conducted to benefit a charity, one could well expect that participating bidders gain utility based on the total amount of money raised by the charity. This issue has been examined using field data from charity auctions in Isaac and Schnier (2002) to examine evidence in favor of such preferences. For modeling purposes, we will claim that pure preferences for charitable giving involve bidder preferences that are focused specifically on the revenue the seller raises and therefore we should have $\alpha_i = \beta_i$. There is, however, no reason to believe that all n bidders in an auction are alike in their charitable preferences thus we allow for $\beta_i \neq \beta_j$. The source of these preferences could be either pecuniary or non-pecuniary, but the most likely story is

[1] We note that, in the process of working on this project, we found that many of the results for this case have been separately derived in Engers and McManus (2002).

that they are non-pecuniary[2]. This case is captured by EW, but restricted to the case of inter-personal symmetry.

A variant of these baseline preferences for charitable giving concerns bidders who gain utility from seller revenue only when they are the winning bidder. A more informal description of such motivations is that a bidder wishes to bid in the auction to be seen as a generous person. We will refer to such preferences as preferences to See and Be Seen (SBS). Such bidders would possess $\beta_i > \alpha_i \geq 0$. Isaac and Schnier discussed this concept, and Engers and McManus (2002) extended EW to allow for $\beta_i > \alpha_i$ but their model retains the assumption that parameters are identical across individuals. That specification implies that all bidders are attempting to be as demonstrative as everyone else. We believe a more useful and realistic representation is that some bidders are significantly more demonstrative than others. Further, the demonstrative bidders may or may not be aware that they are "different from the norm" while the less demonstrative bidders may not be aware of the existence of these other bidders. We will capture this phenomenon by modelling a stark case in which $\alpha_i = 0$ for all bidders, and $\beta_i > 0$ for only a subset of bidders who "want to be seen" as being generous to the charity. We will also examine cases in which beliefs are mis-specified. The motivation of this phenomenon for charitable giving is again almost certainly non-pecuniary.

There is both theoretical and experimental evidence that casts doubt on the usefulness of our examination of winner pay auction formats for charitable giving. Goeree, Maasland, Onderstal, and Turner (2004) argues that all pay auctions and lotteries or raffles are substantially superior to winner pay auctions in terms of raising revenue for charitable causes with all pay auctions also dominating lotteries. Davis, Razzolini, Reilly, and Wilson (2003) provides experimental evidence that lotteries can be more successful at raising money than English auctions. The strongest argument to motivate why we are concerned with auctions instead of lotteries is that in many places around the country charity lotteries are either illegal or carry significant and perhaps prohibitive transactions costs due to legal restrictions regarding their use[3]. Second, charitable auctions are quite often conducted by religious institutions and many of these institutions object to lotteries on the basis that they represent a form of gambling. While all-pay auctions may not suffer the same legal and religious problems as lotteries, there still seems to be reluctance on the part of auctioneers to use them except on low value items. Due to these issues and the prevalence of winner pay auctions run on the behalf of charities as discussed in Isaac and Schnier (2002), we feel justified in claiming that there is still value to be found in examining first and second price or ascending winner-pay auctions.

The last case we will examine involves preferences quite different from those to give to

[2]Pecuniary based preferences in this case might be derived from charity auctions attended by parents held on behalf of their child's school. Part of the money raised by the auction may be used to benefit their child thus granting them pecuniary benefits from the auctioneer raising more money.

[3]The types of regulatory costs include geographically restrictive, expensive, prolonged or discretionary licensure (e.g. Georgia and local option Illinois), licensing, training, or bonding requirements for individuals in the organization (e.g. Colorado), duration of business requirements (e.g. Texas, Colorado, local option Illinois) restrictions on advertising (Texas), frequency limits (e.g. Georgia), restrictions on allowable prizes (e.g. Texas), reporting and record requirements (e.g. Arizona, Georgia), specifications for ticket printing (e.g. Colorado), gross receipts subject to sales tax (e.g. Iowa), and funds segregation (e.g. Colorado). And, of course, in any state there are federal record keeping and reporting requirements for tax purposes that may daunt smaller organizations. More specific citations on these legal issues are available from authors upon request.

charity, but rather preferences to damage a rival. We will refer to this case as preferences for Raising a Rival's Cost (RRC). Part of the circumstantial evidence presented by the government in the 1946 decision against the three largest tobacco companies is that the companies deliberately acted to raise the cost of rivals through the ubiquitous auctions of leaf tobacco[4]. Similar concerns have reappeared with the expansion of B-to-B auction sites on the web: "If you can use your clout in a B-to-B exchange to force your competitors to pay more for a key component, making it impossible for them to compete effectively, you might have broken antitrust laws."[5] Such motivations have also been proposed to explain British Telecom 3G's bidding behavior in the U.K.'s UMTS[6] auction by Klemperer (2002a). The structure of these preferences is almost certainly asymmetric across firms and it also stands to reason that "prey" firms may not realize they are being targeted and "predator" firms may not realize there are other predators in the population. We will model this case assuming $\beta_i = 0$ for all firms with $\alpha_i > 0$ for the predators but $\alpha_j = 0$ for the prey. Because the benefits to the predatory firm do not come directly from the seller, we classify the preference as "non-pecuniary" although the predator's preferences may be based upon the hope of future monetary payoffs through other channels. Morgan, Steiglitz, and Reis (2002) develops a model of purely spiteful behavior in auctions which is a closely related yet not the same phenomenon. We will discuss this issue more in section 4.

While these are the 3 cases that we will focus our analysis on, the model is easily generalizable to deal with several other sorts of situations. The bid functions we will examine under the SBS specification are identical to the bid functions one could derive for simple cases of auctions involving subsidized bidders. The Federal Communications Commission regularly subsidizes bidders with β's as high as .45 for certain auctions. Shachat and Swarthout (2002) develops an alternative model of subsidized bidders in which the subsidy is a fixed amount rather than a percentage based on the bid level. Corns and Schotter (1999) constructs a model of asymmetric bidders for procurement contracts in which the disadvantaged or high cost bidders are given a bidding advantage which consists of modifying the bids placed by the advantaged bidder by some amount for the purposes of determining who wins the contract. Because the underlying motivation to subsidize bidders is typically to assist disadvantaged bidders[7], modeling this phenomenon necessitates using asymmetric value distributions which would require a substantial extension of what we will present under the SBS case. To keep our analysis consistent we will therefore not specifically deal with this case below.

McAfee (1992) constructs a similar model and applies it to an environment consisting of two bidders in an auction attempting to divide a personal or professional partnership. The winner pays the loser one half of the winning bid (which may be set by a first-price or second-price rule). The McAfee (1992) specification is a special case of our model using the assumption that $\beta_1 = \alpha_1 = \beta_2 = \alpha_2 = \frac{1}{2}$ with the values of the bidders being drawn from the same distribution. The utility transfer in this case is clearly pecuniary. de Frutos (2000) extended this model to allow for asymmetric value distributions but still enforced $\beta_1 = \alpha_1 = \beta_2 = \alpha_2 = \frac{1}{2}$.

[4] *American Tobacco Company v. United States*, 328 U.S. 781 (1946).

[5] David A. Price, American Antitrust Institute, http://www.antitrustinstitute.org/recent/82.cfm.

[6] Universal Mobile Telecommunications System

[7] Most auctioneers would only consider subsidizing a proper subset of the bidders who are deemed to be disadvantaged in relation to another set. The FCC, however, has chosen to subsidize all of the bidders in some of their auctions.

It is also possible to model shill bidding in this framework if one models a shill as a bidder who is paid a share of the seller revenue when he loses, as (purportedly) an incentive to bid up seller revenue. This case is equivalent to the RRC case, with the exception that the preference is now directly pecuniary. As in the case of RRC, this certainly involves asymmetric bidders with only a single bidder possessing $\alpha_i > 0$ and it would almost certainly be the case that the other bidders in the auction would not be aware of the existence of the shill. A shill bidding arrangement might also involve reimbursing the shill completely upon winning or $\beta_i = 1$ for the shill where the v_i for the shill is some constant also set by the auctioneer to pay the shill in the event they win. Due to the special nature of how v is set for the shill, we will not examine this issue directly either.

The next section will develop the specifics of a theoretical model that can be used to derive bid functions for some of these cases for a first price and second price or ascending auction. The fourth section contains a detailed examination of the effect on revenue and efficiency of each of these cases and the final section incorporates concluding thoughts on the insights gained into practical auction design from our results.

