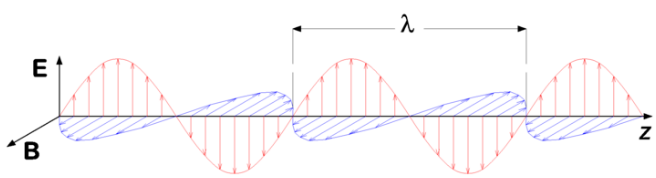
Photon Polarization as an Observable

1

Classical Polarization Refresher

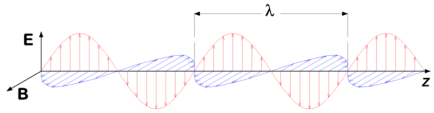
- Classically, light is an time/space-varying electromagnetic (EM) field (*i.e.*, a wave) comprised of an electric field, E, and orthogonal magnetic field, H, that is Standing (time-varying only) or Traveling (time and space varying)
- Traveling waves propagate in a direction Perpendicular to the plane defined by the E and H fields at an instant in time



- Polarization state is defined (by convention) assuming a coordinate system wherein the EM wave propagates toward the positive z-axis
- Thus, the E and H fields are orthogonal in the x-y plane

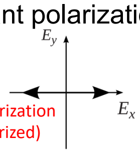
2

Linear Polarization



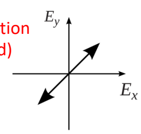
- Polarization state is defined (by convention) assuming a coordinate system wherein the EM wave propagates toward the positive z-axis
- Thus, the E and H fields are orthogonal in the x-y plane
- When the E-field is fixed in orientation in the x-y plane, the wave is said to be linearly polarized
  - collinear with the x-axis is horizontal polarization
  - collinear with the y-axis is vertical polarization
  - 45° from the x-axis and -45° from the y-axis is positive slant polarization, vice versa is negative slant polarization

Arbitrary Slant Polarization (linearly polarized)



Horizontal Polarization (linearly polarized)

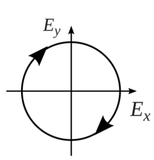
$+45^{\circ}$ Slant Polarization (linearly polarized)



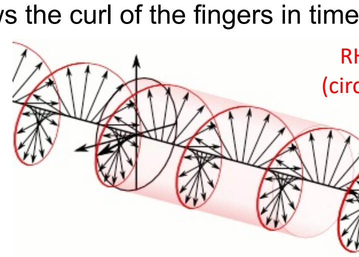
3

Classical Circular Polarization

- When the E-field position varies in orientation in the x-y plane, the wave is said to be elliptically polarized
  - circular polarization is a special case when the elliptical semi-major and semi-minor axes are equal; i.e., the ellipse is a circle
  - RHC follows the right-hand rule where the thumb points in the direction of propagation from transmitter to receiver and the E-field orientation follows the curl of the fingers in time
  - LHC follows the left-hand rule where the thumb points in the direction of propagation from transmitter to receiver and the E-field orientation follows the curl of the fingers in time



Circular Polarization

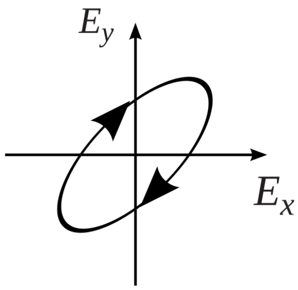


RHC Polarization (circularly polarized)

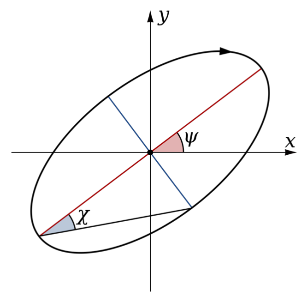
4

Elliptical Circular Polarization

- When the E-field position varies in orientation in the x-y plane, the wave is said to be elliptically polarized
  - elliptical semi-major and semi-minor axes are unequal; i.e., the ellipse is a circle
  - Most general form of EM-wave polarization state



