

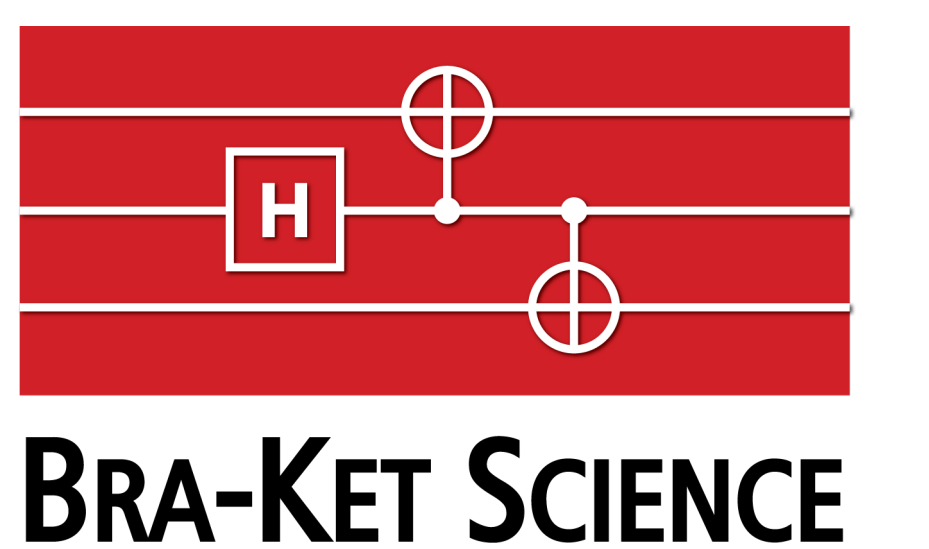
Single Photon Quantum State Oscillator

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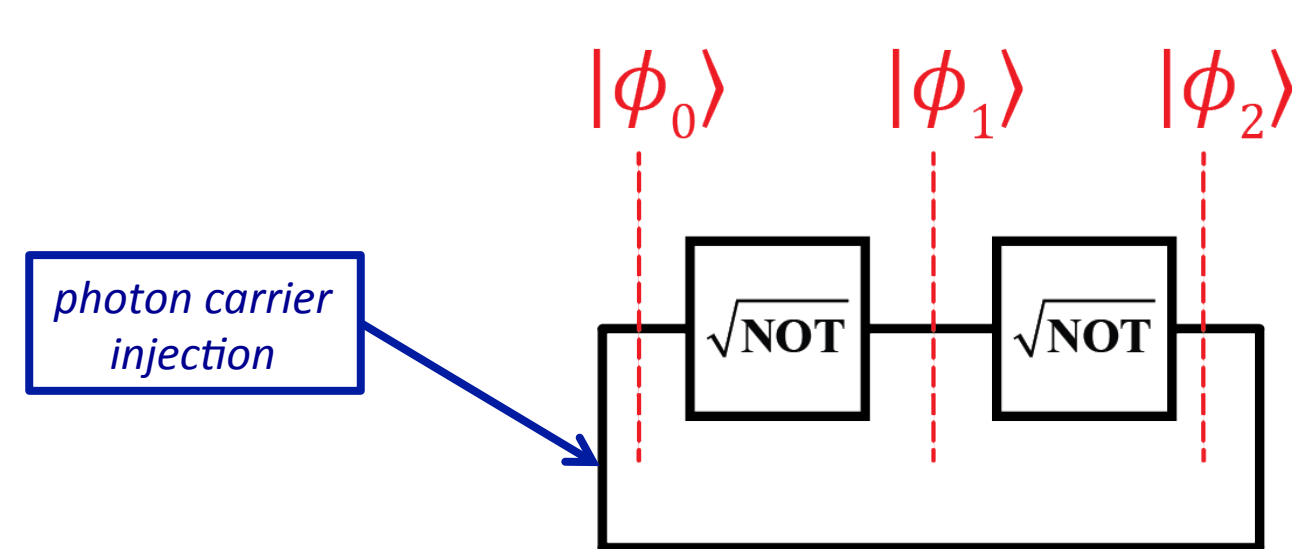
OBJECTIVES: Provide an architectural building block to support quantum computation and information processing at room temperature. Exploits the principle of continuous quantum state regeneration to reduce the required coherence time of the photonic information carrier.

