Data Compression Performance

- **Compression Ratio (CR):**
  \[
  CR = \frac{\text{total number of bits before compression}}{\text{total number of bits after compression}}
  \]

- **Percent Root Mean Square Difference (PRD):**
  \[
  PRD = \sqrt{\frac{\sum_{n=0}^{M-1} (x[n] - \hat{x}[n])^2}{\sum_{n=0}^{M-1} x[n]^2}} \times 100
  \]
Case Studies

- Jalaleddine
- Aydin
ECG Signal Processing

- ECG Component Detection
- ECG Arrhythmia Detection
- High Fidelity ECG via Signal Averaging
- Measurement of Heart Rate
ECG Component Detection

P: atrial depolarization
R: ventricular depolarization
T: ventricular repolarization

0.6 s  typical HR: 70 bpm
R-Wave Detection

Automated R-wave detection useful for:
- heart rate measurement.
- detection of tachycardia (high HR), bradycardia (low HR), or ventricular fibrillation.
Noise Sources in ECG

- motion artifact (can be wide-band)
- 60 Hz interference
- baseline drift (generally amplifier related)

These noise sources can significantly degrade the quality of the ECG. Simple amplitude thresholding does not produce good R-wave detection.
General Approaches to R-Wave Detection

- Non-Syntactic: rely on characteristic features of QRS complexes to perform the detection—simple.
- Syntactic: based on grammatical inference—can get complicated.
- Hidden Markov Models: based on a probabilistic model of R-wave occurrence times—complicated.

We’ll cover all three approaches.
Non-Syntactic QRS Detection

Bandpass Filter

lowpass filter: \[ H(z) = \frac{(1 - z^{-6})^2}{(1 - z^{-1})^2} \]

Highpass Filter

\[ H(z) = \frac{(-1 + 32z^{-16} + z^{-32})^2}{(1 - z^{-1})} \]


Squaring Function:

\[ y[n] = x[n]^2 \]
Moving Average (MA) Filter

\[ y[n] = \frac{1}{N_w} (x[n - (N_w - 1)] + x[n - (N_w - 2)] + \cdots + x[n]) \]

\( N_w \): length of MA (FIR) filter

Threshold Devices:

\[ x[n] \]

\[ T \rightarrow \quad b \]

\[ b = \begin{cases} 
H, & x[n] \geq T \\
L, & x[n] < T 
\end{cases} \]

(comparator)
Rules for Non-Syntactic Detector

- Thresholds $T_1$ and $T_2$ are adjusted from beat-to-beat in order to compensate for trends in R-wave peak amplitude.
- Processing delays are compensated for.
- If an R-wave is not detected after a set time interval, the algorithm reprocesses the data going back to the previous detection, using a new lower set of thresholds.
- Algorithm also detects non-QRS peaks such as P and T waves using a different lower set of thresholds.
Detection Assessment

- False Positive: R-wave detected when none was present.
- False Negative: R-wave not detected when one was present.
- Results of Pan-Tompkins algorithm applied to the MIT/BIT arrhythmia database:

  0.437% false positive rate (5% typical)
  0.239% false negative rate

- Some subjects show high detection failure.
- No noise was added to the data to simulate poor recording conditions.
Syntactic Detection of R-Waves

- Based on theory of formal languages called \textit{grammars}.
- Grammars are used to describe all signals (sentences) belonging to a given class (language).
- A class is represented by a given set of known sentences called a \textit{training set}.
- We wish to estimate the class associated with a given sentence based on what we know from the training set.
- There are a number of different grammars, we will consider only \textit{finite state grammars}. 
Finite State Grammars

Grammar has associated with it a finite set of states,

\[ \{S_0, S_1, \ldots, S_{N-1}, T_1, T_2, \ldots, T_M\} \]

and a finite set of symbols:

\[ \{a, b, \ldots, f\} \]

Sentences generated by this grammar are composed of the symbols \( \{a, b, \ldots, f\} \) and are determined by a set of rules which are best represented by a graph.

Graph starts at start state \( S_0 \) and ends at one or more terminal states \( T_1, T_2, \ldots, T_M \).
Graph for Finite State Grammar

possible sentences: b,
    abbbbaa,
    abbbbbbab,
    aaa, etc.
Finite State Automata

- Given a sentence from an unknown finite state grammar, would like to be able to determine whether that sentence comes from a certain grammar.
- This can be done with a finite state automaton.
- The automaton has associated with it a finite number of states

\[ \{Q_0, Q_1, \ldots, Q_{N-1}\} \]

- The automaton reads in symbols from a sentence, the symbols define state transitions. If certain final states are arrived at, starting from \( Q_0 \), the sentence is accepted as belonging to a certain grammar.
Finite State Automaton for Grammar on slide 330

Any sentence ending in state $Q_3$ or $Q_4$ is accepted, otherwise it is rejected.

abbbbbbbaa: accepted
abb: rejected