Full Duplex Multiuser MIMO MAC Protocol (FD-MUMAC)

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Abstract—A new medium access control (MAC) protocol for full-duplex multiuser multi input multi output (FD-MUMAC) is proposed. FD-MUMAC is an 802.11 infrastructure-based MAC protocol where the communication in the network is moderated by a full-duplex access point (AP) that can handle multiple simultaneous uplink and downlink streams from half-duplex users. FD-MUMAC is the first MAC protocol that jointly determines the selection of uplink and downlink users and controls the transmission rate by considering transmit and receive beamforming, channel states, and multiuser-interferences. To demonstrate the usefulness of the proposed protocol, we introduce a joint fairness selection algorithm that ensures that each user obtains a fair allocation of time to utilize the channel and balances its traffic between the uplink and downlink directions. In one specific instantiation, FD-MUMAC achieves a throughput gain of 59%, 177% and 94% compared with a single antenna FD MAC protocol and two half-duplex MU-MIMO protocols, respectively.

Keywords—FU-MUMAC, MU-MIMO, MAC, WLAN.

I. INTRODUCTION

The rapid increase in traffic demand has led the investigation for new techniques to increase the spectral efficiency. One candidate technique to fulfill this demand is the in-band full-duplex (IBFD) technology. IBFD enables a node to transmit and receive simultaneously using a single frequency band, and it has the potential to double the spectral efficiency [1]. However, IBFD’s main challenge is the suppression of the self-interference (SI). Numerous SI reduction techniques have been proposed that combine propagation with analog and digital domain cancellation techniques [2][3]. IBFD can also be combined with a multi-antenna access point (AP) to simultaneously support multiple uplink and downlink users. However, such a system requires appropriate medium access control (MAC). The MAC protocol must also enable the selection of users based on the underlying channel conditions, the time-varying traffic, user demands, and the level of the inter-user interference (IUI) from the uplink users [4] [5][6].

IEEE 802.11ax is the first half-duplex (HD) standard that supports MU-MIMO for uplink and downlink transmission[4]. There are multiple IBFD MAC protocols [7][8][9] that are designed for an AP with IBFD capability and HD users. These protocols serve only one uplink user and one downlink user simultaneously. Also, there are multiple HD MU-MIMO MAC protocols [10][11] that integrate multiuser functionalities for uplink and downlink traffic. However, only a few MAC protocols are designed specifically for IBFD MU-MIMO infrastructure-based systems. The first method uses the idle time in a full-duplex transmission to construct a secondary transmission[12]. The resulting protocol, however, is not a simultaneous multiuser FD transmission. The second method uses a trigger frame to activate IBFD MU-MIMO after the AP selects uplink and downlink clients. However, this method assumes the AP has full knowledge of interference and lacks a selection mechanism, which is not practical and also unfair for the users. Most of these prior works and this paper only consider HD clients due to the challenges of implementing complex SI cancellation at the mobile nodes. The main contributions of this paper are as follows.

- The proposed FD-MUMAC protocol provides an integrated selection approach for uplink and downlink users. The protocol also provides a mechanism for collecting the required channel state and interference information for uplink and downlink MU beamforming.

- FD-MUMAC allows the system designer to determine the trade-off between maximizing the total rate and achieving the desired fairness metric. The protocol also allows trade-off in fairness or rate in one traffic direction (such as downlink) to determine the choice of users in the other direction (such as uplink). As an example, for an HD user who has not received much downlink data, their uplink RTS will not receive a CTS from the AP to ensure that downlink data can be sent to that user. This coupling is an essential difference between the current protocol and other existing MAC protocols. For illustration, we use the Jain’s fairness index [13] to measure the fair allocation of resources across users.

- We compare the performance of FD-MUMAC with A-duplex [9], which is a single antenna FD MAC protocol, Uni-MAC [10] and PD-MAC [11], which are MU-MIMO half-duplex MAC protocols. FD-MUMAC increases the average throughput by 58%, 104% and 58.25%, respectively, over A-Duplex, Uni-Mac and PD-MAC.