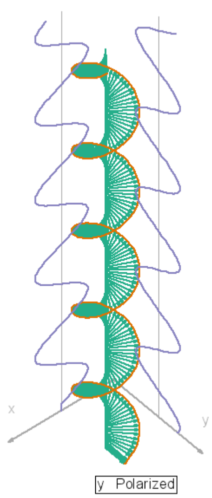
Elliptical Polarization



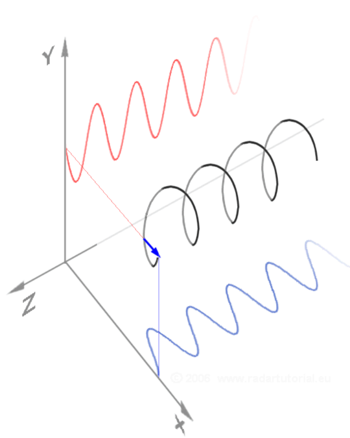
Elliptical Polarization

5

Polarization Animations



Various Linear/Circular Polarization States



Circular Polarization by E- and H-fields being 90deg Phase-shifted

6

Photon is Smallest Quantum of EM-field

- Photons are Wave Packets of EM Energy that also have a Polarization state
- Photon Polarization is an Observable due to Angular Momentum
- Photon polarization can be in Quantum Superposition with respect to a basis
  - can be a sum of orthogonal linear polarizations

$$\left(|\leftrightarrow\rangle,|\updownarrow\rangle\right)=\left(|H\rangle,|V\rangle\right)=\left(|+Z\rangle,|-Z\rangle\right) \qquad \left(|\swarrow\!\!\!\nearrow\rangle,|\nwarrow\!\!\!\searrow\rangle\right)=\left(|D\rangle,|A\rangle\right)=\left(|+X\rangle,|-X\rangle\right)$$

  - can be a sum of circular polarizations

$$\left(|L\rangle,|R\rangle\right)=\left(|\circlearrowleft\rangle,|\circlearrowright\rangle\right)=\left(|+Y\rangle,|-Y\rangle\right)$$

  - elliptical can be sum of both linear and circular polarizations

7

Photonic Measurement Example

- The Measurement symbol (previous slide) must be accompanied with a *Measurement Basis* or, alternatively, an *Observable* to make sense.
- Usually *Observable* is the *Computational Basis* or Pauli-$\mathbf{Z}$ of if not specified otherwise
  - can encode computational basis as orthogonal polarization states
- Variety of ways to specify a measurement basis, including:
  - Specify the Observable operator matrix
  - Given a specific technology, use a basis that is in reference to the observables of the technology.
- **EXAMPLE**: *Observable is Photon Polarization*
  - Linear (vertical and horizontal), denoted as:

$$\left(|\leftrightarrow\rangle,|\updownarrow\rangle\right)=\left(|H\rangle,|V\rangle\right)=\left(|+Z\rangle,|-Z\rangle\right)$$

  - Slant (left or +45, right or -45), denoted as:

$$\left(|\swarrow\!\!\!\nearrow\rangle,|\nwarrow\!\!\!\searrow\rangle\right)=\left(|D\rangle,|A\rangle\right)=\left(|+X\rangle,|-X\rangle\right)$$

  - Circular (LHC or RHC), aka photon spin, denoted as:

$$\left(|L\rangle,|R\rangle\right)=\left(|\circlearrowleft\rangle,|\circlearrowright\rangle\right)=\left(|+Y\rangle,|-Y\rangle\right)$$

8

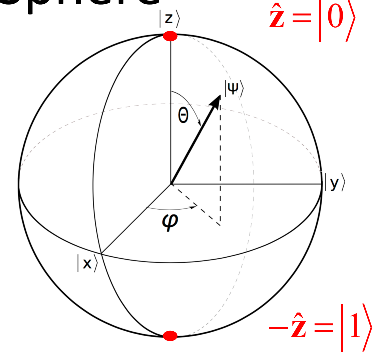
Recall Bloch Sphere

- Unit Radius Sphere
- Point on Surface Represents the State of a Qubit
- Qubit: Quantum State of Single "Information Carrier"
- Observable yields the information content of qubit
- The qubit:

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

- In General,

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\alpha|^2 + |\beta|^2 = 1$$

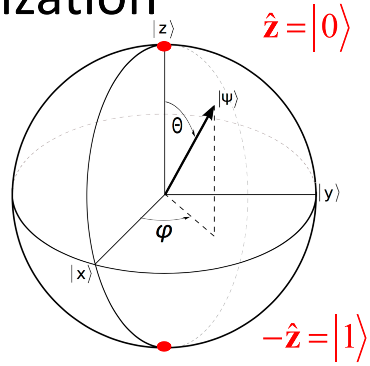


9

Photon Polarization

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

- Can be defined in terms of θ and ϕ:
  - θ=0°, ϕ=0°: Horizontal
  - θ=180°, ϕ=0°: Vertical
  - θ=90°, ϕ=0°: Slant 45°
  - θ=-90°, ϕ=0°: Slant -45°
  - $\theta=\theta_0$, ϕ=0°: Linear with pol. angle θ_0
  - θ=90°, ϕ=90°: LH Circular
  - θ=-90°, ϕ=-90°: RH Circular
  - $\theta=\theta_0$, ϕ>0°: Elliptical (RH helicity)
  - $\theta=\theta_0$, ϕ<0°: Elliptical (RH helicity)



$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

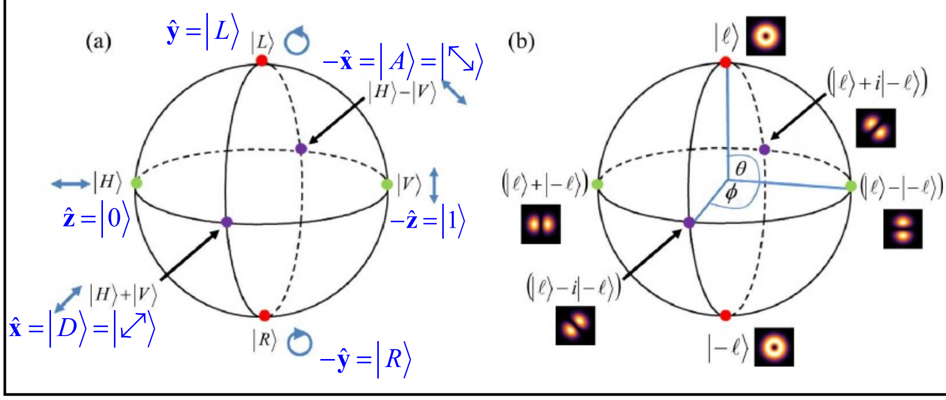
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

10

Poincare’ Sphere

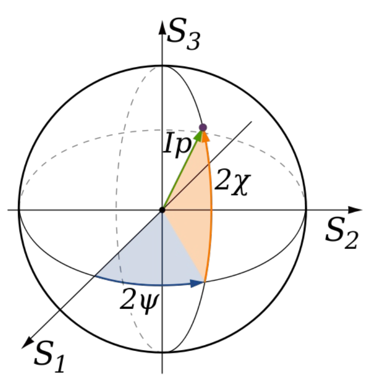
- In Optics/EMAG, Poincare’ Sphere is Bloch Sphere with z-axis rotated counterclockwise to left
- Blue Font has Bloch Sphere Unit Vector
  - Annotated with photon polarization notation

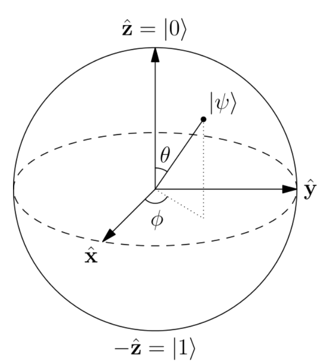


11

Bloch/Poincare’ Sphere: Stokes Parameters

- In optics, the Bloch sphere is the Poincare’ sphere
- Assumes Unity-radius with Normalized Power
  - Stokes Parameters: S_0, S_1, S_2, and S_3
  - Normalizing by S_0 results in the Bloch Sphere