3 Model

We will be assuming a common structure throughout the theoretical analysis in this paper. Each auction will consist of n risk neutral bidders who each draw an independent and privately known value for winning the auction, v_i, from a commonly known distribution $F(v)$ with pdf $f(v)$ which will be assumed for convenience to have support over the range $[0,1]$. Each bidder will possess two parameters β_i and α_i which describe the utility the bidder gains for each additional unit of revenue the auctioneer raises when the bidder wins and loses the auction, respectively. We will be assuming that $\alpha_i, \beta_i \geq 0$ and it would seem quite unreasonable to assume anything other than $\alpha_i, \beta_i \leq 1$.

If we assume for the first price auction that there exists some symmetric bid function $b_f^*(v, \beta_i, \alpha_i)$ that is monotonically increasing and differentiable in v and assume that $\alpha_i = \alpha_j$ and $\beta_i = \beta_j$ for all i and j, then we can show what such a bid function must look like and show that it exists. The fact that we are working with symmetric bid functions allows us to make use of the fact that the probability of winning if i bids as if his value were r is $\Pr(b_f^*(r, \beta_i, \alpha_i) > b_f^*(v_j, \beta_i, \alpha_i)$ for all $j) = \Pr(r > v_j$ for all $j) = F(r)^{n-1}$. This means that the first price auction problem is defined as:

$$\max_r S(v_i, r) = (v_i - (1-\beta_i) b_f^*(r, \beta_i, \alpha_i)) F(r)^{n-1} + \alpha_i \int_r^1 b_f^*(t, \beta_i, \alpha_i)(n-1) f(t) F(t)^{n-2} dt \quad (1)$$

With the equilibrium condition as

$$\frac{\partial (S(v_i, r))}{\partial r}\Big|_{r=v_i} = 0 \quad (2)$$

The first term represents the utility that the bidder receives when he wins the auction multiplied by the probability of that event. The second term is his utility when he loses the auction which requires integrating over all possible prices that the actual winner of the auction might pay multiplied by their probability of occurring. This model is a straightforward extension of the one solved in EW and it can be solved with similar methods. We omit most of the proofs supporting these results to conserve space because they are similar to those in EW, but we have included some of the details of the derivation in the appendix. The solution can be shown to be:

$$b_f^*(v_i, \beta_i, \alpha_i) = \frac{\int_0^{v_i} t\,(n-1)\,f(t)F(t)^{\frac{(n+\beta_i(2-n)+\alpha_i(n-1)-2)}{1-\beta_i}}dt}{(1-\beta_i)(F(v_i)^{n-1})^{\frac{1-\beta_i+\alpha_i}{1-\beta_i}}} \tag{3}$$

In the case of the uniform distribution on the range $[0, 1]$, the bid function becomes

$$b_f^*(v_i, \beta_i, \alpha_i) = \frac{n-1}{n(1-\beta_i+\alpha_i)-\alpha_i} v_i \tag{4}$$

Similarly, the second price problem is

$$\begin{aligned} \max_r S(v_i, r) = \int_0^r &(v_i - (1-\beta_i)b_s^*(t, \beta_i, \alpha_i))(n-1)f(t)F(t)^{n-2}dt + \\ &\alpha_i b_s^*(r, \beta_i, \alpha_i)(n-1)F(r)^{n-2}(1-F(r)) + \\ &\alpha_i \left(\int_r^1 b_s^*(t, \beta_i, \alpha_i)(n-2)(n-1)F(t)^{n-3}(1-F(t))f(t)dt \right) \end{aligned} \tag{5}$$

The first term represents the utility that bidder i gets in the event that he wins the auction and we must integrate over the possible prices he would pay which are the bids an opponent would be expected to make as defined by $b_s^*(t, \beta_i, \alpha_i)$ multiplied by the probability of t being the second highest value. The second term defines the utility i would receive from placing second highest bid as he would set the price and would thus get $\alpha_i * b_s^*(r, \beta_i, \alpha_i)$ times the probability his bid is second highest. The final term represents the utility from coming in less than second as he will get α_i times whatever the second highest bidder bids. Solving this results in a general solution of

$$b_s^*(v_i, \beta_i, \alpha_i) = \begin{cases} \frac{1}{\alpha_i} \frac{\int_{v_i}^1 t(1-F(t))^{\frac{(1-\beta_i)}{\alpha_i}} f(t)dt}{(1-F(v_i))^{\frac{1+\alpha_i-\beta_i}{\alpha_i}}} & if \quad \alpha_i > 0 \\ \frac{v_i}{1-\beta_i} & if \quad \alpha_i = 0 \end{cases} \tag{6}$$

Again, if we simplify this to the case of the uniform distribution on the range $[0, 1]$, the bid function becomes

$$b_s^*(v_i, \beta_i, \alpha_i) = \frac{v_i(1-\beta_i+\alpha_i)+\alpha_i}{(1-\beta_i+2\alpha_i)\,(1-\beta_i+\alpha_i)} \tag{7}$$

Note that since the bid functions for the second price auction do not depend on either n or the lower bound of the value distribution, this function can be used to find the equilibrium drop out prices for an English clock auction. This bid function could also be interpreted to deliver the highest price a bidder would be willing to stay in for in a non-clock English auction. It would be incorrect, however, to assert this drop-out price as an equilibrium strategy for the non-clock ascending auction for the reasons discussed in Isaac, Salmon, and

Zillante (2005).

For many of the cases we are interested in, however, these solutions will not be sufficient since not all bidders will possess the same preference parameters. This requires introducing asymmetry into the model and results in significant additional complications. In particular, we can no longer assume that probability of winning is just $F(r)^{n-1}$. Letting β_{-i} and α_{-i} reflect the vector of all bidders' preference parameters except i, i now wins if $b^*(v_i, \beta_i, \alpha_i, \beta_{-i}, \alpha_{-i}) > b^*(v_j, \beta_j, \alpha_j, \beta_{-j}, \alpha_{-j})$ for all j. We cannot simply invert both bid functions and be left only with values because the parameters may be different. Further, we must make additional assumptions on the amount of information that each i has in regard to the parameters of the other bidders. As discussed above, in certain situations the environment requires dropping a standard assumption that the bidders know either the exact values of the parameters in use or even that they know the distribution of possible parameters. Either of those assumptions would allow us to find the equilibrium bidding strategies by computationally solving n differential equations (as done in Corns and Schotter (1999) and other papers), but either would violate the fundamental premise of many of the issues we are interested in examining.

We will instead take an approach that will allow us to investigate issues that the more standard approach would not allow. Our approach has an attractive side benefit which is that it is also more computationally tractable, but that is not our main motivation for using it. We will assume that each bidder assumes all other bidders have some commonly known parameters, $\bar{\alpha}, \bar{\beta}$, which might be assumed to be the collective beliefs about the average parameter values in the population. In the event that we use the actual averages, our approach will result in an approximation of the true Nash equilibrium bid functions for the auction. We can also investigate the empirically interesting possibility of mis-specified beliefs by setting $\bar{\alpha}$ and $\bar{\beta}$ to other values. For example, in the See and Be Seen case, it is quite reasonable to propose that bidders with high β_i's underestimate the average value of β in the population. In the Raising Rivals' Cost case, it is quite probable that the non-predatory bidders are unaware of the existence of the predators and that even the predators might not suspect there are other predators around. These mis-specified beliefs also seem a reasonable assumption because many of the auctions we are interested in modeling would occur infrequently, making it difficult for bidders to learn the true parameters in the population or even the distribution. Bidders should, however, be able to form a vague model of what the average opponent they face will do and play a best response to that.

Of course the bid functions we derive through this method will not necessarily be equilibrium bid functions (though in some of the cases we consider, the bid functions will be full equilibrium bid functions despite our simplification). While we are sensitive to this concern, we believe there is still substantial value to what we are doing. This is due in part to the oft observed fact that bidding behavior in experiments has a tendency to diverge from straight equilibrium predictions (see Kagel (1995) for a good overview and discussion of this literature). Our intention is to derive a range of possible bidding behavior to explore the impacts of these potential biases in anticipation of observing data in which they might be present. Further, a similar approach was shown to work well in fitting empirical data in Ivanova-Stenzel and Salmon (2004) in regard to heterogenous risk preferences in first price auctions. Our approach here will allow us to investigate the implications of these potential biases in the beliefs of bidders which we will argue have important consequences for the design and conduct of real auctions that would be overlooked with a more traditional investigation. Further it will turn out that our results involving the uniform distribution are in virtually all cases true equilibrium results. The fact that they will show essentially the same pattern as the results using the normal distribution should increase the validity of the disequilibrium results for the normal case.