- To illustrate the flexibility of FD-MUMAC, we study 3 different implementation variations. In one case, we select users to maximize throughput. In the second case, we select users based on the fairness metric. Finally, we also consider a case where the AP selects users at random. The fairness-based selection achieves 97.1% and 98.7% of the throughput of the max-rate and random selection methods, respectively. Also, it balances the uplink and downlink throughput for each user.
The rest of the paper is organized as follows. Section II shows the system model and problem formulation. We introduce FD-MUMAC in Section III. In Section IV, extensive simulation results of FD-MUMAC are provided. Finally, we conclude the paper in Section V.

Notations: We use boldface capital letters to express matrices. We use $X^T$ and $X^H$ to denote the transpose and the Hermitian form of a matrix $X$. We use $V'$ to denote the complement operation of the set $V$.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we define the system model and then formulate the MAC design for IBFD multiuser systems.

Fig. 1. Full-duplex MU MIMO System model. SI is the self-interference, IUI is the interuser interference, $U_i$ and $D_i$ represent, respectively, the uplink and downlink users.

A. System Model

We consider an infrastructure-based wireless local area network (WLAN) system with one AP equipped with $N$ antennas and $M$ single-antenna users, as shown in Fig. 1. The AP supports IBFD communication and can serve up to $N$ uplink and $M$ downlink streams simultaneously. The $M$ users are divided into uplink users $U_L = \{U_1, .., U_J\}$ and downlink users $D_L = \{D_1, .., D_K\}$, where $K+J \leq M$ during the transmission time. The received signal, $Y_{AP}$, at the AP from the uplink users is:

$$Y_{AP} = \sum_{j=1}^{J} H_{Uj} x_{Uj} + \sum_{k=1}^{K} G_{Dk} x_{Dk} + n_u$$  \(1\)

Here, $H_{Uj} \in \mathbb{C}^{N \times 1}$ is the channel matrix for uplink user $j$; $x_{Uj}$ transmitted signal of user $j$; $G_{Dk} \in \mathbb{C}^{N \times N}$ represents the channel between the antennas of the AP, which is the self-interference channel; $x_{Dk} \in \mathbb{C}^{N \times 1}$ represents the transmitted signals from the AP to user $k$; $n_u \in \mathbb{C}^{N \times 1}$ is the additive white gaussian noise (AWGN) with zero mean and covariance $I$. The received signal, $Y_k$, at node $k$ is:

$$Y_k = H_{Dk} x_D + \sum_{j=1}^{J} H_{jk} x_{Uj} + n_k$$  \(2\)

Here, $H_{Dk} \in \mathbb{C}^{1 \times N}$ is the channel between user $k$ and the AP; $x_D \in \mathbb{C}^{N \times 1}$ is the transmitted signal from AP antenna to the downlink users; $n_k$ is the AWGN with zero mean and unit variance; $H_{jk}$ is the channel between uplink user $j$ and downlink user $k$ and $x_{Uj}$ is the uplink signal from user $j$. We use the channel model $H_{L,U} = \partial_{L,U} H_{U,U}$ where $\partial_{L,U}$ is the path loss between node $i$ and node $i'$ and $\overline{H_{L,U}}$ is a complex independent and identically distributed random variable with zero mean and unit variance. We use the beamforming technique and the pathloss model in [14] for our model. Then, we calculate the signal to interference and noise ratio (SINR) and the received signal strength indicator (RSSI) for each user after the beamforming to select the proper transmission rate for each user.