- 1) Qubit storage
- 2) Quantum State Oscillator
- 3) True Random Number Generator (TRNG)
- 4) Metrology applications

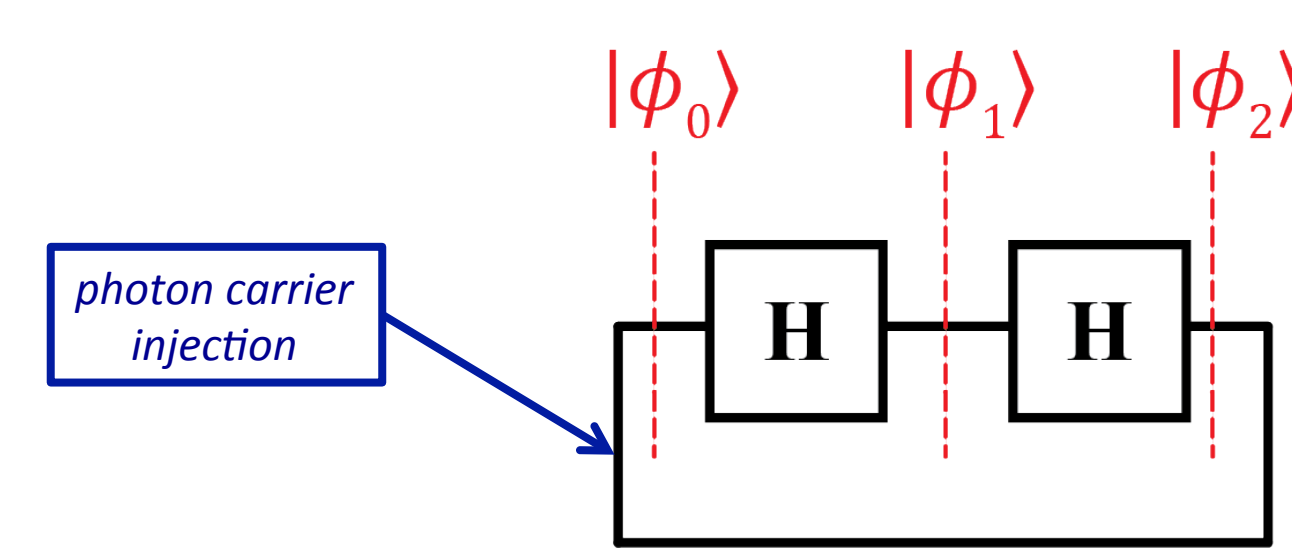
APPROACH: Quantum ring structure to trap photonic information carrier. The trap structure can be configured as a quantum state ring oscillator. Uses the principle of continuous regeneration that is analogous to the refresh cycle of an electronic DRAM cell.

*Injection of photonic information carriers is not shown

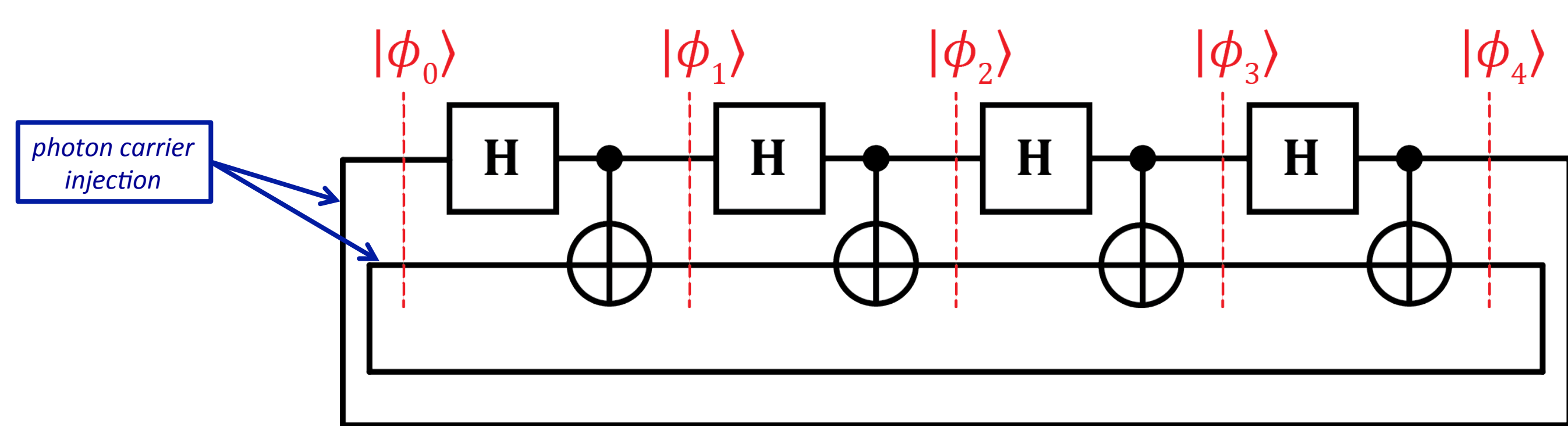
Photonic Structure "Data Paths"