Of course the other disadvantage of this approach is that when assuming bidder heterogeneity we will be unable to obtain analytical expressions for the bid functions for distributions other than uniform and we will be unable to derive expected revenue and expected efficiency equations even for uniformly distributed values. Thus most of our results on expected revenue and efficiency will be based on computationally solving for the bid functions. It is important to note that even were we to compute the full equilibrium bid functions, that process and the comparisons would still have to be done computationally. While this computational approach will limit the generalizibility of our results, we believe that it will allow us to clearly demonstrate the general principles at work in these environments.

Solving for the bid functions in these cases involves finding bidder i's best response to the belief that he is bidding against $n-1$ bidders who possess parameters of $\bar{\alpha}$ and $\bar{\beta}$ and who believe that they themselves are facing $n-1$ bidders possessing parameters of $\bar{\alpha}$ and $\bar{\beta}$. This assumption allows us to assert that in a first price auction, bidder i expects his $n-1$ opponents to bid according to the bid function $b_f^*(v_j, \bar{\beta}, \bar{\alpha})$ and $b_s^*(v_j, \bar{\beta}, \bar{\alpha})$ in a second price auction. We then solve for the bid functions for each individual bidder in the asymmetric case which we will denote as $B_F^*(v_i, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha})$ and $B_S^*(v_i, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha})$.

To find the probability of winning in this case, we can use the fact that if bidder i bids as if he had value of r, then his perceived probability of winning is $\Pr(B_F^*(r, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha}) > b_f^*(v_j, \bar{\beta}, \bar{\alpha}) \forall v_j) = \Pr(b_f^{*-1}(B_F^*(r, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha})) > v_j \ \forall v_j) = F(b_f^{*-1}(B_F^*(r, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha})))^{n-1}$. To simplify the notation we will let this be represented as just $\phi(r)$. The problem for the first price auction becomes

$$\max_r S(v_i, r) = (v_i - (1-\beta)B_F^*(r, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha}))\phi(r) + \alpha \int_q^1 b_f^*(t, \bar{\beta}, \bar{\alpha}) d\phi(t) \tag{8}$$

where q is defined as the minimum value another bidder must have in order to beat $B_F^*(r, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha})$. This is the q that solves $b_f^*(q, \bar{\beta}, \bar{\alpha}) = B_F^*(r, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha})$ or $q = b_F^{*-1}(B_F^*(r, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha}))$. We can use the standard lower bound in solving this of $B_F^*(0, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha})$, but there will also be an upper bound equal to $b_f^*(1, \bar{\beta}, \bar{\alpha})$. This is because if a bidder places a bid of $b_f^*(1, \bar{\beta}, \bar{\alpha}) + \varepsilon$ they expect to win with probability of 1 since they never expect an opponent to bid above this level.

Solving this problem in the case of uniformly distributed values is fairly trivial and can be done analytically giving us $B_F^*(v_i, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha}) = \frac{n-1}{n(1-\beta_i+\alpha_i)-\alpha_i} v_i$ with an upperbound of $\frac{n-1}{n(1-\bar{\beta}+\bar{\alpha})-\bar{\alpha}}$. Except for the upper bound, this is exactly what we obtained in the symmetric case, indicating that beliefs about opponents parameters are largely unimportant when values are distributed uniformly. For all other value distributions, we must solve this computationally and beliefs about the parameters of others may play a larger role. The methods used are discussed below.

The problem that must be solved for the second price auction also becomes more complex. If we let $w = b_s^{*-1}(B_S^*(r, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha}))$ then we can represent the basic problem as follows:

$$\max_r S(v_i, r) = \int_0^w (v_i - (1-\beta_i) b_s^*(t, \bar{\beta}, \bar{\alpha})) f(t)(n-1)F(t)^{n-2} dt + \alpha_i B_S^*(r, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha})(n-1)F(w)^{n-2}(1-F(w)) + \alpha_i \left(\int_w^1 b_s^*(t, \bar{\beta}, \bar{\alpha})(n-2)(n-1)F(t)^{n-3}(1-F(t))f(t)dt \right) \quad (9)$$

The general construction of this problem is identical to the previous second price case, but allowing for the fact that in the event i does not come in second, he expects that the price will be set by a bidder bidding according to $b_s^*(t, \bar{\beta}, \bar{\alpha})$ while if i comes in second the price is set according to $B_S^*(r, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha})$. Also, the bounds of integration require a term, w, which is the lowest value a competitor can have and beat bidder i's bid.

At least this is the problem that must be solved if $\alpha_i > 0$. If $\alpha_i = 0$, then the solution is just $B_S^* = \frac{v_i}{1-\beta_i}$ regardless of $F()$ and beliefs about the parameters of others. If $\alpha_i > 0$ then we do know one thing about the solution, which is that it has a lower bound of $b_s^*(0, \bar{\beta}, \bar{\alpha})$ as this is the minimum bid that i believes his opponent will ever make. Consequently, i believes that he can drive up the price to at least this price level. If $\alpha_i > 0$ then this lower bound is required. In the event that a bidder possesses $\alpha_i = 0$, he is indifferent between bidding any amount between this and $\frac{v_i}{1-\beta_i}$ when the bound is above $\frac{v_i}{1-\beta_i}$. Since a bidder might realize his beliefs are not perfect and there is no added value from bidding like this to someone with $\alpha_i = 0$, we will assume that such bidders ignore this lower bound and break the tie in favor of bidding $\frac{v_i}{1-\beta_i}$.

Again, we can derive an analytical solution to the asymmetric problem assuming uniformly distributed values which is similar to the solution for the symmetric case but with two changes. First is the addition that there will be a lower bound equal to $\frac{\bar{\alpha}}{(1-\bar{\beta}+2\bar{\alpha})(1-\bar{\beta}+\bar{\alpha})}$ for bidders possessing $\alpha_i > 0$. The second change is that some of the α's and β's in the sloped part of the bid function turn out to be the α and β believed to be held by the opponent. The complete bid function for uniformly distributed values is:

$$B_S^*(v_i, \beta_i, \alpha_i, \bar{\beta}, \bar{\alpha}) = \begin{cases} \frac{v_i}{(1-\beta_i)} & if \quad \alpha_i = 0 \\ \max\left\{ \frac{\bar{\alpha}}{(1-\bar{\beta}+2\bar{\alpha})(1-\bar{\beta}+\bar{\alpha})}, \frac{v_i(1-\bar{\beta}+\bar{\alpha})+\alpha_i}{(1-\beta_i+2\alpha_i)(1-\bar{\beta}+\bar{\alpha})} \right\} & if \quad \alpha_i > 0, \end{cases} \quad (10)$$

For the numerical examples below we will report results for the uniform distribution on the range $[0, 1]$ and also for the normal distribution with $\mu = .5$ and $\sigma = .15$ using the same bounds. While the technical bounds on this distribution are not $[0, 1]$ this range contains 99.91% of the mass so we introduce very little error by using this simplification. We specifically chose a normal distribution as a second example that was very tight both to allow the use of these simple bounds but also to make a contrast with the uniform to ensure that a tighter value distribution does not yield markedly different results.

When computing revenue and efficiency for the cases with uniformly distributed values, we can use these analytical bid functions but can not compute analytical expected efficiencies and revenues because the integrals are not continuous. For the normal distribution cases, we have computed approximations to the bid functions using the standard minimization procedures in Matlab by computing the best responses for a grid of values between 0 and 1 and then we fit a fourth order polynomial (e.g. $\widetilde{b}(v) = \gamma_1 + \gamma_2 v + \gamma_3 v^2 + \gamma_4 v^3 + \gamma_5 v^4$) to the points. This produces excellent fits to the best response curves with R^2 generally around .999 and with relatively little error when compared to cases for which there are analytically computable results. Expected revenue and efficiencies are then calculated by drawing 1000 value vectors, generating bids according to these bid functions and computing the resulting average revenue and efficiency.

4 Results

We will present results comparing the revenue and efficiency produced by first and second price auctions from three of the cases described in the introduction. In regard to efficiency, we consider only allocative efficiency; in other words, an auction will be 100% efficient if the bidder with the highest v_i wins. We ignore the price preference elements of a bidder's total value in this regard, because incorporating non-pecuniary preferences can lead to the efficient allocation requiring an infinite price. We first present the results achievable in the baseline charity case with comparisons to the standard non-charity case to determine the effects on revenue from the addition of these preferences. We will then examine the See and Be Seen and Raising Rivals' Cost cases.

