ECE 5/7383 Introduction to Quantum Informatics
Homework 3

1. Operations over finite groups and fields can be generically denoted by the asterisk, "*." If the operator requires two elements of a set as operands, it is a "binary" operator. In general, a binary operator can be defined by a table that lists all the elements of a set along the outer column and row and contains the results after applying the operator to the middle portion of the table. In honor of the famous mathematician, Arthur Cayley, these are sometimes called Cayley tables.

   A binary operation, *, on the set $A=\{a, b, c, d\}$ is partially defined by the following Cayley table. (NOTE: values in the leftmost column precede the * operator and values on the top row follow the * operator, as in $a*b=d$).

*	*a*	*b*	*c*	*d*
a		*d*		
b			*d*	
c				
d			*b*	

   a) If * is known to be associative, fill in the $a*d$ entry (only) and show any work you did to find the answer.

*	*a*	*b*	*c*	*d*
a		*d*		
b			*d*	
c				
d			*b*	

   b) If * is known to be both associative and commutative, fill in as many of the missing entries as possible.

*	*a*	*b*	*c*	*d*
a		*d*		
b			*d*	
c				
d			*b*	

2. Using step-by-step derivations (meaning that you **start with the leftmost side of the equation and apply a SINGLE algebraic manipulation at a time to the righthand side**).

   a) Show that:

$$i^i = e^{-\frac{\pi}{2}}$$

   b) Show that (note *ln*(*x*) is the logarithm base-*e*, or "natural log" of *x*):

$$\ln(i) = i\frac{\pi}{2}$$

   c) (FOR 7383 STUDENTS ONLY) Show that:

$$\pi = -2i\ln(i)$$

d) (FOR 7383 STUDENTS ONLY) Show that:

$$e^{\pi} = \frac{1}{\left(i^{i}\right)^{2}}$$

3. Answer the following questions about matrices.

a) If matrices A and B are symmetric, show that $AB=BA$.

b) If matrices C and D are orthogonal, show that CD is also orthogonal.

c) If matrices E and F are Hermitian, then show that EF is not Hermitian unless EF commute under direct multiplication.

d) (For 7383 students only) If matrices G and H are Unitary, show that the product GH is also unitary.

4. Let D represent an operator for a first order derivative with respect to time (d/dt) and let y be a function of t denoted $y(t)$. Given the following differential equation:

$$-36y + D^2y = 0$$

And, assuming that the general form of the solution is:

$$y(t) = c_1 e^{at} + c_2 e^{bt}$$

a) Find the numerical values of the a and b terms and give the expression for $y(t)$.

b) Find the numerical values of the c_1 and c_2 terms when the initial conditions are $y(0)=0$ and $Dy=(dy/dt)=12$, and give the expression for $y(t)$.