12

Poincare’ Sphere is Bloch Sphere

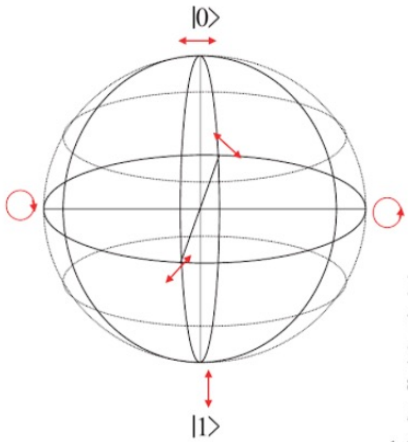
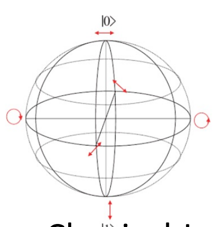


Fig. 3.8 The Poincaré sphere. Note that it is usual to place right circular polarization at the north pole of the sphere and left at the south pole so that all linear polarizations lie on the equator. We have rotated the sphere here so as to match the Bloch sphere.

*S. Barnett, Quantum Information, Oxford Univ. Press, 2009

13



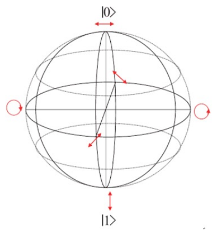
Jones Vectors

- Consider the Jones Vector of a Single Photon versus the Jones Vector of a Classical Jones Vector

- Classical Jones vector:
  - v_x and v_y (vector components) are Proportional to the Electric Field Present
  - Passing Light through Linear Polarizer Reduces Reduces Intensity of Light by Factor $|v_x|^2$
- Single Photon Jones Vector:
  - v_x and v_y are the Probability Amplitudes
  - Passing the Photon through Linear Polarizer, Detector will Detect Photon with Probability $|v_x|^2$ (assumes Detector with x-eigenbasis)
- *Probability of Detection is based on* $|v_x|^2 + |v_y|^2 = 1$

*S. Barnett, Quantum Information, Oxford Univ. Press, 2009

14



Jones Matrix

up to an arbitrary global phase. The effect of a single polarization-dependent element is obtained by multiplying the Jones vector $\underline{v}$ by the corresponding Jones matrix $\mathbf{J}$:

$$\underline{v} \to \mathbf{J}\underline{v}. \tag{3.53}$$

A sequence of n such devices will modify the Jones vector as follows:

$$\underline{v} \to \mathbf{J}_n \cdots \mathbf{J}_2\mathbf{J}_1\underline{v}, \tag{3.54}$$

Rotation of these devices is described by means of an orthogonal transformation of the associated Jones matrices. If a device is rotated through an angle θ then the corresponding Jones matrix $\mathbf{J}$ transforms as

$$\mathbf{J} \to \mathbf{R}^{\mathrm{T}}(\theta)\mathbf{J}\,\mathbf{R}(\theta),$$

where $\mathbf{R}(\theta)$ is the rotation matrix

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}.$$

	Horizontal	$\|0\rangle$	Z
	Vertical	$\|1\rangle$	
	Diagonal up	$\frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$	X
	Diagonal down	$\frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$	
	Left circular	$\frac{1}{\sqrt{2}}(\|0\rangle + i\|1\rangle)$	Y
	Right circular	$\frac{1}{\sqrt{2}}(\|0\rangle - i\|1\rangle)$	

Fig. 3.9 Linear and circular polarizations and the associated single-photon qubit states.