4.1 Baseline Charity

EW and Engers and McManus (2002) prove that in the symmetric cases involving $\alpha_i = \beta_i$ or even $\alpha_i \neq \beta_i$ with the parameters symmetric across all bidders, the second price or ascending clock auction yields at least as much revenue as the first price. It is also easy to see that in these symmetric cases, efficiency will be 100% in all auctions and it will therefore be equal across institutions.

Figures 1 and 2 contain results from the cases in which $\alpha_i = \beta_i = \alpha_j = \beta_j \in \{0, .15, .3\}$ for auctions involving 2, 4 and 6 bidders for the uniform and normal value distributions respectively[8]. Figure 2 contains more comparisons because with the assumption of normally distributed values we can investigate the effects of beliefs on the revenue. We only present results about mis-specified beliefs for the normal distribution because they result in no important effects in the uniform case. In the second price auction, although beliefs regarding the parameters of other bidders technically enter into the bid function it is clear to see from equation 10 that the beliefs cancel out and have no effect in the event that $\bar{\beta} = \bar{\alpha}$. In the first price auction with uniformly distributed values, beliefs about the parameters of others only figure in through the upper bound. There is no appreciable effect on revenue for the parameters tested. In figure 2, the "0" bar represents the case in which the bidder beliefs are that their opponents parameters are their own multiplied by 0, while the "1" bar represents the case in which the bidders believe their opponents parameters are their own multiplied by 1. We only present different revenue estimates under the different belief specifications for the first price auction as they have no discernible effect on the revenue from the second price auctions.

[8] We have chosen to use graphs in presenting these revenue comparisons to allow for easy visual comparisons in levels. We have a prepared a separate appendix listing all of the numbers behind these tables and claims along with tests of significance available at http://garnet.acns.fsu.edu/~tsalmon.

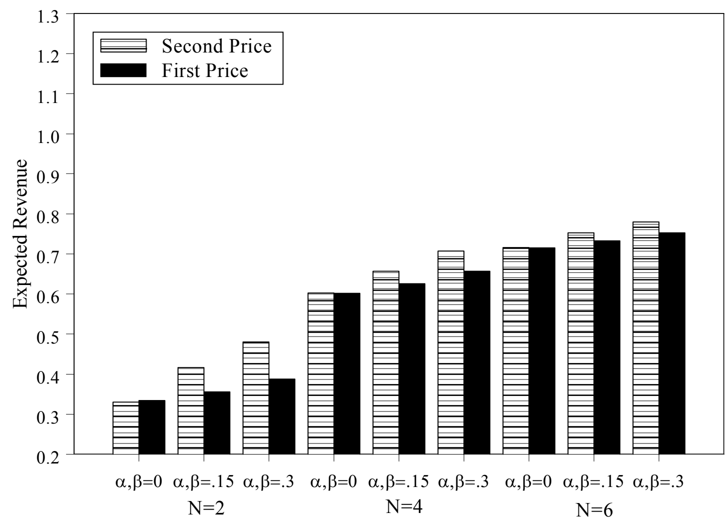

Figure 1: Revenue comparisons between first and second price auctions assuming uniformly distributed values for 2, 4 and 6 bidder auctions. The cases considered involve fully symmetric bidders with the parameters indicated.

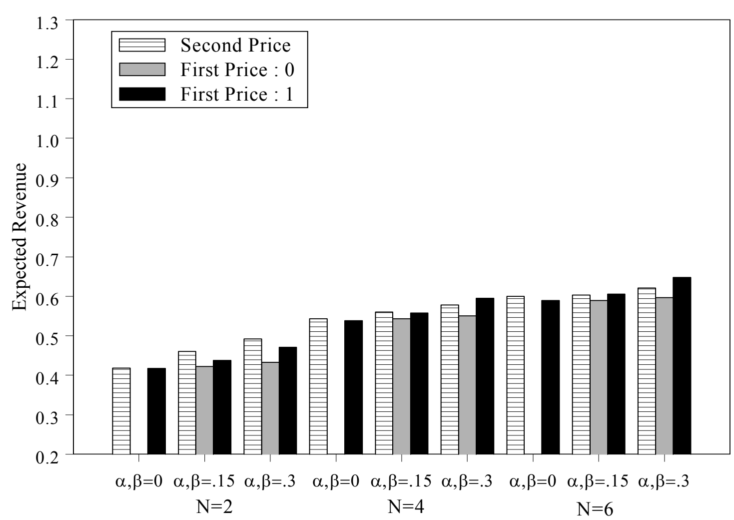

Figure 2: Revenue comparisons between first and second price auctions assuming normally distributed values for 2, 4 and 6 bidder auctions. The cases considered involve fully symmetric bidders with the parameters indicated. The first price 0 and 1 bars repesent cases in which the bidders believe $\bar{\beta}$ and $\bar{\alpha}$ are equal to 0 and 1 times the true parameters.

We see two important results from these graphs. The first is that while the claim that $E[R_S] \geq E[R_F]$ is indeed technically valid, the revenue difference between the first and second price auctions is quite small in most cases. The one case in which the second price does markedly better is the $n = 2$ case with uniform values, but the edge for the second price auction disappears as more bidders are added. In the normally distributed case, the differences are small across the entire range[9]. The small size of the difference is important because it has been observed in countless laboratory experiments that bidders will bid higher than the risk-neutral Nash equilibrium prediction in first price auctions. Consequently, the size of these revenue differences could easily be overwhelmed by such behavior.

Figure 2 contains the second important result which is that in the case of normally distributed values the revenue outcome from a first price auction is highly contingent upon the beliefs of the bidders. The result is that revenue significantly decreases if the bidders underestimate the preferences for charitable giving among the rest of the population. The change in revenue cause by the change in beliefs is of approximately the same order as the change in revenue derived from changing auction formats.

There is an important point embedded into these results that echoes a similar issue discussed in Klemperer (2002b). The argument in Klemperer (2002b) is that auctioneers should not be heavily influenced in their choice of auction designs based on the result found in Milgrom and Weber (1982) that ascending auctions will raise more money than first price auctions when values are affiliated. The reason is that while the Milgrom and Weber (1982) result is technically correct, Riley and Li (1999) shows that even the theoretical revenue difference is of a very small magnitude. This means that if there are any small deviations in the bidding strategies used or in the environment between the standard Milgrom and Weber (1982) theory and a field application, the revenue ranking could very easily be upset. Our claim is identical in that we show that while in the case of these charitable preferences it is possible to show theoretically that the second price raises at least as much money as a first price, the theoretical difference is small and could be easily upset by other environmental factors not included in the basic theory.

4.2 See and Be Seen

The SBS model involves some bidders possessing $\beta_i > 0$ while others possess $\beta_j = 0$ and all bidders possess $\alpha_i = \alpha_j = 0$. This case is intended to represent cases of charitable bidding in which a bidder derives utility from having others see him as giving generously to support the charity while he derives no utility just from the fact that the charity gets more money. A straightforward application of the standard Revenue Equivalence Theorem will show that if all bidders possessed equal β_i's with $\alpha_i = 0$ then expected revenue would be constant across both mechanisms. To see this clearly, consider the case of uniformly distributed values. The equilibrium bid functions for the second price auction and first price auctions are $\frac{v_i}{1-\beta_i}$ and $\frac{n-1}{n}\frac{v_i}{(1-\beta_i)}$ respectively[10]. The effect of the β_i is just to make a bidder who has a value of v_i bid as if his true value were $\frac{v_i}{1-\beta_i}$. Thus with symmetric β_i's, all bidders inflate their bids by the same proportional amount over the case with all β_i's equal to 0 which can be

[9] There are two of the high parameter value cases in the graph showing the first price achieving more revenue than the second price. The actual differences are small, in the range of .01, and are largely from a small degree of rounding in some of the numerical calculations.

[10] Recall that in the standard case, without price preference valuations, the bid functions are v_i and $\frac{n-1}{n}v_i$.

thought of as just an upward shift of the value distribution with people otherwise bidding "normally". No conditions of the RET are violated by such a monotonic transformation of values leading to both mechanisms yielding equivalent revenue and perfect efficiency. Of course with the upward shift in the value distribution, the revenues when $\beta_i > 0$ will not be equivalent to the case when $\beta_i = 0$.

