\[ p : f \text{ is integrable.} \]
\[ q : g \text{ is differentiable.} \]
\[ r : h \text{ is differentiable.} \]
\[ s : f \text{ is bounded.} \]

The whole argument may be symbolized as
\[ p \rightarrow (q \lor r) \]
\[ \neg q \rightarrow (\neg p \land s) \]
\[ s \rightarrow (q \lor r) \]
\[ \therefore q . \]

Now assume that there is an assignment of truth values to \( p \), \( q \), \( r \), and \( s \) such that the truth value of each of the statement forms \( p \rightarrow (q \lor r) \), \( \neg q \rightarrow (\neg p \land s) \), and \( s \rightarrow (q \lor r) \) is \( T \), but the truth value of \( q \) is \( F \).

Because the truth value of each of \( \neg q \) and \( \neg q \rightarrow (\neg p \land s) \) is \( T \), it follows that for this assignment \( (\neg p \land s) \) has the truth value \( T \), so the truth value of \( \neg p \) and \( s \) is \( T \). This implies that the truth value of \( p \) is \( F \) and the truth value of \( s \) is \( T \). Then, from \( s \rightarrow (q \lor r) \), we find that the truth value \( (q \lor r) \) is \( T \). But the truth value of \( q \) is \( F \). Thus, the truth value of \( r \) is \( T \).

Hence, we find that there is an assignment \( F,F,T,T \) for \( p,q,r,s \), respectively, such that the truth value of each of the statement forms \( p \rightarrow (q \lor r) \), \( \neg q \rightarrow (\neg p \land s) \), and \( s \rightarrow (q \lor r) \) is \( T \), but the truth value of \( q \) is \( F \). Therefore, the given argument is not valid.

**Exercise 5:** Show that \( \neg p, (\neg q \lor p), \neg r \lor q \models \neg r \).

**Solution:** We write the following argument:

\[ B_1 : \neg p \] hypothesis
\[ B_2 : (\neg q \lor p) \] hypothesis
\[ B_3 : (\neg q \lor p) \rightarrow (q \rightarrow p) \] tautology
\[ B_4 : q \rightarrow p \] \( B_1, B_3, B_4 \) is a logically valid argument, by modus ponens.
\[ B_5 : \neg q \] \( B_2 \) and \( B_4 \) and modus tollens
\[ B_6 : \neg r \lor q \] hypothesis
\[ B_7 : \neg r \lor q \rightarrow (r \rightarrow q) \] tautology
\[ B_8 : r \rightarrow q \] \( B_3, B_5, B_6 \) is a logically valid argument, by modus ponens.
\[ B_9 : \neg r \] \( B_6, B_7, B_8 \) is a logically valid argument, by modus tollens.

Hence, we find that there exists an argument \( B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9 \) satisfying the conditions of the Definition 1.3.5. Hence, \( \neg p, (\neg q \land p), \neg r \lor q \models \neg r \).

## SECTION REVIEW

### Key Terms

- proof
- argument
- conclusion
- premise
- modus ponens
- modus tollens
- disjunctive syllogisms
- hypothetical syllogism
- dilemma
- conjunctive simplifications
- disjunctive additions
- conjunctive addition
- logically valid

### Some Key Definitions

1. A finite sequence \( A_1, A_2, A_3, \ldots, A_{n-1}, A_n \) of statements is called an argument. The final statement, \( A_n \), is the conclusion, and the statements \( A_1, A_2, A_3, \ldots, A_{n-1} \) are called the premises of the argument.
2. An argument \( A_1, A_2, A_3, \ldots, A_{n-1}, A_n \) is called logically valid if the statement formula

\[ (A_1 \land A_2 \land A_3 \land \cdots \land A_{n-1}) \rightarrow A_n \]

is a tautology.
1. Use a truth table to determine whether the following argument form is valid.

\[ p \rightarrow q \]
\[ p \rightarrow r \]
\[ \therefore p \rightarrow (q \lor r) \]

2. Use a truth table to determine whether the following argument form is valid.

\[ p \rightarrow q \]
\[ \sim (p \lor r) \]
\[ \therefore \sim p \]

3. Use a truth table to determine whether the following argument form is valid.

\[ \sim p \lor q \]
\[ r \rightarrow (\sim q) \]
\[ \therefore p \rightarrow (\sim r) \]

4. Use a truth table to determine whether the following argument form is valid.

\[ (p \lor q) \]
\[ p \rightarrow (\sim q) \]
\[ p \rightarrow r \]
\[ \therefore r \]

5. Determine whether the following argument form is valid.

\[ p \rightarrow q \]
\[ \sim p \]
\[ \therefore \sim q \]

6. Prove that the following argument form is invalid.

\[ p \rightarrow q \]
\[ q \]
\[ \therefore p \]

7. Test the validity of the following argument: For a particular real number \( x \): \( x \) is positive or \( x \) is negative. If \( x \) is positive, then \( x^2 > 0 \). If \( x \) is negative, then \( x^2 > 0 \). Therefore, \( x^2 > 0 \).

In Exercises 8–21, test whether the given arguments are logically valid.

8. If the budget is not cut, then prices remain stable if and only if taxes will be raised. If the budget is not cut, then taxes will be raised. If prices remain stable, then taxes will not be raised. Therefore, taxes will not be raised.

9. If Rita works hard and has talent, then she will get a good job. If she gets a good job, then she will be happy. Hence, if Rita is not happy, then she did not work hard or she does not have talent.

10. If it snows, then the streets become slippery. If the streets become slippery, then accidents happen. Accidents do not happen. Therefore, it does not snow.

11. If it rains, the prices of vegetables go up. The prices of vegetables go up. So it rains.

12. If capital investment remains unchanged, then government spending will increase or unemployment will result. If government spending does not increase, taxes can be reduced. If taxes can be reduced and capital investment remains unchanged, then unemployment will not result. Hence, government spending will increase.

13. If Chris studies, then he will pass the class test. If Chris does not play cards, then he will study. Chris did not pass in the class test. Therefore, Chris played cards.

14. If Lisa’s job performance for the year is good, she will get a bonus. If she gets a bonus, she will take a vacation. If she takes a vacation, she will take a cruise. Lisa did not take a cruise. Therefore, Lisa did not get a bonus.

15. If I do all the exercises in this chapter, I will understand the material. If I understand the material, I will do well on the exam. If I do well on the exam, I will pass. I passed the exam. Therefore, I did all the exercises in the chapter.

16. During the summer Laurie will go to New York or Paris. If she goes to New York, she will not visit the Eiffel Tower. If she does not visit the Eiffel Tower, she will visit the Statue of Liberty. She did not go to Paris. Therefore, she visited the Statue of Liberty.

17. If I save money, I will buy a house. I did not buy a house. Therefore, I did not save money.

18. If interest rates go up, then the prices of houses go down. The prices of houses did not go down. Therefore, interest rates went up.

19. Shelly is a computer science major or a chemistry major. If Shelly is a chemistry major, then she must take the organic chemistry course. Therefore, Shelly is a computer science major or she must take organic chemistry.

20. Anne plays golf or Anne plays basketball. Therefore, Anne plays golf.

21. I met Brandon at our university library or I met him at the football field. If I met Brandon at the football field, then I talked about our football team. If I met Brandon at our university library, then I talked about the discrete structure course. I did not talk about the discrete structure course. Prove that I talked about our football team.