Physics Review and the History of Quanta

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Fundamentals: Force, Work, Energy, and Power

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Force

- Force, **f**, is the capacity to do work
  - both a magnitude (Newtons, pounds) and a direction: a vector quantity, **f**
  - equal and opposing (in direction) forces cancel out thus no work is performed, mechanical statics
  - net, non-zero forces applied to an object cause work to be performed, W
  - a force **f** applied over a distance **x** where **f** and **x** are vector quantities
  - Newton's "laws"* of motion

*__MAT opinion__: The term "laws" is often used to refer to mathematical physical models. This is misleading since "laws" are often considered as absolute rules. A more appropriate term would be "models" since the so-called "physical laws" in mathematical form are merely human "models" of reality and have been shown, time and again, to be imprecise and idealistic.

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Newton's "Laws"*

Isaac Newton
1643 - 1727

- Foundation of Classical Mechanics
  - 1st law: the law of inertia
    An object either remains at rest or continues to move at a constant velocity unless acted on by a force
  - 2nd law: conservation of momentum
    The net force (vector sum) on an object is equal to its mass multiplied by its acceleration (rate of change of velocity). Assumes mass is constant
  - 3rd law:
    When one body exerts a force on a second body, the second body simultaneously exerts a force in equal magnitude, but opposite direction on the first body

*These laws have been superseded by Einstein's theory of special relativity. They still work well as long as the velocities involved are much slower than the speed of light.

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Momentum

- (kinetic) Momentum, **p**, is the energy exhibited by an object
  - both a magnitude (Kg-m/s) and a direction: a vector quantity, **p**, same direction as velocity
  - a conserved quantity
  - generalized momentum is an important concept in QM
  - Mathematical model: (product of mass and velocity)

$$\mathbf{p} = m\mathbf{v} \Rightarrow \mathbf{p} = m\frac{d\mathbf{x}}{dt} \Rightarrow \frac{d\mathbf{p}}{dt} = \frac{d}{dt}\left(m\mathbf{v}\right)$$

$$\frac{d\mathbf{p}}{dt} = \frac{d}{dt}\left(m\mathbf{v}\right) = m\frac{d^2\mathbf{x}}{dt} = m\mathbf{a} \Rightarrow \frac{d\mathbf{p}}{dt} = \mathbf{f}$$

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Momentum Conservation

Demonstration with "Newton's Cradle"

https://www.youtube.com/watch?v=nPUL8Zcb2P0

$$\mathbf{p} = m\mathbf{v} \Rightarrow \mathbf{p} = m\frac{d\mathbf{x}}{dt} \Rightarrow \frac{d\mathbf{p}}{dt} = \frac{d}{dt}\left(m\mathbf{v}\right) \qquad \frac{d\mathbf{p}}{dt} = \frac{d}{dt}\left(m\mathbf{v}\right) = m\frac{d^2\mathbf{x}}{dt} = m\mathbf{a} \Rightarrow \frac{d\mathbf{p}}{dt} = \mathbf{f}$$

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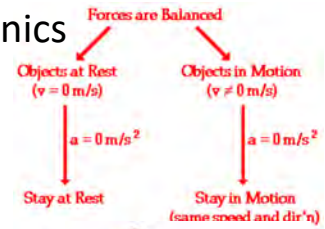
Newton's "Laws": Mathematical Form

- Foundation of Classical Mechanics
  - 1st law: the law of inertia

$$\sum_{i=1}^{n} \mathbf{f}_i = \mathbf{0} \Rightarrow \frac{d\mathbf{v}}{dt} = 0$$

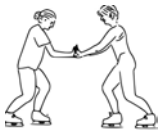


  - 2nd law: the law of acceleration

$$\mathbf{f} = m\mathbf{a} \Rightarrow \frac{d\mathbf{p}}{dt} = \frac{d\left(m\mathbf{v}\right)}{dt} = m\frac{d\mathbf{v}}{dt}$$

  - 3rd law: the law of action-reaction

$$\mathbf{f}_A = -\mathbf{f}_B \Rightarrow \left\|\mathbf{f}_A\right\| = \left\|\mathbf{f}_B\right\| \Rightarrow \mathbf{f}_A + \mathbf{f}_B = 0$$



*These laws have been superseded by Einstein's theory of special relativity. They still work well as long as the velocities involved are much slower than the speed of light.