*S. Barnett, Quantum Information, Oxford Univ. Press, 2009

15

Projective Measurement of Photon Polarization as an Observable

16

Measurement Basis Example

- Assume Photon Polarization is the Observable used to Represent Quantum Information
- Horizontally Polarized Light Represents Computational Basis* Vector $|0\rangle$ and Vertically Polarized Light Represents Computational Basis $|1\rangle$:

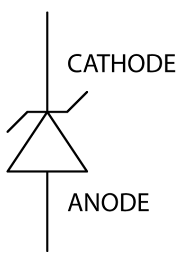
$$\big(|0\rangle,|1\rangle\big) = \big(|\leftrightarrow\rangle,|\updownarrow\rangle\big) = \big(|H\rangle,|V\rangle\big) = \big(|+Z\rangle,|-Z\rangle\big)$$

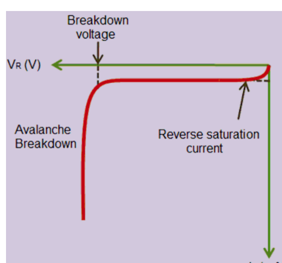
- A quantum photonic measurement device can be constructed by using two devices:
  – a polarizer, a filter that only passes light of a particular polarization
  – an energy detector such as a *Single-Photon Avalanche Diode* (SPAD), a type of photodetector that converts photon energy to an output voltage
- Polarizer only passes Horizontally or Vertically Polarized light that is Incident on Circuit with SPAD

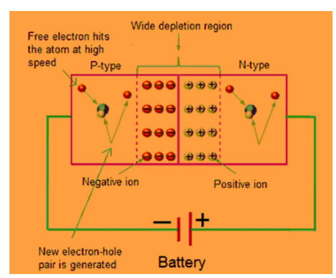
*Although the Pauli-$\mathbf{Z}$ basis is generally is used for the computational basis, it is actually an arbitrary choice as to which basis is used to encode ket-zero and ket-one.

17

Avalanche Diode





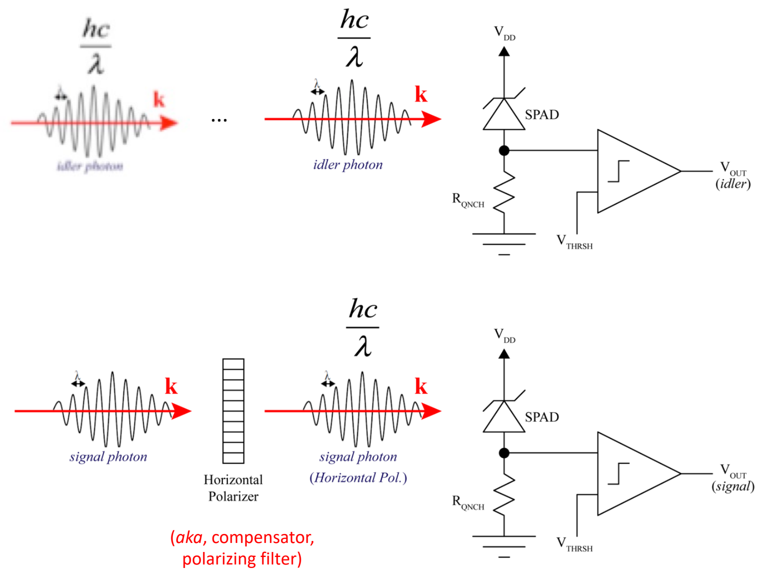


- 2-terminal doped Si device that depends upon avalanche breakdown effect; used in a reverse biased configuration; a zener diode with heavier doping levels
- High-gain detector in low-light situations that supports a high high reverse-bias current in the breakdown region
- Electron-hole pair created by single incident photon that in turn cause more and more pairs to be created causing an “avalanche” of increasing numbers of pairs to produce a larger and larger photocurrent