B. Problem Formulation

We assume that the AP and $M$ users are backlogged continuously (i.e., they have data to send all the time). The AP wants to send data to $K$ downlink users while receiving data from the uplink users. However, the AP has a limit of receiving a maximum of $N$ streams and cannot receive from all uplink users at once if $M > N$. Furthermore, the AP cannot transmit data for the selected half-duplex users if they are sending an uplink data. These assumptions and restrictions introduce the following three essential requirements for deploying IBFD MU-MIMO. (i.) First, the MAC protocol and physical layer transceiver must be able to distinguish multiple uplink users from the received signals at the AP by using a multiuser detector (MUD). In addition, the downlink users must be able to decode their signals from the AP and mitigate the active uplink users’ IUIs. This problem is called multiuser interference cancelation (MUIC). The MUD and MUIC problems can be solved by using zero-forcing beamforming or minimum mean squared error (MMSE) beamforming. (ii.) The second requirement is the acquisition of channel state information (CSI) by the beamformer, which is the AP in our case. The AP can fulfill this requirement by acquiring the CSI explicitly (i.e., asking each user to report its channel) or implicitly by calculating the CSI from any received frame. (iii.) Finally, the MAC protocol must address the selection method for the uplink and downlink users. This method should attain the metrics that are defined by the network designer. We consider maximizing the throughput, reducing the average packet delay, and ensuring fairness among the metrics for developing the user selection in FD-MUMAC.

III. FD-MUMAC

In this section, we introduce the frame structure of the FD-MUMAC protocol. Then, we define the management packets format. Finally, we introduce the controlled fair selection algorithm that sustains fairness in FD-MUMAC.

A. FD-MUMAC Frame Structure

FD-MUMAC is a multi-stage protocol where the AP controls the length of each stage based on the 802.11 DCF. In the first stage, the AP tentatively selects downlink users from $M$ available users. Then, the AP broadcast a beacon to all users that waiting for the back-off timer and sensing an idle channel. The AP acquires the CSI of the uplink user during its RTS transmission while other users consider this transmission as a possible IUI if the AP selects this uplink user. The length, $T_{UC}$, of the second stage is:
where $T_{SIFS}$ is the short inter-frame space, and $C$ is a tunable scalar parameter that determines the length of the contention stage. This scalar defines the maximum number of RTSs that the AP can receive during the contention stage. Scalar $C$ must be selected carefully because the AP cannot control the number of received RTSs due to the nature of the DCF back-off timer and the possibility of RTSs collisions during the contention stage. Hence, scalar $C$ cannot be lower than $N$ to serve $N$ uplink users. However, choosing a very high $C$ (e.g., larger than $2N$) will introduce an additional temporal overhead, thereby reducing effective goodput and affecting the system performance because the AP can only serve a maximum of $N$ uplink users. Therefore, the value of $C$ is selected between $N$ and $2N$. In Section IV.B, we will investigate the effect of $C$ on the throughput.

In the third stage, the AP selects up to $N$ uplink users from the contention winners. Then, the AP sends a clear to send to the selected uplink contention winners and request to send to the selected downlink users by using (C/RTS) frame. The size of the C/RTS frames is:

$$S_{C/RTS} = 14 + 6(K + J) \text{ (bytes)}$$

where $K$ is the number of selected downlink users, and $J$ is the number of the uplink users.

In the fourth stage, the selected downlink users start sending a CTS to the AP sequentially that includes the IUI from uplink users. Furthermore, the AP in this stage acquires the downlink users’ CSI from their CTSs. The length of this stage is:

$$T_{DC} = K(T_{SIFS} + T_{CTS})$$

In the fifth stage, the AP uses the collected information to set the maximum transmission rate for each user. Then, the AP starts transmitting data to the selected $K$ downlink users while simultaneously receiving data from $J$ uplink users.