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Work

- Work is the amount of energy <u>transferred</u> per unit of time
  - no "direction" so it is a scalar and not a vector
  - units are Joules
  - other units: Watt-hours, foot-pounds, British thermal units (BTU)
  - mathematical symbol is W
  - a force $\mathbf{f}$ applied over a distance $\mathbf{x}$ where $\mathbf{f}$ and $\mathbf{x}$ are vector quantities

$$W = \mathbf{f} \bullet \mathbf{x} \qquad W = \int_{\mathbf{x}_0}^{\mathbf{x}} \mathbf{f} \bullet d\mathbf{x}$$

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Power

- Power is the rate of doing work
  - no "direction" so it is a scalar and not a vector
  - units are Joules per Second, or Watts (SI)
  - other units: ergs/sec, hp, ft-lbs/min, dBW (log scale), dBm (log scale), calories/hr, BTU/hr (tons of refrigeration)
  - mathematical symbol is P

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\mathbf{f} \bullet \mathbf{x}\right) = \mathbf{f} \bullet \frac{d\mathbf{x}}{dt} = \mathbf{f} \bullet \mathbf{v}$$

$$P_{avg} = \frac{\Delta W}{\Delta t}$$
(average power)

$$P = \lim_{\Delta t \to 0} P_{avg} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$
(instantaneous power)

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Energy

- Energy is the property that must be transferred to an object in order to perform work on it
  - energy is a **conserved property**: it can be converted in form but it cannot be created nor destroyed
  - converted forms: heat, kinetic, potential, elastic, radiant, thermal, others
  - units: joule (same as work), 1 joule is energy transferred to object by work of moving it a distance of 1 meter against a force of 1 Newton
  - other units: erg, BTU, kW-hours, Kcalories, electronvolt, food calorie
  - In quantum mechanics (QM), energy is defined in terms of an **energy operator**, where an operator is a mathematical mapping

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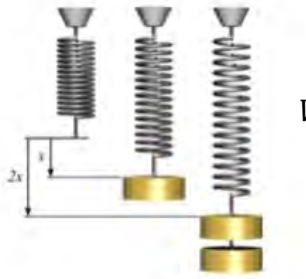
Potential Energy of Linear Spring

- Hooke's law, $\mathbf{f}_s = -K\mathbf{x}$
- Stiffness constant, K, (units are N/m)
- Work, rate of energy transfer in force times displacement, is preformed by gravity (here) to stretch the spring
- Total amount of energy transferred and stored in spring after it has stretched, $V(x)$, is the integral of work from position 0 to position x



$$W = \int_{\mathbf{x}_0}^{\mathbf{x}} \mathbf{f} \bullet d\mathbf{x}$$

$$W = \int_{\|\mathbf{x}_0\|}^{\|\mathbf{x}\|} \|\mathbf{f}\| \, \|d\mathbf{x}\| \cos(\pi) = \int_0^{|x|} K|x| \; d|x|$$

$$V(x) = \int_0^{|x|} -K|x| \; d|x|(-1) = \frac{1}{2}Kx^2$$

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Energy of Linear Spring

- Hooke's law describes the force exerted by a mechanical spring as a linear function of the spatial displacement, x
- The "slope" of the line describing the linear displacement is the "stiffness constant," K
- Consider a linear spring attached to the ceiling with a mass, m, attached to the free end
- Gravity exerts a force on the mass that causes the spring to undergo a displacement, x
- When the mass is attached to the end of the spring, work is accomplished via the transfer of energy from the earth's gravitational field to the spring as it elongates
- After the spring has elongated and is not stretching anymore, it has an amount of potential energy stored that is a function of its displacement , x
- The potential energy is then $V(x)$

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Blackbody Radiation and the Photoelectric Effect

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Short Youtube Video:

Blackbody Radiation and the Ultraviolet Catastrophe

https://www.youtube.com/watch?v=7BXvc9W97iU

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The Notion of Quanta

"Planck had put forward a new, previously unimagined thought, the thought of the atomistic structure of energy."