*https://www.elprocus.com/avalanche-diode-construction-and-working/

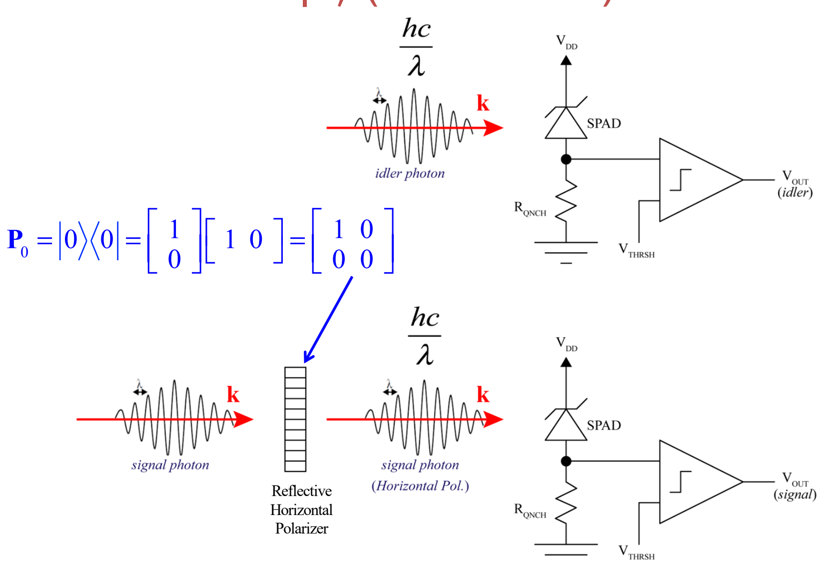
18

Polarizer/SPAD Photon Measurement Circuit for $|0\rangle$ Detection



19

Polarizer/SPAD Photon Measurement Circuit for $|0\rangle$ (Horiz. Pol.) Detection



20

Polarization Observable Eigenvectors

- $\mathbf{Z}$-basis Polarization, horizontal & vertical:

$$|\leftrightarrow\rangle=\begin{bmatrix}1\\0\end{bmatrix},|\updownarrow\rangle=\begin{bmatrix}0\\1\end{bmatrix}$$

- $\mathbf{X}$-basis Polarization, diagonal & anti-diagonal:

$$|\swarrow\!\!\!\nearrow\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix},|\nwarrow\!\!\!\searrow\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

- $\mathbf{Y}$-basis Polarization, LHC & RHC:

$$|\circlearrowleft\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix},|\circlearrowright\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$$

- Observables can be Formed from these Bases:
  - Note: these observables correspond to the Polarizing filters
  - Photonics community refers to the Projectors as "Jones Matrices"

21

Polarization Observables

- $\mathbf{A_Z}$ Observable, Linear Hor./Ver., $\{\lambda_{\leftrightarrow},\lambda_{\updownarrow}\}=\{-1,+1\}$:

$$\mathbf{A}_{\mathrm{Z}}=\lambda_{\leftrightarrow}|\leftrightarrow\rangle\langle\leftrightarrow|+\lambda_{\updownarrow}|\updownarrow\rangle\langle\updownarrow|=(1)\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1&0\end{bmatrix}+(-1)\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0&1\end{bmatrix}=\begin{bmatrix}1&0\\0&-1\end{bmatrix}$$

- $\mathbf{A_X}$ Observable, Linear Slant, $\{\lambda_{\square},\lambda_{\square}\}=\{+1,-1\}$:

$$\mathbf{A}_{\mathrm{X}}=\lambda_{\swarrow\!\nearrow}|\swarrow\!\!\!\nearrow\rangle\langle\swarrow\!\!\!\nearrow|+\lambda_{\nwarrow\!\searrow}|\nwarrow\!\!\!\searrow\rangle\langle\nwarrow\!\!\!\searrow|=\lambda_{\swarrow\!\nearrow}\left(\frac{1}{\sqrt{2}}\right)^2\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}1&1\end{bmatrix}+\lambda_{\nwarrow\!\searrow}\left(\frac{1}{\sqrt{2}}\right)^2\begin{bmatrix}1\\-1\end{bmatrix}\begin{bmatrix}1&-1\end{bmatrix}$$

$$=\lambda_{\swarrow\!\nearrow}\frac{1}{2}\begin{bmatrix}1&1\\1&1\end{bmatrix}+\lambda_{\nwarrow\!\searrow}\frac{1}{2}\begin{bmatrix}1&-1\\-1&1\end{bmatrix}=(1)\frac{1}{2}\begin{bmatrix}1&1\\1&1\end{bmatrix}+(-1)\frac{1}{2}\begin{bmatrix}1&-1\\-1&1\end{bmatrix}=\begin{bmatrix}0&1\\1&0\end{bmatrix}$$