The ACK stage is the final stage to ensure the successful reception of the transmitted frames. The downlink users will each send an ACK to the AP simultaneously. Then, the AP will send a group ACK to all uplink users. The length, $T_{ACK}$, of this stage is:

$$T_{ACK} = 2(T_{SIFS} + T_{ACK})$$

FD-MUMAC can also operate in a half-duplex mode, as shown in Fig. 2(b) and 2(c) if the traffic demand is either uplink traffic or downlink traffic only. FD-MUMAC operates in half-duplex downlink traffic only mode if there are no received RTSs during the contention stage. In this case, FD-MUMAC follows the same process described previously. If the AP has no data to send, FD-MUMAC operates in half-duplex uplink traffic only mode. In this case, the AP sends a group CTS to the users and notifies them to start the uplink transmission immediately after receiving the CTS frame.

### B. Packet Format

In FD-MUMAC, we introduce the AP Beacon frame that the AP broadcast to all users and C/RTS frame that is sent only to the selected users for uplink and downlink transmission. Other frames follow the 802.11 standard packet format except for CTS packets, where we add a field to report the cumulative IUI from the uplink users. Fig. 3 shows the packet format for all the packets that are used in the FD-MUMAC protocol.
C. Collecting the Channel States and Interferences

The exchange of the management control packets (RTS and CTS) enables FD-MUMAC to collect the required CSI and IUI for beamforming. When an uplink station sends RTS to the AP, the AP will detect the user’s uplink CSI while all other users listen to this transmission and use their internal buffer to record the level of the interference from this user. The selected downlink users will report the accumulative level of interference from the selected uplink users with their CTS response to the AP’s C/RTS frame. The AP will acquire the downlink users’ CSI from their CTS response.

Algorithm 1 Controlled fair selection

Input: \( N, n_u^D(i), n_d^D(i), i \in M \)

while current time < simulation time do

Choose first 2N downlink users with the highest deficit

\[ DL \leftarrow \{DL(1) \ldots DL(2N)\}; \text{ where } n_d^D(1) > n_d^D(2) > \ldots > n_d^D(2N) \]

Sort the potential J uplink users according to their UU deficit

\[ UL \leftarrow \{UL(1) \ldots UL(J)\}; \text{ where } n_u^U(1) > n_u^U(2) > \ldots > n_u^U(J) \]

\[ Q = DL \cap UL \]

if \( Q = \emptyset \) then

\[ DL \leftarrow \{DL(1) \ldots DL(N)\} \]

return DL;

if \( J \leq N \)

return UL;

else

\[ UL \leftarrow \{UL(1) \ldots UL(N)\} \]

return UL;

\[ temp_deficit = r_u^D(i) - r_d^D(i) \]

for \( i \in temp_deficit \) do

if \( temp_deficit(i) > 0 \)

remove i from DL;

else

remove i from UL;

\[ DL \leftarrow \{DL(1) \ldots DL(N)\}; \]

return DL;

if \( J \leq N \)

return UL;

else

\[ UL \leftarrow \{UL(1) \ldots UL(N)\} \]

return UL;

\[ t_u^D(M)_{\text{max}} = t_u^D_{\max} \]

\[ t_d^D(M)_{\text{max}} = t_d^D_{\max} \]

D. Controlled Fair Selection Algorithm:

Before sending the C/RTS frame, the AP selects the uplink and downlink users. However, the AP can only select the uplink users from the contention winners (i.e., the users who transmit RTS successfully during the uplink contention period). Furthermore, the AP cannot select any uplink user as a downlink user because these users are HD users. A simple approach is to make the AP selects first N uplink contention winners as an uplink user and choose N different users for downlink transmission. While this approach is easy to implement and can solve the user assignment problem, it may introduce unfairness in the throughput. To solve this problem, we introduce a selection algorithm that we call controlled fair selection algorithm (CFSA) based on the deficit round-robin algorithm [15]. The basic idea of Algorithm 1 is to ensure that all uplink and downlink users obtain their fair time of using the channel to balance the uplink and downlink throughput for each user based on their uplink and downlink deficit \( \tau_u^D(i) \) and \( \tau_d^D(i) \) respectively.

