EE 5345
Biomedical Instrumentation
Lecture 13: slides 257-276

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slides can be viewed at:
http://www.seas.smu.edu/~cd/ee5345.html
Square Transducers

far-field impulse response:

\[ h(x_z, y_z) = \frac{K}{z} \sin \left( \frac{Dx_z}{\lambda z} \right) \sin \left( \frac{Dy_z}{\lambda z} \right) \]

isonification field:

\[ u(x_z, y_z, t) = h(x_z, y_z) \times p \left( t - \frac{z}{c} \right) e^{-j\omega_0 t} * a(t) \]

- \( p(t) \): pulse envelope (short-duration)
- \( \omega_0 \): ultrasound frequency (rad/s)
- \( a(t) \): \( \frac{d}{dt} \) filter
Far-field Impulse Response of Square Transducer

(D = 1)

\[ h(x, y) \] is equivalent to the isonification intensity at a fixed distance \( z \), for a D = 1 transducer centered at (0,0).
Far-field Impulse Response of Square Transducer

(D = 0.5)
Far-field Impulse Response of $D = 1$ Square Transducer (x-axis)

$h(x_z, 0)$

$X_z$
Square Transducer Response in Far-field

- Smaller sized transducers have larger main lobe.
- Isontification pattern is very close to its sensitivity pattern in receive mode.
- Would like main lobe to be small so that reflections are confined to a small lateral (x-y) region for good lateral image resolution.
- Small main lobes are difficult to achieve with a single transducer.
- Transducer must be moved over skin to obtain B-scans.
Phased Array (Transmit Mode)

$p(t)e^{j\omega_0 t}$

pulse generator

delay elements

$\tau_0 \rightarrow \tau_1 \rightarrow \tau_2 \rightarrow \tau_3 \rightarrow \tau_4$

$\omega_0$ pulse generator
Phased Array (Receive Mode)

\[\sum\]

array output

delay elements

\[\tau_0\]
\[\tau_1\]
\[\tau_2\]
\[\tau_3\]
\[\tau_4\]

\(D\)
\(d\)
\(w\)
\(x\)
\(y\)
\(z\)
Phased Array Properties

- The time delays can be adjusted to isonify at a given x-location.
- The time delays can be adjusted so that the array is most sensitive to reflections from a given x-location.
- By adjusting the delays, can scan along the x-direction without having to move the transducer array.
Scanning with Phased Arrays

propagation directions of outgoing wavefronts
Phased Array Impulse Response

- no deflection

\[ h(x_z, y_z) = K(z) \sin c \left( \frac{wx_z}{\lambda z} \right) \sum_{n=-\infty}^{\infty} \sin c \left( D \left( \frac{x_z}{\lambda z} - \frac{n}{d} \right) \right) \]

- with deflection

\[ h(x_z, y_z) = K(z) \sin c \left( \frac{wx_z}{\lambda z} \right) \sum_{n=-\infty}^{\infty} \sin c \left( D \left( \frac{x_z}{\lambda z} - \frac{n}{d} - \frac{\beta}{\lambda} \right) \right) \]

\[ \tau_n = \beta \frac{nd}{c} \]
Array Response $\beta = 0$ (all transducers pulsed simultaneously)

\[ \sin c \left( \frac{\omega x_z}{\lambda z} \right) \]

grating lobes

\[ \frac{-\lambda}{D}, \frac{\lambda}{D}, \frac{\lambda}{d}, \frac{2\lambda}{d} \]
Array Response $\beta \neq 0$:

$$\sin c\left(\frac{wx_z}{\lambda_z}\right)$$
Can Reduce Grating Lobes by:

- Limit angular scan $\beta$
- For a given array width, $D$, reduce transducer separation, $d$, must therefore increase number of transducers in array.

Most commercial ultrasonic imaging systems are based on phased arrays.
Nuclear Medicine

- Radioactive Decay and Radiopharmaceuticals
- Basic Imaging Configuration
- Scintillation Counter
- Gamma (Auger) Camera
- Pinhole vs Parallel-Hole Collimators
- Positron Emission Tomography (PET)
Radioactive Decay and Radiopharmaceuticals

- X-rays discovered by Roentgen in 1895.
- Marie Curie discovered element *radium*, coined “radioactivity”.
- **atomic weight** \( (A) \): \# protons and neutrons in nucleus.
- **atomic number** \( (Z) \): \# protons in nucleus.
- **isotopes**: atoms having different \( A \) but same \( Z \).
- \( \alpha \) particle: 2 protons and 2 neutrons (nucleus of He atom).
- radioactive elements: decay spontaneously into other elements, decay releases \( \alpha \) particles, \( \beta \) particles, or \( \gamma \) rays.
Stability of Atomic Nuclei

- **ground state**: most stable.
- **metastable state**: nucleus is unstable, has long lifetime before transforming to another state.
- **excited state**: nucleus very unstable, exists in this state for small time period before transforming.

radioactive decay \{ \text{excited state} \rightarrow \text{ground state} \}

\text{radiation (α, β, or γ)}
Types of Radioactive Decay

notation: \[ {A \choose Z} E \quad \text{or just} \quad {A} E \]

element

- \( \alpha \) decay: not used for imaging

\[ ^{226} R \rightarrow ^{222} Rn \quad (\text{radium decays to radon}) \]

\( \alpha \)

\( Z \) decreased by 2

\( A \) decreased by 4
Types of Radioactive Decay (cont.)

- $\beta^-$ (negatron) decay:
  - neutron converts to negatron and proton
  - $\beta^-$ is emitted
  - $Z$ increases by 1
  - $A$ stays same
  - ex) $^{32}P \rightarrow ^{32}S \beta^-$
Types of Radioactive Decay (cont.)

- \( \beta^+ \) (positron) decay:
  - proton converts to neutron and positron
  - \( Z \) decreased by 1
  - \( A \) stays same
  - \( Q \)
  - ex)

\[
^{11}C \rightarrow ^{11}B + e^-
\]

\[\gamma \quad \beta^+ \quad \gamma\]

\(180^\circ\)

Event happens within 1 ns, \( \gamma \) rays are at 510 keV