- $\mathbf{A_Y}$ Observable, LHC & RHC, $\{\lambda_{\circlearrowleft},\lambda_{\circlearrowright}\}=\{+1,-1\}$:

$$\mathbf{A}_{\mathrm{Y}}=\lambda_{\circlearrowleft}|\circlearrowleft\rangle\langle\circlearrowleft|+\lambda_{\circlearrowright}|\circlearrowright\rangle\langle\circlearrowright|=\lambda_{\circlearrowleft}\left(\frac{1}{\sqrt{2}}\right)^2\begin{bmatrix}1\\i\end{bmatrix}\begin{bmatrix}1&-i\end{bmatrix}+\lambda_{\circlearrowright}\left(\frac{1}{\sqrt{2}}\right)^2\begin{bmatrix}1\\-i\end{bmatrix}\begin{bmatrix}1&i\end{bmatrix}$$

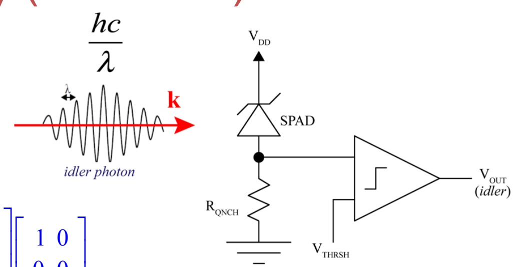
$$=\lambda_{\circlearrowleft}\frac{1}{2}\begin{bmatrix}1&-i\\i&1\end{bmatrix}+\lambda_{\circlearrowright}\frac{1}{2}\begin{bmatrix}1&i\\-i&1\end{bmatrix}=(1)\frac{1}{2}\begin{bmatrix}1&-i\\i&1\end{bmatrix}+(-1)\frac{1}{2}\begin{bmatrix}1&i\\-i&1\end{bmatrix}=\begin{bmatrix}0&-i\\i&0\end{bmatrix}$$

22

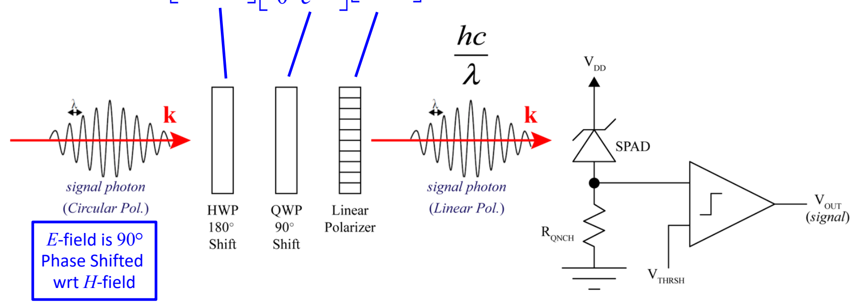
Polarizer/SPAD Photon Measurement Circuit for |0⟩ (LHC Pol.) Detection

"waveplates modify (add constant) to δ)

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

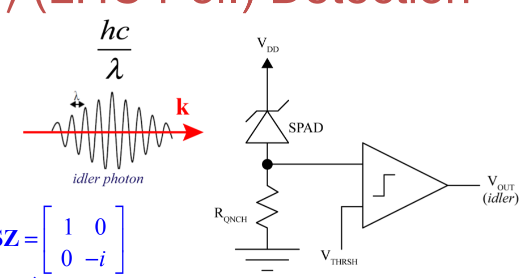


$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

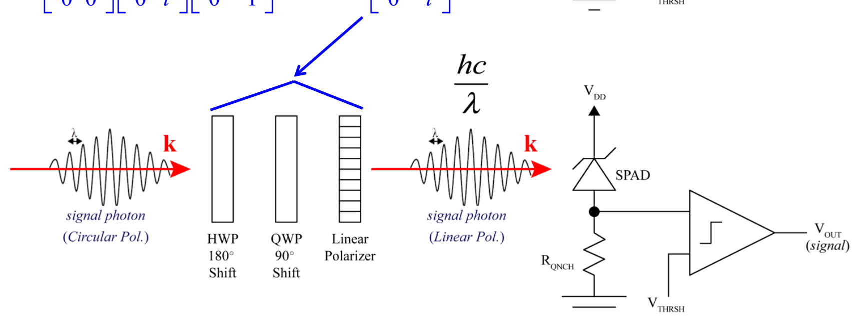


23

Polarizer/SPAD Photon Measurement Circuit for |0⟩ (LHC Pol.) Detection



$$\mathbf{P}_{\circlearrowleft 2\leftrightarrow} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \mathbf{P}_0\mathbf{SZ} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$$