TABLE 1 SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PHY header</td>
<td>20 ( \mu s )</td>
</tr>
<tr>
<td>Slot time</td>
<td>9 ( \mu s )</td>
</tr>
<tr>
<td>SIFS</td>
<td>16 ( \mu s )</td>
</tr>
<tr>
<td>DIFS</td>
<td>24 ( \mu s )</td>
</tr>
<tr>
<td>Frame Size</td>
<td>1500 Bytes</td>
</tr>
<tr>
<td>Burst Packets</td>
<td>5</td>
</tr>
<tr>
<td>Bit Rate (Mbps)</td>
<td>6.5 13 19.5 26 39 52 58.5 65</td>
</tr>
<tr>
<td>Min.SNR (dB)</td>
<td>5 8 12 14 18 21 23 28</td>
</tr>
<tr>
<td>Max.SNR (dBm)</td>
<td>79 76 74 71 67 62 62 61</td>
</tr>
</tbody>
</table>

IV. SIMULATION RESULTS

We conduct extensive simulations for the FD-MUMAC protocol by using MATLAB as a simulation platform. We use the parameters in Table1 for our simulations unless otherwise stated. We use an area of 100m x 100m as the simulation area. We place the AP in the center of this area and randomly distribute the users around the AP. We vary the number of users from 5 to 20. The results represent the average of 10 simulation runs with different random user placement in the grid. We use the IEEE 802.11ac frame burst to send multiple consecutive frames with a SIFS between the frames. We set the maximum transmission power to 25 dBm and 20 dBm for downlink and uplink transmission, respectively. All users use a fully backlogged traffic pattern (i.e., users have data to send all the time). We assume the AP can cancel 83 dB from its self-interference [6]. We use the channel model and path loss model that we described previously. We use an SNR-based rate adaptation algorithm [16] for picking the user’s transmission rate, as shown in Table 1. For instance, if the SNR and RSSI are equal to 24 and -63, respectively, for user \( i \), then we pick 52 Mbps as the transmission rate for this user. We use a state of art single-antenna FD MAC protocol and two half-duplex MU-
MIMO protocols to compare to the performance of FD-MUMAC. We also define four variations within FD-MUMAC:

- Max Rate Selection scheme: The AP will serve N users from the uplink contention winners and N downlink users who maximize the total throughput (i.e., uplink and downlink).
- Random Selection scheme: The AP will serve first N uplink contention winners. Then, the AP randomly selects N downlink users.
- Throughput Fair selection: The AP will follow the proposed CFSA for uplink and downlink user selection method, where the deficit of each user is based on throughput.
- Temporal Fair selection: The AP will follow the proposed CFSA for uplink and downlink user selection, where the deficit of each user is based on time allocation.

A. Throughput Results

In this section, we compare the performance of FD-MUMAC with throughput fair selection to different MAC protocols. First, we compare our results with A-Duplex[9], which is a single user FD MAC protocol that utilizes the capture effect to maximize the transmission rate. We set the maximum rate to 18 Mbps and vary the users from 5 to 20. Fig. 4 shows that FD-MUMAC improves throughput over A-duplex by 58.37% with just one additional antenna and without enabling frame burst, and it outperforms A-Duplex by 156.4% when we enable frame burst. However, A-Duplex has a slight improvement compared to FD-MUMAC with a single antenna due to the addition of the AP beacon frame and the contention stage in the FD-MUMAC. FD-MUMAC overcomes this problem and outperforms A-Duplex by a gain of 26.45% when we enable frame burst.

![Fig. 4. FD-MUMAC throughput and A-Duplex throughput vs. the number of stations M.](image)

Then, we compare FD-MUMAC with throughput fair selection performance to a half-duplex MU-MIMO MAC protocols Uni-MAC[10] and PD-MAC[11]. The first is a half-duplex MU-MIMO MAC protocol based on 802.11 DCF. The latter is a half-duplex MU-MIMO MAC protocol that reduces the size of the contention window for each user to equal the total number of the users. Fig. 5 shows that FD-MUMAC achieves an average throughput gain of 104% and 58.25% compared with Uni-MAC and PD-MAC, respectively. The gain increases with an increasing number of users. For example, FD-MUMAC throughput with 20 users achieves a gain of 177% and 94.15% compared with Uni-MAC and PD-MAC, respectively.

