Random Variables and Minimum Mean-Squared Error Estimation

1. In this assignment, we will work with random variables $x_1, x_2,$ and $x_3$. Assume each of these are independent and uniformly distributed on $[0, 1]$. Assume also that a third random variable is given by $y = 0.3x_1 + 0.5x_2 + 0.1x_3 + z$, where $z$ is a zero-mean random variable independent of $x_1, x_2,$ and $x_3$. Determine $R$, $P$, and $\hat{y}$, the linear combination of $x_1, x_2,$ and $x_3$ which is closest to $y$ in the mean-squared error sense.

2. What is the minimum mean-squared error for the problem in 1?

3. What is $E[(y - \hat{y})x_i], i = 1, 2, 3$? Why?

4. Implement the LMS algorithm in Simulink and apply it to the system identification problem as shown in the figure. Plot the error signal and the input signal, also display the estimates of the unknown system parameters as shown. Demo your implementation to the class instructor. You may want to view the Simulink tutorials on the Mathworks web page

The diagram shows a signal processing system.

1. The input signal, $x(n)$, is fed into a random number generator.
2. The output of the random number generator, $x(n)$, is then passed through an unknown filter described by the transfer function $\frac{0.2+0.5z^{-1}+0.3z^{-2}}{1}$.
3. The output of the unknown filter, $y(n)$, is then compared to an input signal $x_n$.
4. The error signal, $e(n)$, is calculated and fed into an adaptive filter.
5. The adaptive filter uses the LMS algorithm to update its coefficients, $w_{(n-1)}$.
6. The updated coefficients are then used to process the input signal $x_n$.
7. The final output $y_{(n)}$ is displayed.

The coefficients used in the LMS algorithm are $0.20$, $0.50$, and $0.30$. The display box shows these values.
LMS Algorithm

\[ w_{n} = w_{n-1} + \mu e(n) x_n \]