24

LHC Polarization to Horizontal (|0⟩) Circuit

$$\mathbf{P}_{\circlearrowleft 2\leftrightarrow} = \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = \mathbf{P}_0\mathbf{SZ} = \begin{bmatrix}1 & 0\\ 0 & -i\end{bmatrix}$$

Incident Photon is LHC: $\left|\Psi_{LHC}\right\rangle = \dfrac{\left|0\right\rangle + i\left|1\right\rangle}{\sqrt{2}}$

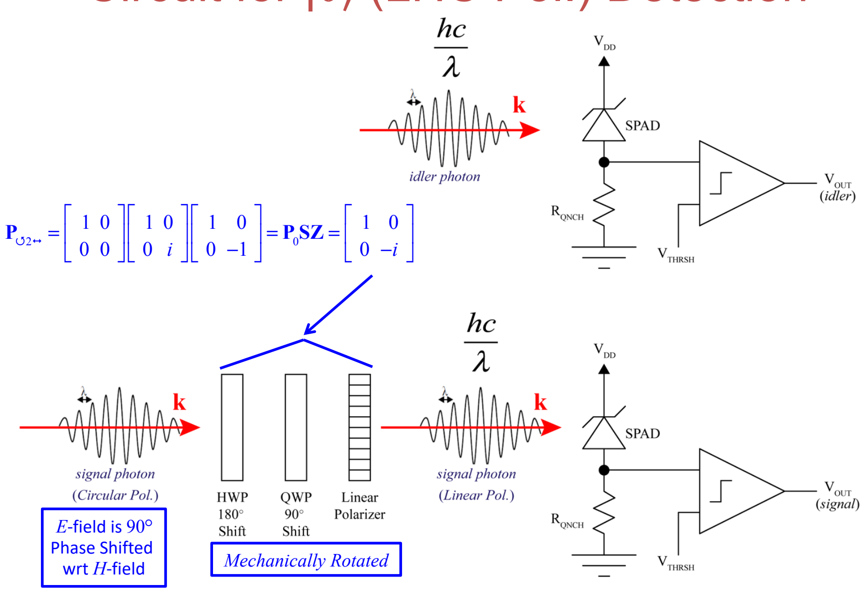
$$\mathbf{P}_{\circlearrowleft 2\leftrightarrow}\left|\Psi_{LHC}\right\rangle = \begin{bmatrix}1 & 0\\ 0 & -i\end{bmatrix}\left(\frac{1}{\sqrt{2}}\right)\begin{bmatrix}1\\ i\end{bmatrix} = \left(\frac{1}{\sqrt{2}}\right)\begin{bmatrix}1\\ 1\end{bmatrix}$$

This is +45°, so we mechanically rotate the three components about the axial axis -45°:

$$\mathbf{P}_{\circlearrowleft 2\leftrightarrow}\left|\Psi_{LHC}\right\rangle = \begin{bmatrix}1 & 0\\ 0 & -i\end{bmatrix}\left(\frac{1}{\sqrt{2}}\right)\begin{bmatrix}1\\ i\end{bmatrix} = \left(\frac{1}{\sqrt{2}}\right)\begin{bmatrix}1\\ 1\end{bmatrix}$$

25

Polarizer/SPAD Photon Measurement Circuit for |0⟩ (LHC Pol.) Detection



26

Measurement and Bases as Related to the Heisenberg Uncertainty Principle

https://www.youtube.com/watch?v=M1htf8KjKP4 (8:54)

27