![Fig. 5. FD-MUMAC, Uni-mac and PD-MAC throughput with four antennas versus. the number of stations M.](image)

B. The Effect of The Contention Length

In this section, we investigate the effect of the contention scalar C. Fig. 6 shows the throughput as a function of C for different numbers of antennas and users. We empirically determine the value of C that achieves the highest throughput. Fig. 6 shows that the highest throughput occurs when C is higher than or equal to 1.5N. Therefore, we select the value of C to be between 1.5N and 2N.

![Fig. 6. FD-MUMAC throughput with varying the contention period length.](image)

C. Effect of Controlled Fair Selection Algorithm

Fig. 7 shows the throughput versus the number of users for the four selection schemes with 2, 4 and 6 antennas. Max rate achieves the highest throughput as expected. However, the max rate implementation is computationally expensive since it requires an exhaustive search through the contention winners and the potential downlink users to achieve the highest throughput. Furthermore, we evaluate the fairness of each implementation by using Jain’s fairness index [13]. We calculate the average and total Jain’s fairness index. The total fairness index is the calculated fairness index at the end of the simulation and represents the long-term fairness. The average fairness index is the average of the calculated fairness index after every short period of time (10k slots or 90 ms in our simulations) and represents the short-term fairness. Note that, while a scheme that achieves short-term fairness also automatically ensures fairness over a longer time period, the reverse implication is not always valid. Tables 2 shows that the fairness algorithm of the proposed CFSA algorithm
outperforms the max rate and random scheme at the cost of reducing the total throughput.

The downlink fairness index for CFSA with a different implementation is the highest since the AP has full control of downlink selection, unlike the uplink selection, which depends on the received RTSs from the uplink users. Furthermore, FD-MUMAC with CFSA balances the uplink and downlink throughput, unlike the other two implementations. For example, the uplink/downlink throughput ratio for an AP with two antennas and five users is 4 for a random scheme implementation and 1.02 for the CFSA implementation.

V. CONCLUSIONS

We proposed a MAC protocol for full-duplex MU-MIMO (FD-MUMAC). FD-MUMAC enables a FD AP to serve multiple uplink and downlink users simultaneously by coordinating the communication and collecting the required information to enable full-duplex MU-MIMO communication. Also, a controlled fairness selection algorithm (CFSA) is proposed, which ensures that all users obtain a fair share in the channel utilization and balance the uplink/downlink traffic for each user. In future work, we will modify FD-MUMAC to support full-duplex clients.

TABLE 2 PERFORMANCE OF UPLINK AND DOWNLINK TRANSMISSIONS

<table>
<thead>
<tr>
<th></th>
<th>Uplink Throughput (Mbps)</th>
<th>Downlink Throughput (Mbps)</th>
<th>Uplink Fairness Index</th>
<th>Downlink Fairness Index</th>
<th>Average Uplink Fairness Index</th>
<th>Average Downlink Fairness Index</th>
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</thead>
<tbody>
<tr>
<td>Random N=2 M=5</td>
<td>40.416</td>
<td>76.4577</td>
<td>0.9976</td>
<td>0.9996</td>
<td>0.9792</td>
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<td>Max N=2 M=5</td>
<td>41.742</td>
<td>73.8252</td>
<td>0.7991</td>
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<td>0.8783</td>
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<tr>
<td>CFSA(Time) N=2 M=5</td>
<td>40.7302</td>
<td>76.4658</td>
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<tr>
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<td>0.9999</td>
<td>0.9185</td>
<td>0.9969</td>
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REFERENCES