

# APPENDIX E

## MISCELLANEOUS

### E.1 L'Hôpital's Rule

If  $\lim f(x)/g(x)$  results in the indeterministic form  $0/0$  or  $\infty/\infty$ , then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{\dot{f}(x)}{\dot{g}(x)} \quad (\text{E.1})$$

### E.2 Taylor and Maclaurin Series

$$f(x) = f(a) + \frac{(x-a)}{1!} \dot{f}(a) + \frac{(x-a)^2}{2!} \ddot{f}(a) + \dots$$

$$f(x) = f(0) + \frac{x}{1!} \dot{f}(0) + \frac{x^2}{2!} \ddot{f}(0) + \dots$$

### E.3 Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots \quad x^2 < \frac{\pi^2}{4}$$

$$Q(x) = \frac{e^{-x^2/2}}{x\sqrt{2\pi}} \left( 1 - \frac{1}{x^2} + \frac{1 \cdot 3}{x^4} - \frac{1 \cdot 3 \cdot 5}{x^6} + \dots \right)$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + \binom{n}{k}x^k + \dots + x^n$$

$$\approx 1 + nx \quad |x| \ll 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots \quad |x| < 1$$

## E.4 Sums

$$\sum_{m=0}^k r^m = \frac{r^{k+1} - 1}{r - 1} \quad r \neq 1$$

$$\sum_{m=M}^N r^m = \frac{r^{N+1} - r^M}{r - 1} \quad r \neq 1$$

$$\sum_{m=0}^k \left(\frac{a}{b}\right)^m = \frac{a^{k+1} - b^{k+1}}{b^k(a - b)} \quad a \neq b$$

## E.5 Complex Numbers

$$e^{\pm j\pi/2} = \pm j$$

$$e^{\pm jn\pi} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$e^{\pm j\theta} = \cos \theta \pm j \sin \theta$$

$$a + jb = re^{j\theta} \quad r = \sqrt{a^2 + b^2}, \quad \theta = \tan^{-1} \left(\frac{b}{a}\right)$$

$$(re^{j\theta})^k = r^k e^{jk\theta}$$

$$(r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

## E.6 Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j \sin x$$

$$\cos x = \frac{1}{2}(e^{jx} + e^{-jx})$$

$$\sin x = \frac{1}{2j}(e^{jx} - e^{-jx})$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin x$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos x$$

$$2 \sin x \cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \frac{1}{4}(3 \cos x + \cos 3x)$$

$$\sin^3 x = \frac{1}{4}(3 \sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2}[\sin(x - y) + \sin(x + y)]$$

$$a \cos x + b \sin x = C \cos(x + \theta)$$

$$\text{in which } C = \sqrt{a^2 + b^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{-b}{a}\right)$$

### E.7 Indefinite Integrals

$$\int u dv = uv - \int v du$$

$$\int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax$$

$$\int \cos ax dx = \frac{1}{a} \sin ax$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x \sin ax dx = \frac{1}{a^2}(\sin ax - ax \cos ax)$$

$$\int x \cos ax dx = \frac{1}{a^2}(\cos ax + ax \sin ax)$$

$$\int x^2 \sin ax dx = \frac{1}{a^3}(2ax \sin ax + 2 \cos ax - a^2 x^2 \cos ax)$$

$$\int x^2 \cos ax dx = \frac{1}{a^3}(2ax \cos ax - 2 \sin ax + a^2 x^2 \sin ax)$$

$$\int \sin ax \sin bx dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \quad a^2 \neq b^2$$

$$\int \sin ax \cos bx dx = -\left[ \frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)} \right] \quad a^2 \neq b^2$$

$$\int \cos ax \cos bx \, dx = \frac{\sin (a-b)x}{2(a-b)} + \frac{\sin (a+b)x}{2(a+b)} \quad a^2 \neq b^2$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}$$

$$\int x e^{ax} \, dx = \frac{e^{ax}}{a^2} (ax - 1)$$

$$\int x^2 e^{ax} \, dx = \frac{e^{ax}}{a^3} (a^2 x^2 - 2ax + 2)$$

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx)$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$$

$$\int \frac{1}{x^2 + a^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int \frac{x}{x^2 + a^2} \, dx = \frac{1}{2} \ln (x^2 + a^2)$$