Adding asymmetry to the bidders' parameters will break revenue equivalence and lead to inefficiency but interestingly enough, it will not break efficiency equivalence between the mechanisms. Efficiency equivalence is maintained because the effect of the β_i's is effectively to just re-scale the value that a bidder is bidding as if they possess resulting in a new set of what might be considered "modified" values. Both auction formats turn out to be "efficient" with respect to these modified valuations as the bidder with the highest modified valuation will win in both. That bidder may not have the highest allocative value or v_i though and since we are considering only the v_i's for measuring true efficiency, we will measure the auctions as being occasionally inefficient.

Figures 3 and 4 show the revenue comparisons between first and second price auctions for both low and high β cases for 2, 4 and 6 bidder auctions. Under both low and high[11] β cases, half of the bidders possess $\beta_i = .15$ or $\beta_i = .5$ while the other half possess $\beta_j = 0$. The results from the normal distribution include the results from three different belief specifications. The 0 case involves all bidders expecting their opponents to have parameters equal to 0 times the actual parameters of the bidders with $\beta_i > 0$. The .5 case involves beliefs where these values are multiplied by .5 and the 1 case, multiplied by 1. Seeing all of these cases allows us to examine what happens when people under and overestimate the average parameters in the population as well as get them approximately correct. Again, we only present the mis-specified belief revenue results for the first price case in the normal distribution because there was no effect in the second price revenue totals for either the uniform or normal distributions.

Figure 3 contains the revenue comparison between the auction formats under the assumption of uniformly distributed values. In the low β case, $\beta_i = .15$, the first price auction achieves slightly more revenue than does the second price but in the high β_i case, $\beta_i = .5$, the first price achieves strikingly more revenue than the second price. The reason for this is that in both first and second price auctions, the bidders with $\beta_i > 0$ inflate their bids over what the $\beta_j = 0$ bidders would bid at the same values and these $\beta_i > 0$ bidders win much more often. In the first price case, the auctioneer gets the full benefit of this inflated bid. In the second price case, the auctioneer may see a very high bid from the $\beta_i > 0$ bidder, but the price the auctioneer receives might be set by the $\beta_j = 0$ bidder which is not inflated. This can cause the revenue in the second price auction to be significantly lower than the first price for large values of β_i. The effect is stronger in the two bidder case as it is almost always the $\beta_j = 0$ bidder setting the price in the second price auction while in the 4 and 6 bidder cases, the prices are occasionally set by the other $\beta_i > 0$ bidders.

In the case of normally distributed values, the same effect holds to some extent, but as figure 4 shows, the beliefs on the part of the bidders are absolutely crucial to the revenue ranking. In the cases involving the bidders underestimating the charitable giving of their

[11] We chose to use a higher value for the high β value for the SBS case and RRC case we will present later because of a belief that such extreme values are more plausible in these cases and we wanted to explore the effect of such extreme preferences. We retained the same lower value for consistency with the baseline charity case.

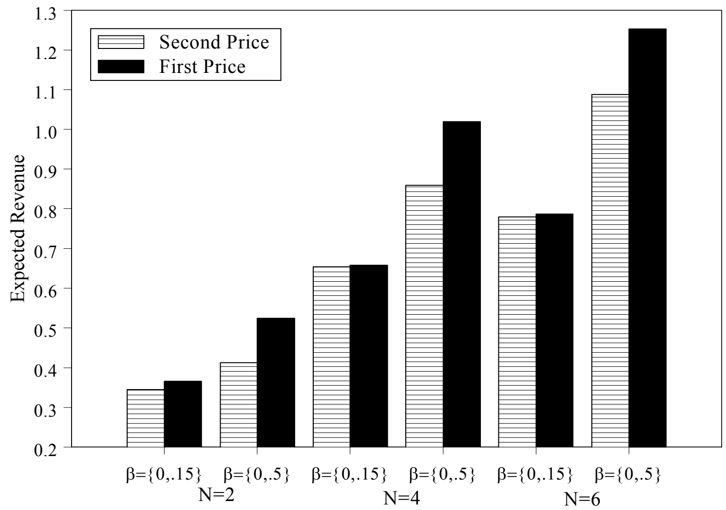

Figure 3: Revenue comparisons for SBS model assuming uniformly distributed values for auctions with 2, 4 and 6 bidders. The cases shown involve half of the bidders possessing each of the indicated β's while $\alpha_i = \alpha_j = 0$.

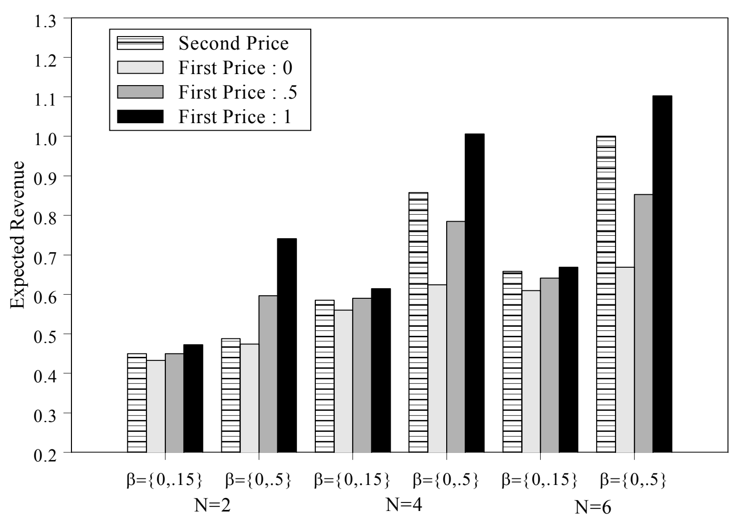

Figure 4: Revenue comparisons for SBS model assuming normally distributed values for auctions with 2, 4 and 6 bidders. The cases shown correspond to half the bidders possessing each of the noted β's while $\alpha_i = \alpha_j = 0$. For each case, there are three belief specifications with bidders believing that $\bar{\beta}$ is equal to 1,.5 or 0 times the high β.

opponents, the revenue of the first price plummets to well below the revenue achieved by the second price. As the perceived preferences for charitable giving increase, revenue increases such that when this is overestimated, the revenue from the first price is significantly greater than the second price. When expectations are approximately accurate, the revenue ranking is indeterminate. In practical terms it appears that for the first price auction to dominate the second price, all participants must think that just about all of the other participants are "show-offs" or high β types.

The reason for this pattern is that in a first price auction, a bidder bids by estimating the minimum amount he must bid in expectation to shut out his closest rival. If he believes his closest rival to have a low β, then he can shade his bid by much more than if he expects his opponent to have a high β. Bidders in second price auctions again are unconcerned with the preferences of their opponents so long as their α's are zero and therefore revenue is unchanged across the changing beliefs. These results demonstrate that asymmetries across bidders and the beliefs possess concerning the preferences of other can have important impacts on revenue that are completely overlooked by the revenue equivalence result obtainable with symmetric β_i's.

While efficiency is identical across both institutions, the asymmetric parameters do introduce inefficiency. In the low β_i case, efficiencies are around .98 while the high β_i case drops the efficiencies down to around .91 . These efficiency numbers are approximately equivalent for both value distributions and only drop slightly as the number of bidders increases.

4.3 Raising Rivals' Cost

When bidders are interested in raising the costs of their rivals (RRC), it seems that simple "common sense" would lead one to suppose that this should lead to increased auction revenue. Our results will very clearly show the exact opposite, at least when this motivation is modeled as we have done here. Understanding the reason for this result will prove quite interesting.

Figure 5 shows the revenue comparisons for the RRC case under the assumption of uniformly distributed values. We have again presented examples of low and high values of the parameters or $\alpha_i = .15$ and $\alpha_i = .5$. We have also constructed each case such that half of the bidders possess this $\alpha_i > 0$ while the other half possess $\alpha_j = 0$ and all bidders possess $\beta_i = \beta_j = 0$. This is the first time that the different belief structures have an effect on revenues in the second price auction and thus we have included bars for the second price revenue under the 0,.5 and 1 specifications. The different belief specifications, as usual, have no discernible impact on the revenue in the first price auction when assuming uniformly distributed values. For ease of comparison, we have also included a baseline bar of the revenue achievable in a standard auction with $\alpha_i = \beta_i = 0$ for all bidders. This allows us to see very clearly that when $n > 2$, revenue in both auctions is decreasing in α_i. When $n = 2$ and α_i is low, this can lead to slightly more revenue than the standard case. We can also clearly see that revenue is decreasing in the beliefs about the opponents value of $\bar{\alpha}$ for the second price auctions. This should be expected as $\frac{\partial\left(\frac{v_i(1+\bar{\alpha})+\alpha_i}{(1+2\alpha_i)(1+\bar{\alpha})}\right)}{\partial\bar{\alpha}} = -\frac{\alpha_i}{(1+2\alpha_i)(1+\bar{\alpha})^2} < 0$. Note that since all of the $\alpha_i > 0$ bidders will have $\alpha_i \geq \bar{\alpha}$ and all others have $\alpha_j = 0$, the lower bound is never an issue.