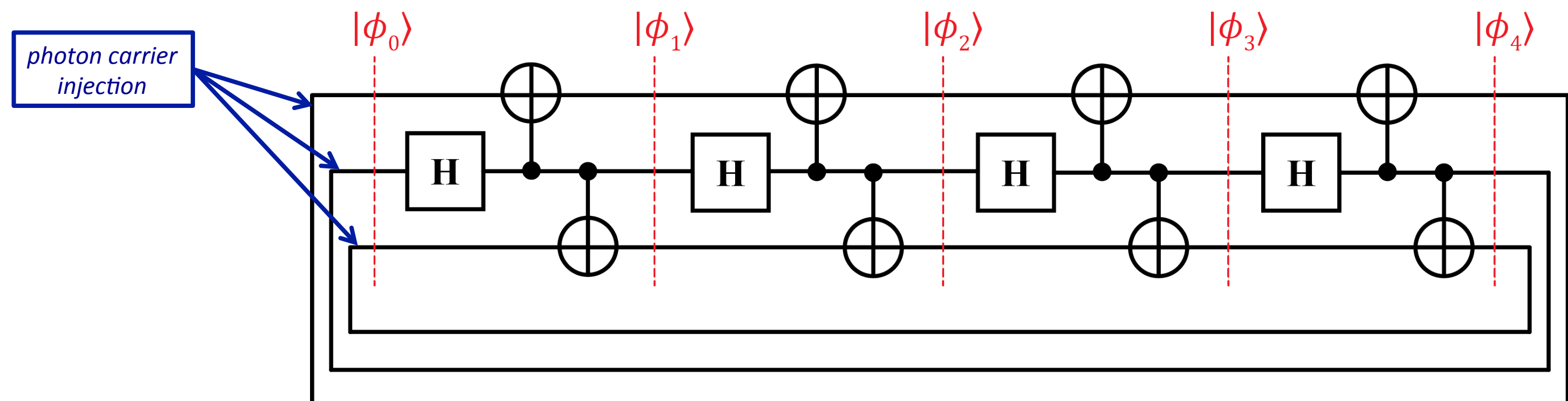
Quantum Ring Oscillator (Single-Photon QRO)



Non-oscillating Photon Trap (NPT)



Bell State Oscillator (Two-Photon BSO)



Greenberger-Horne-Zeilinger Ring Oscillator (Three-Photon GSO)

Analysis

$$\sqrt{NOT} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$$

$$|\phi_0\rangle = |0\rangle, |\phi_1\rangle = (\sqrt{NOT})|0\rangle = \frac{(1+i)|0\rangle + (1-i)|1\rangle}{\sqrt{2}}, |\phi_2\rangle = (\sqrt{NOT})\left(\frac{(1+i)|0\rangle + (1-i)|1\rangle}{\sqrt{2}}\right) = |1\rangle$$

$$|\phi_0\rangle = |1\rangle, |\phi_1\rangle = (\sqrt{NOT})|1\rangle = \frac{(1-i)|0\rangle + (1+i)|1\rangle}{\sqrt{2}}, |\phi_2\rangle = (\sqrt{NOT})\left(\frac{(1-i)|0\rangle + (1+i)|1\rangle}{\sqrt{2}}\right) = |0\rangle$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|\phi_0\rangle = |0\rangle, |\phi_1\rangle = H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, |\phi_2\rangle = H\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) = |0\rangle$$

$$|\phi_0\rangle = |1\rangle, |\phi_1\rangle = H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}, |\phi_2\rangle = H\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) = |1\rangle$$

