Least Squares

1. Consider the least squares approximation in Euclidean vector space \((N \times 1)\) vectors. Let 
   \(x_1 = \begin{bmatrix} 2 & 3 & 0 \end{bmatrix}^T, \ x_2 = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}^T\) and \(y = \begin{bmatrix} -3 & 1 & 2 \end{bmatrix}^T\). Determine, intuitively, 
   the vector \(\hat{y}\) in the subspace spanned by \(x_1\) and \(x_2\) that is closest to \(y\).

2. Solve problem 1 using the least squares normal equations derived in lecture. Use 
   Matlab to do the computations. Print out the Matlab command window showing your 
   work.

3. Find the projection matrix used to solve problem 1.

4. Show that any projection matrix is both symmetric \((P_X = P_X^T)\) and idempotent 
   \((P_X P_X = P_X)\).