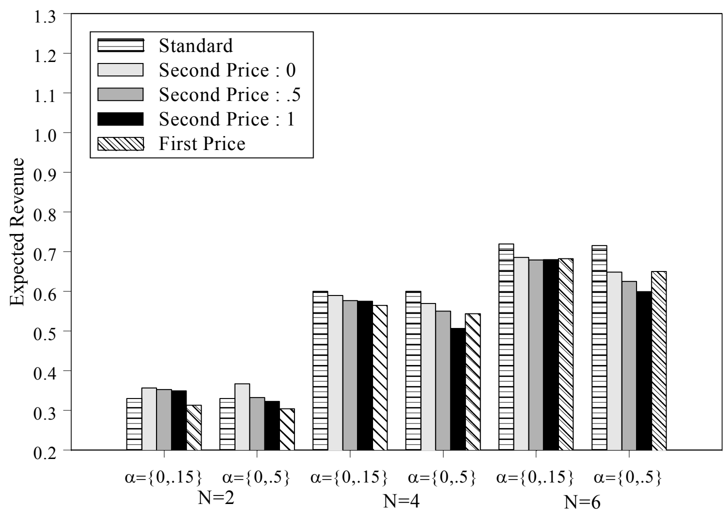

Figure 5: Revenue comparisons for RRC model assuming uniformly distributed values for auctions with 2, 4 and 6 bidders. The cases shown correspond to half the bidders possessing the α's indicated while $\beta_i = \beta_j = 0$. The standard bar is the revenue expected without price preferences while the three second price bars correspond to expectations that $\bar{\alpha}$ is equal to 0, .5 or 1 times the high α.

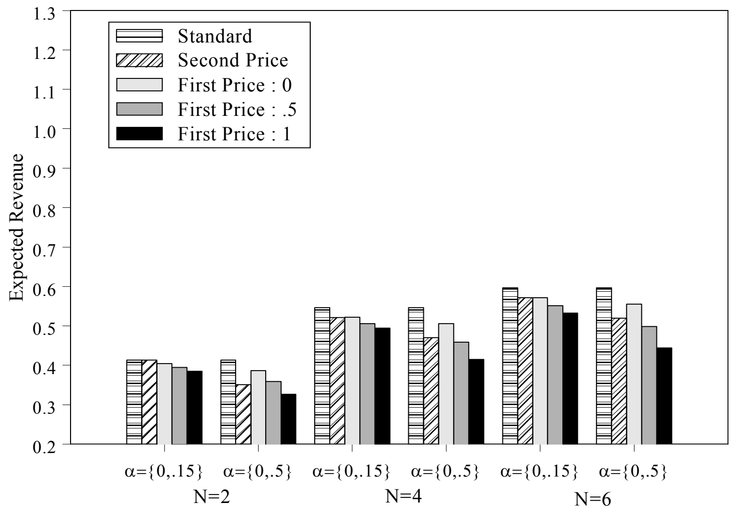

Figure 6: Revenue comparisons for RRC model assuming normally distributed values for auctions with 2, 4 and 6 bidders. The cases shown correspond to half the bidders possessing the α's indicated with $\beta_i = \beta_j = 0$. The standard bar is the revenue expected without price preferences while the three first price bars correspond to expectations that $\bar{\alpha}$ is equal to 0, .5 or 1 times the high α.

Neither auction format performs significantly better in insulating the auctioneer against this drop in revenue, though the best case for the auctioneer is for all bidders to believe that no other bidders possess a positive value of $\alpha > 0$. These results are echoed in the case of normally distributed values as shown in figure 6. This figure contains the revenue achievable by a second price auction and then that achievable by a first price auction under three different belief conditions along with the standard revenue from assuming $\alpha_i = \beta_i = 0$. The different belief specifications had no effect on the revenue in the second price auctions, but they did affect the first price auctions. The results show again that revenue is generally decreasing in α and that the best case for the auctioneer in the event that positive α's exist is for all bidders to assume otherwise.

The reason for higher α_i's reducing revenue in first price auctions is fairly straightforward. In a first price auction, if a bidder derives utility from his opponent paying a higher price, there is unfortunately nothing that bidder can do to make his opponent pay a higher price. The effect of the α term serves only to make losing more attractive. The higher the α, the more attractive is losing and thus the lower a bidder will bid in an attempt to win. This effect is easy to show in the case of uniformly distributed values as $\frac{\partial\left(B_F^*(v_i,\beta_i,\alpha_i,\bar{\beta},\bar{\alpha})\right)}{\partial\alpha_i} = -\frac{(n-1)^2}{(\alpha_i-n-n\alpha_i+n\beta_i)^2}v_i < 0$.

In the second price auction, the effect of α_i is more complex. For example, in the case of uniformly distributed values, $\frac{\partial\left(B_S^*(v_i,\beta_i,\alpha_i,\bar{\beta},\bar{\alpha}))\right)}{\partial\alpha_i} = \frac{1-2v_i-2v_i\bar{\alpha}}{(1+2\alpha_i)^2(1+\bar{\alpha})}$ possesses an indeterminate sign. It is positive so long as $v_i \leq \frac{1}{2+2\bar{\alpha}}$ and negative otherwise. To see the effect under

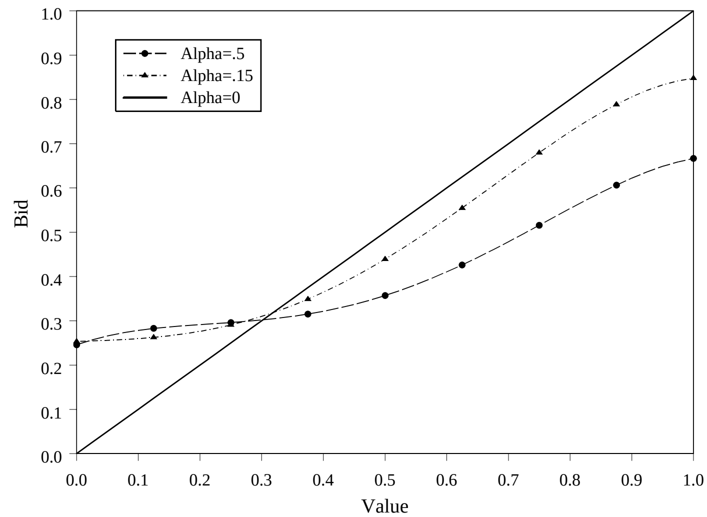

Figure 7: Bid functions for second price auctions assuming normally distributed values for three different values of α with $\beta = 0$.

normally distributed values it is easier to examine some examples of bid functions. Figure 7 contains sample bid functions for normally distributed values for $\alpha_i = 0$, $\alpha_i = .15$ and $\alpha_i = .5$ with the bidders assuming that $\bar{\alpha} = \alpha_i$ [12]. These show that for values less than a threshold, the α causes the bidder to inflate their bid over what an $\alpha_i = 0$ bidder would use but for values greater than this point, the bids are lower. Also, the $\alpha_i = .5$ bids are the lowest.

The curious part is why this is so and whether or not this can be rationalized with the idea of a bidder being interested in raising a rival's cost. The issue is made more complicated when viewed in light of the results in Morgan, Steiglitz, and Reis (2002) (MSR). In MSR, the authors construct a model in which bidders are motivated by spite. Their model assumes that bidder i suffers a utility loss of $\alpha(v_j - p)$ when they lose the auction to bidder j. In other words, bidders have a goal of minimizing the surplus that their rival achieves. MSR shows that such a motivation leads bidders to regularly bid above their value in second price and ascending auctions. This is a curious juxtaposition of results, as both our framework and the MSR framework would seem suited to modeling similar behavior. A careful understanding of the reasons the two approaches arrive at the different results though reveals that the two approaches are modeling subtly different behavior.