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}, |\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}, |\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}, |\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

$$B = (CNOT)(H \otimes I) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}, B^2 = \left(\frac{1}{\sqrt{2}}\right)^2 \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$|\phi_0\rangle = |00\rangle, |\phi_1\rangle = B^2|00\rangle = |01\rangle \rightarrow |\phi_0\rangle; |\phi_0\rangle = |01\rangle, |\phi_1\rangle = B^2|01\rangle = |00\rangle \rightarrow |\phi_0\rangle$$

$$|\phi_0\rangle = |11\rangle, |\phi_1\rangle = B^2|11\rangle = |10\rangle \rightarrow |\phi_0\rangle; |\phi_0\rangle = |10\rangle, |\phi_1\rangle = B^2|10\rangle = |11\rangle \rightarrow |\phi_0\rangle$$

$$|\phi_0\rangle = |00\rangle, |\phi_1\rangle = B|00\rangle = |\Phi^+\rangle, |\phi_2\rangle = B|\Phi^+\rangle = \frac{|\Phi^+\rangle + |\Psi^-\rangle}{\sqrt{2}}, |\phi_3\rangle = B\left(\frac{|\Phi^+\rangle + |\Psi^-\rangle}{\sqrt{2}}\right) = \frac{|\Phi^+\rangle - |\Phi^-\rangle + |\Psi^+\rangle + |\Psi^-\rangle}{2}, |\phi_4\rangle = B\left(\frac{|\Phi^+\rangle - |\Phi^-\rangle + |\Psi^+\rangle + |\Psi^-\rangle}{2}\right) = |01\rangle$$

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \left(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$G^2 = (G)(G)(G)(G) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

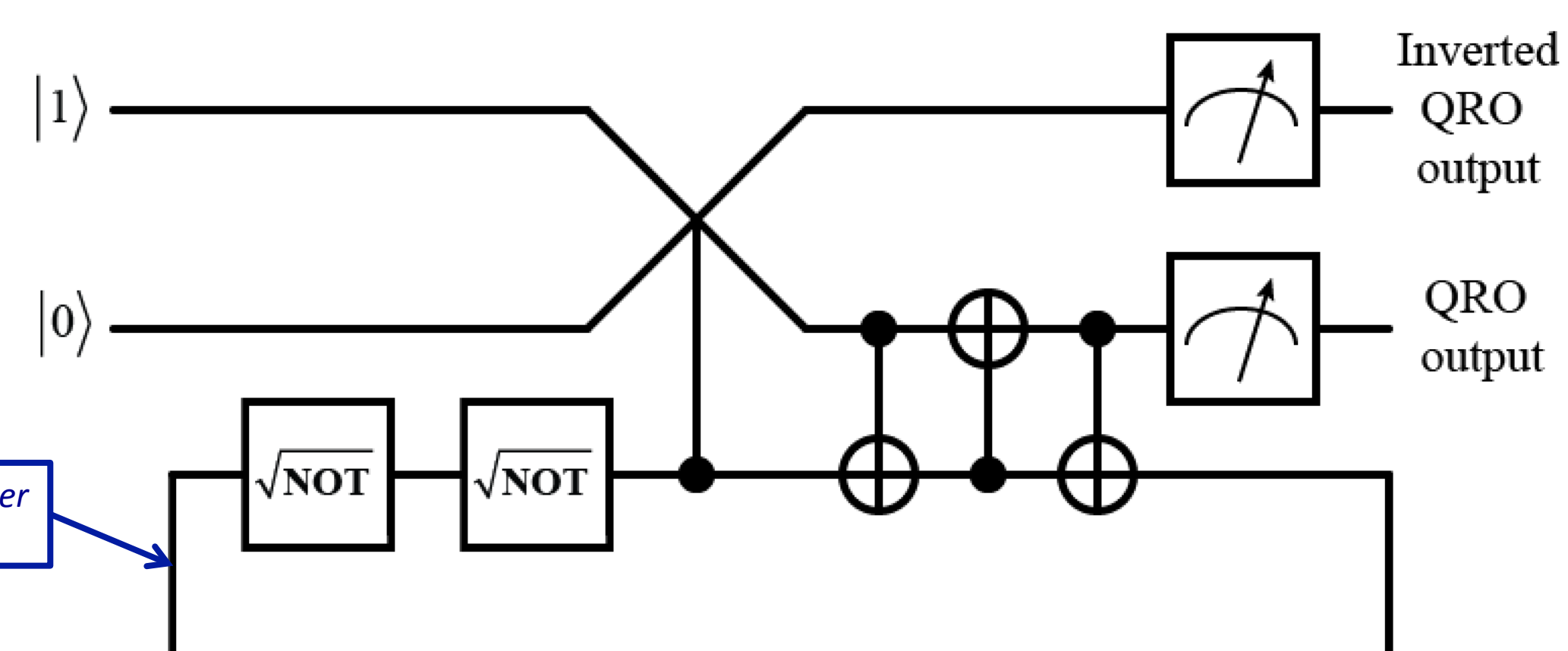
$$c_{|000\rangle} = \frac{1}{\sqrt{2}}(c_{|000\rangle} + c_{|111\rangle}), c_{|100\rangle} = \frac{1}{\sqrt{2}}(c_{|011\rangle} + c_{|100\rangle})$$

