A Vector Space Model for Boolean Switching Networks

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Abstract

Boolean problems are conventionally formulated and solved using the algebraic framework originally proposed by George Boole for application to mathematical logic problems. Boole's algebra was used by Claude Shannon for modeling networks of switching relays and later more general manipulation of information in the form of binary digits, or bits. An alternative approach is to model data as an element within a vector space. The vector space model then allows for the rules of linear algebra to be used for solving problems instead of Boolean algebraic axioms and postulates. The formulation of a vector space model for information representation and manipulation is given and accompanying derivations are provided. Problems often solved in the Boolean space such as simulation and justification are then solved within the vector space. The paper concludes with a discussion of more practical aspects of the linear algebraic modeling of information such as efficient data structures for representing elements and operations within the vector space and methods for converting conventional Boolean representations of switching networks into the vector space model.

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