In our framework, if a bidder has a low value and is likely to lose, he bids specifically to make his rival pay a higher price. For a bidder with a higher value who would likely win, his focus changes. As the price rises, losing becomes relatively more attractive. He ends up

[12] This figure shows what might be thought of as a counterintuitive result which is that both $\alpha_i = .15$ and $\alpha_i = .5$ bid functions possess approximately the same lower bound. This is not an error. The true lower bounds are 0.253 for the $\alpha_i = .15$ case and 0.249 for $\alpha_i = .5$ which can be derived from the analytical symmetric bid function.

bidding under his value because if he bids a little less than v_i and loses but would have won bidding v_i, he does not decrease his expected value much because there are few such cases and the profit he would have made in them is small. The bidder instead gains higher utility from the $\alpha_i p$ term by allowing the other bidder to win at the high price. The predatory interpretation of this might be that they are avoiding the cases in which they win with little profit, preferring instead for their rival to win at a higher price. If one supposes that the motivation for having an $\alpha_i > 0$ is to harm a competitor the harm might be that if these two rivals are in repeated competition, the $\alpha_i > 0$ bidder has forced the other bidder to pay a high price in the current auction and perhaps tie up resources that the other competitor might otherwise put into a subsequent auction. If true, that could allow the $\alpha_i > 0$ bidder to win the next auction at a higher surplus than otherwise possible. Thus while the result of revenue decreasing in α_i is initially somewhat counter-intuitive, it appears to be quite sensible in terms of what might motivate bidders to engage in such behavior.

It is interesting to note that this result is quite similar to the one shown in Pitchik and Schotter (1988). In that paper, the authors derive equilibria for sequential auctions with budget constrained bidders. One of the properties of their bid functions is:

> Property 2: Under Second price rules, bidder i strictly prefers to lose good 1 to bidder $j \neq i$ at higher prices rather than lower prices so long as the higher price is at least as great as c^i.

In this case, the incentive to raise rivals cost was derived endogenously into a multiple period auction model and the effect on the bidding strategy seems quite similar. The advantage of our method of modeling this motivation is that it can be applied to a broader range of situations. One could also think of using a framing like this as a heuristic for deriving approximately accurate bidding strategies for sequential auctions with longer timelines than the two modeled in Pitchik and Schotter (1988). This is because solving for the truly optimal bid functions would get quite difficult as the number of items increases.

If we try to explain the MSR result in a similar manner, it is no longer necessarily true that as your value rises, the idea of losing at a price slightly below your value becomes attractive. If a bidder i bids below his value and if any bidder j wins, i has made j strictly better off. This in turn makes i strictly worse off. Thus it is not attractive to bid below one's value in the MSR framework. In fact, due to the possibility of making the eventual winner less happy, it becomes attractive to bid above your value. Subtly different ways of modeling similar phenomena, therefore, have substantially different effects on bidding behavior and revenue. It appears that the MSR framework is likely best suited to the notion of pure spite while our framework is better suited to an indirect pecuniary motivation to raise the price paid by a rival.

For an auctioneer concerned with efficiency, RRC is the first case that is not efficiency equivalent. In practical terms, however, efficiency is approximately equivalent as neither format consistently outperforms the other.

5 Conclusion

We have investigated the consequences for auction outcomes stemming from situations in which bidders possess preferences over the revenue achieved by the auctioneer. In doing so, we have been careful to also investigate the effects of different belief specifications on the outcome of such auctions. This allows us to point out several issues that may be important to those wishing to conduct auctions in such environments. Ultimately, the importance of these issues is an empirical phenomenon which we will be seeking to carefully test in future work.

The first point to make is that despite the results in EW and Engers and McManus (2002), an auctioneer should be quite skeptical of the purported revenue benefits of using a second price or ascending auction instead of a first price auction for standard charity auctions. While it does seem the case that many charity auctions are run with a roughly ascending format, see Isaac and Schnier (2002) for more details, that should not be taken as *prima facia* evidence that they raise more revenue in the field. There may be many other institutional considerations that make them more attractive. Alternatively, if there are revenue benefits from using them, they may well be derived from other issues such as bidders being more comfortable participating in them rather than sealed bid auctions, which is an effect demonstrated in Ivanova-Stenzel and Salmon (2004).

Secondly, the results based on different belief specifications are useful for practical auctioneers as they may indicate possibilities for the auctioneer to manipulate beliefs or expectations in their favor. For example, in the See and Be Seen case, it was found that revenue was strongly increasing in the belief bidders had about the β's of the other bidders and that first price auctions could have a revenue edge over ascending. If an auctioneer can somehow transmit information to the bidders about how great the preference is of others to give to the charity or in some other way shift bidders' beliefs about the β's of their rivals, then this could benefit the auctioneer. Similarly, if bidders are deriving their utility from being seen as generous, the more the auctioneer can do to facilitate the visibility of such winners, the more willing they are likely to be to bid higher. A design change to take advantage of such preferences might be to have a number of sealed bid auctions that close at different times during the night of the charity auction in which the auctioneer makes public announcements after each auction, naming exactly who was the winning bidder and how much they bid. The general difficulty of running sealed bid auctions for such purposes is that some bidders might leave after placing their bids. Thus doing things like holding several over the course of the night or having other things going on to make sure bidders stay around to make the bid revelation more visible may be helpful.

If, however, an auctioneer believes that there may be bidders in their population who have an interest in raising their rivals' costs, our results suggest other important issues for the auctioneer to consider. At first thought, such bidders would seem helpful to an auctioneer who might be tempted to either encourage or at least not discourage such behavior. Our results suggest that the auctioneer should discourage such behavior as much as possible because revenue is decreasing in α_i for $n > 2$. The auctioneer should also work to convince bidders that such predatory bidders are rare.

In our introduction we briefly mentioned several other situations that are encompassed by our model but that we have not investigated here. Going through all of them is beyond the scope of the paper, but our methods can be extended to each of them. From preliminary investigations with the shill-bidding case, we can say that using a price-preference contract to motivate a shill is almost always revenue decreasing for the auctioneer. The auctioneer can almost always do better with a reserve price than they can with a shill. The reasons should be fairly obvious from the results above based on the effect of raising α_i on the revenue of the auctioneer. This is a topic of further investigation.

Appendix A

Bidding Strategy in Symmetric First Price Auctions

If we hypothesize the existence of the bid function $b_f^*(v_i, \beta, \alpha)$ that is used by the other $n-1$ bidders each of whom possess the same parameters for β and α, we can find what its form must be by determining what value bidder i would choose to submit to such a bid function. Since in this case, the bid function will be symmetric, we can see that the probability of winning if i bids as if their value were r is $\Pr(b_f^*(r, \beta, \alpha) > b_f^*(v_j, \beta, \alpha)$ for all $j) = \Pr(r > v_j$ for all $j) = F(r)^{n-1}$. Letting $\rho(r) = F(r)^{n-1}$ define the probability of winning given that bidder i has bid according to value r, we have:

$$\max_r S(v_i, r) = (v_i - (1-\beta) b_f^*(r, \beta, \alpha))\rho(r) + \alpha \int_r^1 b_f^*(t, \beta, \alpha) d\rho(t) \tag{11}$$

Equilibrium condition is

$$\frac{\partial(S(v_i, r))}{\partial r}|_{r=v_i} = 0 \tag{12}$$

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noting that $\frac{\partial\left(\int_p^y f(x)dx\right)}{\partial p} = -f(p)$

$$v_i \rho'(r) - (1-\beta) b_f^*(r, \beta, \alpha)\rho'(r) - (1-\beta)\rho(r) b_f^{*\prime}(r, \beta, \alpha) - \alpha b_f^*(r, \beta, \alpha)\rho(r) = 0 \tag{13}$$

$$(1-\beta+\alpha) b_f^*(r, \beta, \alpha)\rho'(r) + (1-\beta)\rho(r) b_f^{*\prime}(r, \beta, \alpha) = v_i \rho'(r) \tag{14}$$

If we multiply both sides by $\rho(r)^{\frac{\alpha}{1-\beta}}$ we get:

$$(1-\beta+\alpha) b_f^*(r, \beta, \alpha)\rho(r)^{\frac{\alpha}{1-\beta}}\rho'(r) + (1-\beta_i)\rho(r)^{\frac{1-\beta+\alpha}{1-\beta}} b_f^{*\prime}(r, \beta, \alpha) = v_i \rho'(r)\rho(r)^{\frac{\alpha}{1-\beta}} \tag{15}$$