$$c_{|001\rangle} = \frac{1}{\sqrt{2}}(c_{|001\rangle} + c_{|110\rangle}), c_{|101\rangle} = \frac{1}{\sqrt{2}}(c_{|010\rangle} + c_{|101\rangle})$$

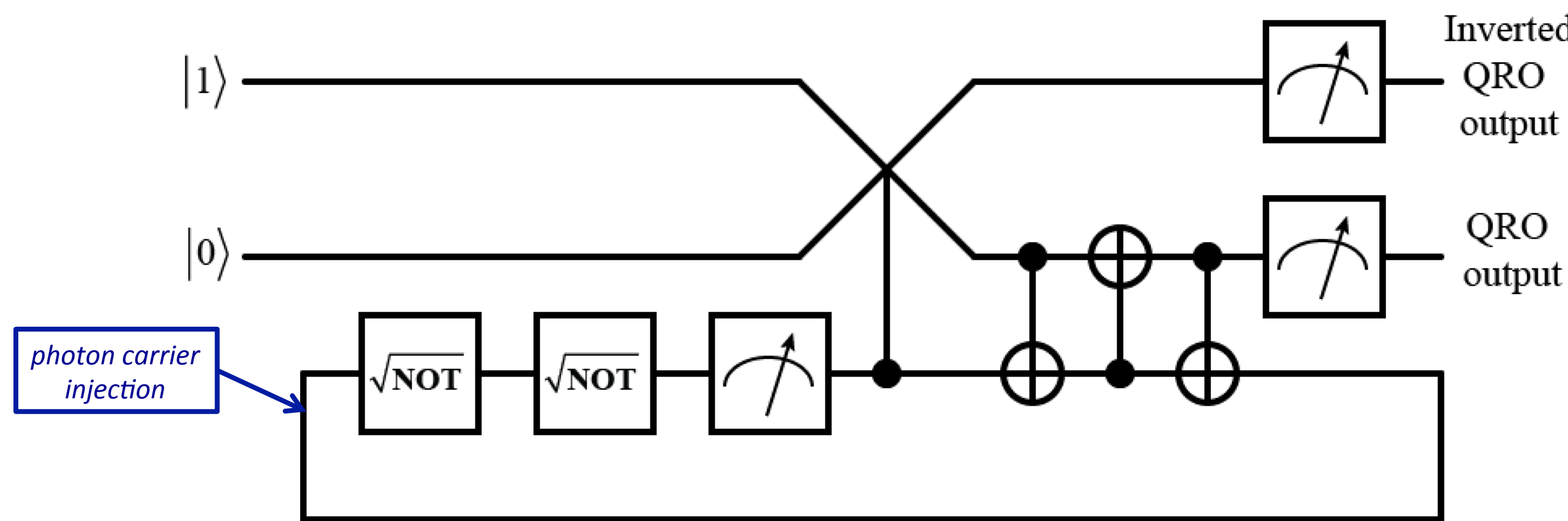
$$c_{|010\rangle} = \frac{1}{\sqrt{2}}(c_{|000\rangle} - c_{|111\rangle}), c_{|110\rangle} = \frac{1}{\sqrt{2}}(c_{|100\rangle} - c_{|011\rangle})$$

