EE 5345
Biomedical Instrumentation
Lecture 25: slides 470-483

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slides can be viewed at:
http://www.seas.smu.edu/~cd/ee5345.html
Symbols for Syntactic ECG Analysis

Symbols: p, qrs, t, b
Finite State Automaton for ECG Analysis

allows for variable P-R, S-T, and T-P segments

symbols must first be extracted from the ECG data.
Case Study on Syntactic Detection of QRS Complexes


symbols used to model ECG:
Automaton for QRS Detection

- **Start State:** T₁
  - Input: Noise classification
  - Next State: Q₀

- **State Q₀**
  - Next States:
    - Input: normup → Q₁
    - Input: normdown → Q₂

- **State Q₁**
  - Next State:
    - Input: normup → Q₂

- **State Q₂**
  - Next State:
    - Input: normup → Q₀

- **Decision Points**:
  - If zero detected, move to Q₀.
  - If else detected, continue alternating through the states.

- **Classification**:
  - Normal QRS classification
  - Noise classification
Symbol Extraction

- They estimate the derivative of the ECG waveform via first differences:
  \[ s[n] = x[n] - x[n-1] \]

- Form consecutive subsequences of \( s[n] \) having equal signs.

ex) \( s[n] = \{ ...1.2, 1.5, 2.1, 0.3, -0.3, -1.2, 0.2, 0.6, ... \} \)
  \[ \{1.2, 1.5, 2.1, 0.3\} \{-0.3, -1.2\} \{0.2, 0.6\} \]

\( S_1 \quad S_2 \quad S_3 \)
Symbol Extraction (cont.)

- The length and sum of elements of each $S_i$ are computed and compared with comparable quantities for normup and normdown sequences for known, normal QRS complexes.

- This comparison is used to classify each $S_i$ as “normup”, “normdown”, “noiseup”, “noisedown”, or “zero”.

- The resulting symbols are then processed by the automaton for QRS detection.

- Entire system was realized on an 8-bit microprocessor.

- Designed to process data off-line at about 10-times real time, from Holter monitor recordings.

- No detection performance stats where reported.
Measurement of Heart Rate

Based on measurement of R-R intervals

\[
\text{HR (bpm)} = \frac{\text{R-R interval}}{60}
\]
Hidden Markov Model-Based QRS Detection

■ Introduction to Probability Theory
■ Introduction to Hidden Markov Models (HMM’s)
■ Three Fundamental Problems
■ The Forward Backward Method
■ The Viterbi Algorithm
■ Baum-Welch Reestimation Formulas
Introduction to Probability Theory

Experiments:

\[ \Omega = \text{sample space: set of all possible outcomes of an experiment (we will assume a finite number of outcomes here).} \]

ex) single coin toss
\[ \Omega = \{H, T\} \]

ex) two consecutive coin tosses
\[ \Omega = \{HH, HT, TH, TT\} \]

ex) single roll of a die
\[ \Omega = \{1, 2, 3, 4, 5, 6\} \]
Basic Definitions

- Events, $A$: any subset of $\Omega$: $A \subseteq \Omega$
- Probability, $\Pr(\ )$: a set function that operates on events.
- Axioms of Probability
  - $\Pr(A) \geq 0, \ \Pr(\Omega) = 1$
  - Let $A \subseteq \Omega, B \subseteq \Omega, A \cap B = \phi$, then
    \[
    \Pr(A \cup B) = \Pr(A) + \Pr(B)
    \]
  - “probability of either event $A$ or event $B$ occurring = probability of $A$ plus the probability of $B$”
Basic Definitions (cont.)

- Independent Events: two events $A \subset \Omega, B \subset \Omega$ are independent if:

  $$\Pr(A \cap B) = \Pr(A) \Pr(B)$$

  $A \cap B \equiv AB$ : event $A$ occurs and event $B$ occurs

Examples:

ex 1) single toss of a fair coin: $\Omega = \{H, T\}$

  $$\Pr(H) = 0.5, \Pr(T) = 0.5, \Pr(H \cup T) = 0.5 + 0.5 = 1$$
Examples (cont.)

ex 2) two consecutive fair coin tosses:

\[ \Omega = \{HH, HT, TH, TT\} \]

- first toss independent of second toss
- \( \Pr(\text{HH}) = 0.5 \times 0.5 = 0.25 \)
- \( \Pr(\text{HT}) = 0.5 \times 0.5 = 0.25 \)
- \( \Pr(\text{TH}) = 0.5 \times 0.5 = 0.25 \)
- \( \Pr(\text{TT}) = 0.5 \times 0.5 = 0.25 \)
Examples (cont.)

ex 3) a fair coin is tossed 3 times:

• possible outcomes: HTH, HHH, etc.

• \[ \Pr(HTH \cup TTH) = 0.5 \times 0.5 \times 0.5 + 0.5 \times 0.5 \times 0.5 \]
  \[ = 0.125 + 0.125 = 0.25 \]

Each of the above examples can be verified experimentally by repeating the experiment, say 100 times; for example 3, count the number of times \( N_{HTH, TTH} \) you get either HTH or TTH. Then:

\[ \frac{N_{HTH, TTH}}{100} \approx 0.25 \]