- Albert Einstein

- 1901 Paper by Max Planck on Black Body Radiation
- Classical Physics leads to Conclusion that Radiation from Hot Object is Very Bright at blue or violet end of Spectrum
- *eg*. A Fireplace Log glowing red ends up Emitting Ultraviolet Rays as well as X-Rays and Gamma Rays - the *Ultraviolet Catastrophe*

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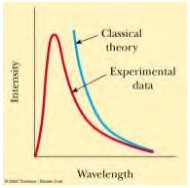
Blackbody Radiation

- Max Planck Formulated a New Law of Blackbody Radiation in 1900
- Blackbody: hypothetical object that completely absorbs all radiant energy falling upon it, reaches some equilibrium temperature, and then reemits that energy as quickly as it absorbs it.
- Classical theory up to time of Planck did not agree with experimental data and predicted the "Ultraviolet Catastrophe"



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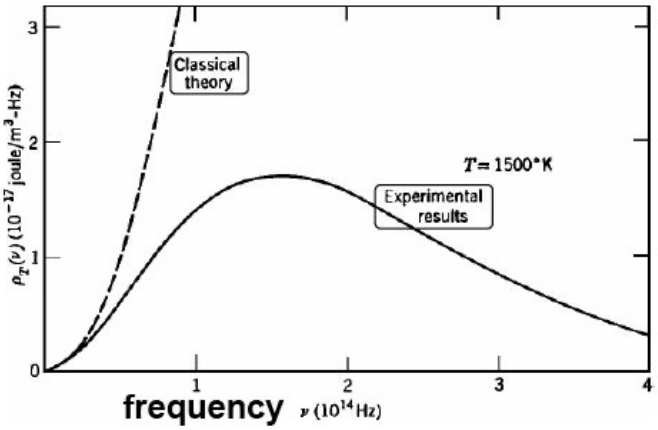
Ultraviolet Catastrophe



- Classical Theory Predicts Radiation Energy Increases with Square of Frequency - Not Seen Experimentally - Infinite Total Radiated Energy!!!

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Blackbody Radiation

- 1901 Planck Made Assumption that Radiation Energy is Emitted in Packets (quanta)
- Out of "Desperation" to Find Theory that Matched Experimental Data
- Each Packet with Energy: $E = h\nu$
- Where h is a Calculated Scale Factor now Called Planck's Constant

$$h = 6.626 \times 10^{-34}\,\mathrm{J \cdot s} \qquad \hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}\,\mathrm{J \cdot s}$$

- ν is the Frequency of the Radiation

The Notion of Quantum Behavior is Introduced in Physics

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Planck's Constant

- 19th Century Physics due to Maxwell and Hertz indicate an Oscillating Charge Produces Radiation
  - v is Oscillating Frequency
  - E is Energy of Oscillating Charge
- Planck Proposed Discrete Energy Levels of Radiation
  - started with Boltzmann's statistical thermodynamics theory

$$E = 0, hv, 2hv, 3hv, 4hv, ..., nhv$$

$$n > 0 \text{ and is an integer}$$

- Theory Successfully Explains why Ultraviolet Catastrophe never Occurs
- When Allotted Energy for an Oscillator is Smaller than a "Package" of Available Energy through Planck's Formula, Radiation Intensity Decreases

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Short Youtube Video:

Photoelectric Effect

https://www.youtube.com/watch?v=MFPKwu5vugg

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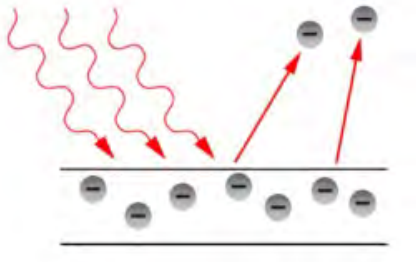
What is the "Photoelectric Effect" ?