$$c_{|011\rangle} = \frac{1}{\sqrt{2}}(c_{|001\rangle} - c_{|110\rangle}), c_{|111\rangle} = \frac{1}{\sqrt{2}}(c_{|101\rangle} - c_{|010\rangle})$$

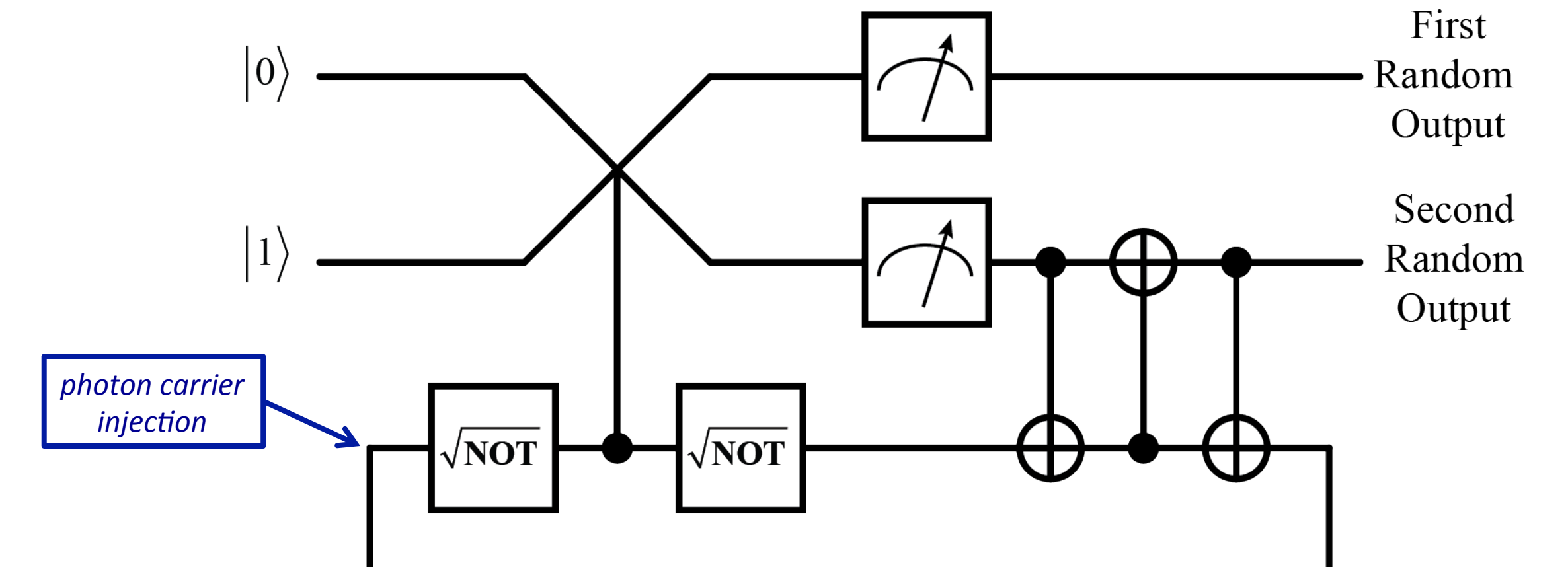
$$\begin{array}{ll} |000\rangle \rightarrow |101\rangle \rightarrow |000\rangle \rightarrow \dots & |100\rangle \rightarrow |001\rangle \rightarrow |100\rangle \rightarrow \dots \\ |001\rangle \rightarrow |100\rangle \rightarrow |001\rangle \rightarrow \dots & |101\rangle \rightarrow |000\rangle \rightarrow |101\rangle \rightarrow \dots \\ |010\rangle \rightarrow |111\rangle \rightarrow |010\rangle \rightarrow \dots & |110\rangle \rightarrow |011\rangle \rightarrow |110\rangle \rightarrow \dots \\ |011\rangle \rightarrow |110\rangle \rightarrow |011\rangle \rightarrow \dots & |111\rangle \rightarrow |010\rangle \rightarrow |111\rangle \rightarrow \dots \end{array}$$



