ECE 5/7383 Introduction to Quantum Informatics

Homework 2

1. Consider the following function that we found to be the solution to the differential equation describing a simple harmonic oscillator for the classical mass and ideal spring system (in class notes).

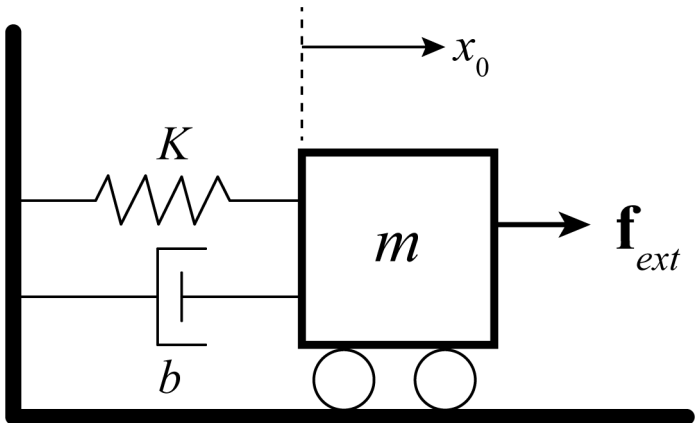
$$x(t) = c_1 \cos(\omega_0 t) + c_2 \sin(\omega_0 t)$$

a) Assume that c_1=3, c_2=2, and ω_0=200. Sketch three plots and make sure that both your t- and x-axes are properly labeled and that tics marks are representative of the proper values. The t-axis should be the horizontal axis and the x-axis should be the vertical axis. The plot should be the following: (i) sketch a sketch of the first term, $c_1\cos(\omega_0 t)$, for at least one complete cycle, ii) sketch a sketch of the second term, $c_1\sin(\omega_0 t)$, for at least one complete cycle, iii) graphically add the two plots in part i) and part ii) to sketch a plot of the overall function, $x(t)$. Your three plots should be vertically aligned (share a common t-axis) so that the graphical addition makes sense when you product the third plot.

b) (7383 STUDENTS ONLY) Show, through derivation including ALL steps, that $x(t)$ can be expressed as follows. To answer this question, you should show how you arrive at expressions for R and ϕ using algebraic and trigonometric relations. That is, begin with the equation given above and use algebra and identities to manipulate it to be in the form below.

$$x(t) = R\cos(\omega_0 t + \phi)$$

2. Consider the following system that consists of an ideal spring (linear with stiffness K) and ideal dashpot (linear with damping, b) that are attached to a wall and connected to a mass that is mounted on frictionless wheels. The positive horizontal direction is to the right and the positive vertical direction is towards the top of the mass.



a) Draw the free body diagram of the mass. Include ALL forces.

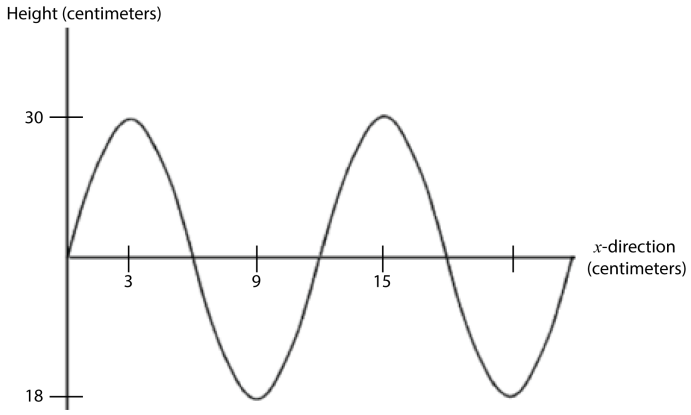
b) Give the force equation when the mass is in equilibrium. Include ALL forces.

c) Give the force equation at time t_0 when the external force is, $\mathbf{f}_{ext} = \| \mathbf{f}_{ext} \| \delta(t_0)$. Include ALL forces.

d) Give the force equation a short after t_0 that is denoted as t_0^+. Include ALL forces.

e) (7383 STUDENTS ONLY) Write the differential equation of motion (in terms of displacement, x) in the time domain at a time t occurring after t_0^+. Use magnitudes only (no vectors), but be sure your arithmetic signs are correct. The direction x is positive in the rightmost direction as indicated by the figure.

3. Consider the wave shown in the diagram below where the horizontal axis is displacement (in centimeters) and the vertical axis is amplitude. This wave has a temporal frequency of 10 s^{-1}. Show details of all calculations including the equations used.



a) What is the amplitude (in cm)?

b) What is the temporal frequency (in Hz) ?

c) What is the wavelength (in cm)?

d) What is the velocity of this wave?

e) (7383 STUDENTS ONLY) What is the temporal period (in seconds)?