Which allows us to write the left side as $\frac{\partial\left((1-\beta_i) b_f^*(r,\beta,\alpha)\rho(r)^{\frac{1-\beta+\alpha}{1-\beta}}\right)}{\partial r}$ and also noting that in equilibrium $r = v_i$ we have

$$\frac{\partial\left((1-\beta) b_f^*(r, \beta, \alpha)\rho(r)^{\frac{1-\beta+\alpha}{1-\beta}}\right)}{\partial r} = v_i \rho'(v_i)\rho(v_i)^{\frac{\alpha}{1-\beta}} \tag{16}$$

Since this condition must hold for all values, we can integrate both sides between 0 and v_i:

$$\int_0^{v_i} \frac{\partial\left((1-\beta) b^*(t, \beta, \alpha)\rho(t)^{\frac{1-\beta+\alpha}{1-\beta}}\right)}{\partial t} dt = \int_0^{v_i} t\rho'(t)\rho(t)^{\frac{\alpha_i}{1-\beta_i}} dt \tag{17}$$

Using boundary condition that $b_f^*(0,\beta,\alpha)=0$, we get

$$(1-\beta)b_f^*(v_i,\beta,\alpha)\rho(v_i)^{\frac{1-\beta+\alpha}{1-\beta}} = \int_0^{v_i} t\rho'(t)\rho(t)^{\frac{\alpha}{1-\beta}}dt \tag{18}$$

$$b_f^*(v_i,\beta,\alpha) = \frac{\int_0^{v_i} t\rho'(t)\rho(t)^{\frac{\alpha}{1-\beta}}dt}{(1-\beta)\rho(v_i)^{\frac{1-\beta+\alpha}{1-\beta}}} \tag{19}$$

since $\rho(x)=F(x)^{n-1}$ then $\rho'(x)=(n-1)f(x)F(x)^{n-2}$ so

$$b_f^*(v_i,\beta,\alpha) = \frac{\int_0^{v_i} t\,(n-1)\,f(t)F(t)^{\frac{(n+\beta(2-n)+\alpha(n-1)-2)}{1-\beta}}dt}{(1-\beta)(F(v_i)^{n-1})^{\frac{1-\beta+\alpha}{1-\beta}}} \tag{20}$$

This will be a valid equilibrium bid function so long as it is differentiable and monotonically increasing which can be shown by extension of Engelbrecht-Wiggans (1994) or as in Engers and McManus (2002).

Bid Function in Symmetric Second Price Auctions

For this case we must consider three possibilities. First is what I expect to get if I win, second is what I get if I come in second and third is what I expect if I come in less than second. We again assume that some monotonic and differentiable bid function $b_s^*(v)$ exists and we wish to check to see if i wants to bid as if they possess some value r instead of v_i.

$$\begin{aligned} S(v_i,r) = \int_0^r (v_i-(1-\beta)b_s^*(t,\beta,\alpha))dF(t)^{n-1} + \\ \alpha b_s^*(r,\beta,\alpha)(n-1)F(r)^{n-2}(1-F(r)) + \\ \alpha\left(\int_r^1 b_s^*(t,\beta,\alpha)(n-2)(n-1)F(t)^{n-3}(1-F(t))dF(t)\right) \end{aligned} \tag{21}$$

Equilibrium condition is again that $\frac{\partial(S(v_i,r))}{\partial r}|_{r=v_i}=0$

Taking the derivative we get:

$$\begin{aligned} v\frac{dF(r)^{n-1}}{dr} + b_s^*(r)(\beta-1)\frac{dF(r)^{n-1}}{dr} - b_s^*(r,\beta,\alpha)\,\alpha\,(1-F(r))\,F(r)\,(n-2)\frac{dF(r)^{n-1}}{dr} \\ + b_s^*(r,\beta,\alpha)\alpha F^{n-2}(r)(n-1)(1-F(r)) - b_s^*(r)\,\alpha\frac{\partial F(r)^{n-1}}{\partial r} \\ + b_s^*(r,\beta,\alpha)\,\alpha F(r)(1-F(r))(n-2)\frac{\partial F(r)^{n-1}}{\partial r} = 0 \end{aligned} \tag{22}$$

This will simplify to

$$vf(r) = b_s^*(r,\beta,\alpha)(1-\beta+\alpha)f(r) - b_s^*(r,\beta,\alpha)\alpha\,(1-F(r)) \tag{23}$$

multiply everything by $-\frac{(1-F(r))^{\frac{1-\beta}{\alpha}}}{\alpha_i}$ and get

$$-vf(r)\frac{(1-F(r))^{\frac{1-\beta}{\alpha}}}{\alpha} = -b_s^*(r,\beta,\alpha)(1-\beta+\alpha)f(r)\frac{(1-F(r))^{\frac{1-\beta}{\alpha}}}{\alpha} + b_s^*(r,\beta,\alpha)\left(1-F\left(r\right)\right)^{1+\frac{1-\beta}{\alpha}} \tag{24}$$

Notice that $(1-F(\overline{x}))=0$ if $\overline{x}$ is the max of the distribution. Also, since this condition must hold for all v, we can integrate both sides:

$$\int_r^1 \frac{\partial(b_s^*(t,\beta,\alpha)(1-F(t))^{\frac{1+\alpha-\beta}{\alpha}})}{dt}dt = \int_r^1 \frac{t}{1+\alpha-\beta}\frac{\partial((1-F(t))^{\frac{1+\alpha-\beta}{\alpha}})}{\partial t}dt \tag{25}$$

In equilibrium $r=v$, and we also need an obvious correction for the case $\alpha_i = 0$.

$$b_s^*(v,\beta,\alpha) = \begin{cases} \frac{1}{\alpha}\frac{\int_v^1 t(1-F(t))^{\frac{(1-\beta)}{\alpha}}dF(t)}{(1-F(v))^{\frac{1+\alpha-\beta}{\alpha}}} & if \quad \alpha > 0 \\ \frac{v}{1-\beta} & if \quad \alpha = 0 \end{cases} \tag{26}$$

References

Corns, A. and A. Schotter (1999). Can Affirmative Action Be Cost Effective? An Experimental Examination of Price-Preference Auctions. *American Economic Review 89*(1), 291–305.

Davis, D. D., L. Razzolini, R. Reilly, and B. J. Wilson (2003). Raising Revenue for Charity: Auctions versus Lotteries. Working Paper Virginia Commonwealth University.

de Frutos, M. A. (2000). Asymmetric Price-Benefits Auctions. *Games and Economic Behavior 33*, 48–71.

Engelbrecht-Wiggans, R. (1994). Auctions with Price Proportional Benefits to Bidders. *Games and Economic Behavior 6*, 339–346.

Engers, M. and B. McManus (2002). Charity Auctions. Working Paper, University of Virginia.

Goeree, J. K., E. Maasland, S. Onderstal, and J. L. Turner (2004). How (Not) to Raise Money. CREED Working Paper.

Isaac, R. M., T. C. Salmon, and A. Zillante (2005). A Theory of Jump Bidding in Ascending Auctions. *Journal of Economic Behavior and Organization*. Forthcoming.

Isaac, R. M. and K. Schnier (2002). Run Silent, Run Cheap? A Study of a Charity Auction Mechanism. Working Paper, Florida State University.

Ivanova-Stenzel, R. and T. C. Salmon (2004). Bidder Preferences Among Auction Institutions. *Economic Inquiry 42*, 223–236.

Kagel, J. H. (1995). Auctions: A Survey of Experimental Research. In J. H. Kagel and A. E. Roth (Eds.), *The Handbook of Experimental Economics*, pp. 501–586. Princeton, New Jersey: Princeton University Press.

Klemperer, P. (2002a). Some Observations on the British 3G Telecom Auction: Comments on Börgers and Dustmann. *ifo Studien 48*, 115–120.

Klemperer, P. (2002b). Using and Abusing Economic Theory - Lessons from Auction Design. Working Paper Oxford University.

McAfee, P. (1992). Amicable Divorce: Dissolving a Partnership with Simple Mechanisms. *Journal of Economic Theory 56*, 266–293.

Milgrom, P. R. and R. J. Weber (1982). A Theory of Auctions and Competitive Bidding. *Econometrica 50*(5), 1089–1122.

Morgan, J., K. Steiglitz, and G. Reis (2002). The Spite Motive and Equilibrium Behavior in Auctions. Working Paper.

Pitchik, C. and A. Schotter (1988). Perfect Equilibria in Budget-Constrained Sequential Auctions: An Experimental Study. *RAND Journal of Economics 19*(3), 363–388.

Riley, J. G. and H. Li (1999). Auction Choice. mimeo, University of California Los Angeles.

Shachat, J. and J. T. Swarthout (2002). Procurement Auctions for Differentiated Goods. Working Paper, IBM Research Labs.