Fixed Polarity Pascal Transforms with Computer Algebra Applications

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***Abstract*—The fixed polarity forms of the Reed-Muller (RM) transform exist in 2^n different polarities. The integer-valued Pascal transform is related to the binary-valued RM transform through the Sierpinski fractal that can be computed from Pascal's triangle and that also appears in the lower triangular portion of the positive-polarity RM transform. We generalize the relationship between the fixed-polarity forms of the RM transform and introduce associated forms of the Pascal transform that are characterized by a polarity value allowing for a family of fixed-polarity Pascal (FPP) transform matrices to be defined. We observe and prove several properties of the FPP transforms and their inverses. An application of FPP transforms in the area of computer algebra that enables very fast decomposition of real-valued polynomials as weighted sums of different binomials raised to a power as compared to manual symbolic manipulation is described. The decomposition weights can be considered to be the FPP spectrum with respect to a real-valued polynomial since they are computed using one of the linear orthogonal FPP transformation matrices.**

I. BODY OF PAPER REDACTED

II. Conclusion

We have introduced the concept of a fixed polarity Pascal (FPP) transformation matrix that is analogous to the fixed-polarity Reed-Muller (FPRM) transformation matrices. This concept resulted in the definition of a family of linear transformations based upon the Pascal transformation that are characterized by an integer value referred to as the "polarity number." We introduced and proved several characteristics of this family of transformation matrices and showed how the inverse FPP transforms can be calculated efficiently in closed form. We applied these results to a computer algebra application resulting in methods that are suitable for implementation as efficient algorithms to both simplify polynomials and to decompose polynomials into weighted sums of binomials raised to a power.

In future work, we will investigate methods that attempt to find the FPP polarity number, and hence the specific FPP transformation matrix that results in a binomial decomposition that has as few binomial terms as possible. This problem is a direct analogy to the well-known and well-studied problem of finding the polarity number for a FPRM transform that results in a switching function expressed in a RM expansion that has as few product terms as possible. We also believe that the FPP transforms can be used as the core operation in many other computer algebra applications and we intend to investigate those as well.

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