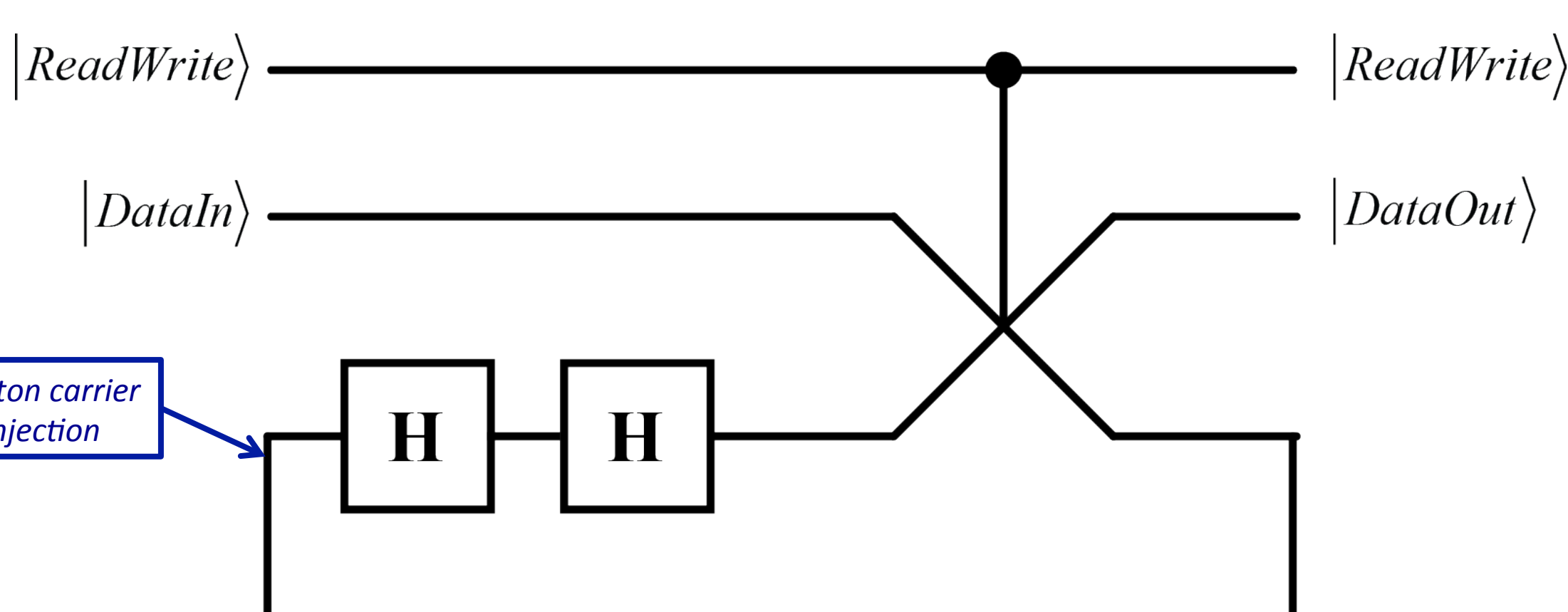
Quantum State Extraction



Quantum State Extraction with Correction

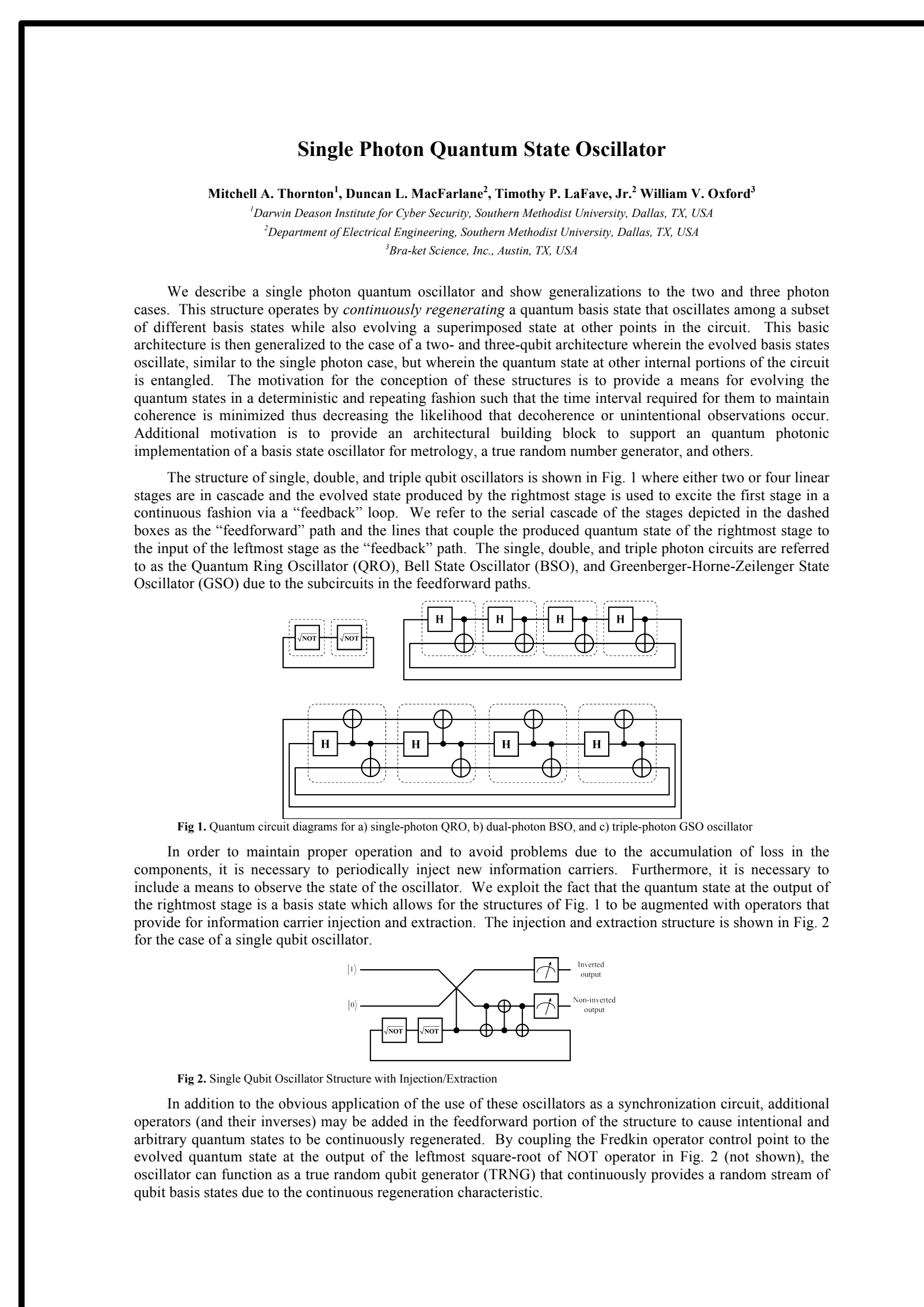


Quantum TRNGs



Quantum State Storage

- Initially Injected Carrier Photon in Basis State
- $|ReadWrite\rangle$ Control when $|0\rangle$:
 - Stored State Circulates and is Continuously Regenerated
 - $|DataOut\rangle$ is Garbage (equal to $|DataIn\rangle$)
- $|ReadWrite\rangle$ Control when $|1\rangle$:
 - Stored State is Present at $|DataOut\rangle$
 - Exchanges/Writes $|DataIn\rangle$ State with Photonic Information Carrier in Ring



*Injection of photonic information carriers is not shown

- Injected Carrier Photon:
 - **Basis State:** Equiprobable Output, $p_0=p_1$, (assumes perfect Hadamard operator)
 - **Arbitrary Basis State:** Bernoulli Distributed Random Variable, $p_0 \neq p_1$
- Controlled-Swap Control in Equal Superposition
- Measurement Collapses and Randomly "chooses" either $|0\rangle$ or $|1\rangle$
- Swap Exchanges Second Random Output State with that of Photonic Information Carrier (optional)