- "Photoelectrons" Emitted from Matter after Absorption of Energy from EM Radiation such as Light
- Demonstrated by Heinrich Rudolf Hertz (Hertz Effect)

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Photoelectric Effect

- Shining a Light on Metal Liberates e^- from Surface
  - Easy for UV Light; harder for Red Light
  - Energy of e^- Depends on Light Frequency
  - Increasing Light Intensity Increases Number of e^-
- Einstein Extended Planck's Hypothesis that Light is Quantized (know known as photons) $E_{photon} = h\nu$
- Each photon Interacts with Single e^-
  - More Photons=More Liberated e^-

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Photoelectric Effect Equations

- Energy of Photon = Energy needed to remove an electron + Kinetic energy of the emitted electron

$$E_{photon} = h\nu \qquad hf = \phi + E_{k_{\max}}$$

- Where
  - h is Planck's Constant
  - f is frequency of incident photon
  - ϕ is the Work Function; minimum energy to remove photoelectron
  - $E_{k\max}$ is the maximum kinetic energy of photoelectron

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Photoelectric Effect Equations

$$hf = \phi + E_{k_{\max}} \qquad \phi = hf_0$$

$$E_{k_{\max}} = \frac{1}{2}mv_m^2 \qquad E = \sqrt{(pc)^2 + (mc^2)^2}$$

- Where
  - f_0 is threshold frequency for photoelectric effect
  - m is rest mass of the ejected electron
  - v_m is velocity of the ejected electron
  - if $hf<\phi$, no electron is emitted
  - p is momentum of particle
  - E related to p by Einstein's Special Theory of Relativity

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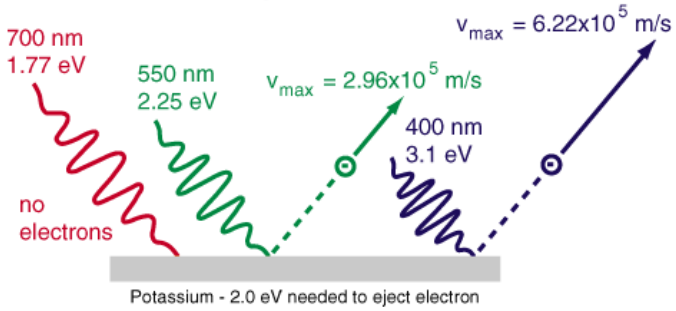
Photoelectric Effect

$E_{photon} = h\nu$



Photoelectric effect

Theory of Discrete (Quantum) Levels Again used in Physics

- Einstein Receives Nobel Prize in 1921 for this Discovery

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Einstein's Planck Medal

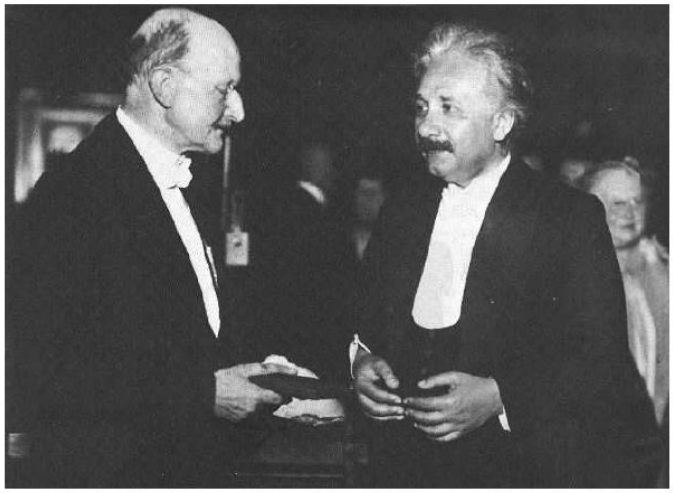

FIGURE 1.2. Planck to Einstein: *I hereby award you the Planck Medal because you expanded my desperate idea of quantum of energy to the even more desperate idea of quantum of light.*

source: unknown and quote is questionable

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