Analysis of a hybrid micro/macro-optical method for distortion removal in free-space optical interconnections

Marc P. Christensen and Predrag Milojkovic

Applied Photonics, 4031 University Drive, Suite 200, Fairfax, Virginia 22030

Michael W. Haney

Department of Electrical and Computer Engineering, University of Delaware, Evans Hall, Newark, Delaware 19716

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Eikonal analyses are applied to a hybrid micro/macro-optical shuffle interconnection approach that minimizes distortion in a multichip smart-pixel shuffle interconnection system. The optical system uses off-axis imaging elements to link clusters of dense arrays of vertical-cavity surface-emitting laser (VCSEL) sources to matching clusters within arrays of detectors. A critical requirement for such a system is that the images of the two-dimensional arrays of the VCSELs must be registered on their associated detector arrays with a precision of the order of 10 \( \mu \text{m} \) across the entire multichip array. The hybrid approach exploits the typical narrow-beam cone angles of VCSELs by use of beam-deflecting micro-optics to create a distortion-canceling symmetry about a central aperture in the optical system for each VCSEL–detector link. The second- and third-order aberrations of the plane-symmetric system created by the global off-axis imaging system are analyzed. The results prove that the hybrid concept cancels distortion and minimizes the spot size at the detector array plane.

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1. BACKGROUND AND MOTIVATION

Global three-dimensional optical shuffle interconnection architectures, utilizing smart pixels, have been proposed to overcome interconnection limitations.\(^1\)–\(^9\) Scalable multiterabit interconnection fabrics may be achieved by use of multiple optoelectronic integrated circuits linked to each other in a global high-bisection-bandwidth pattern.\(^10\) Figure 1 depicts one such high-bisection-bandwidth multichip configuration.\(^11\),\(^12\) In this configuration, each lens links the optical input–output from a single chip, located at the lens’s focal plane, to all chips in the receiving array. Clusters of emitters, such as vertical-cavity surface-emitting lasers (VCSELs), and detectors are imaged onto corresponding clusters on other chips such that many point-to-point links are established in an interleaved optical shuffle pattern across the multichip plane. Monolithically integrated VCSEL–detector arrays, with emitter and receiver elements of 10 and 50 \( \mu \text{m} \), respectively, and with element-to-element spacing as small as 100 \( \mu \text{m} \), have been evaluated in a prototype shuffle system.\(^13\)

With such input–output density and pitch, the global optical interconnection module must provide flat, high-resolution, near-distortion-free image fields across a wide range of ray angles to avoid cross talk and maintain high link efficiency.

2. BEAM-DEFLECTION APPROACH

Although modern optical design and manufacturing techniques provide approaches to achieving high resolution, registration accuracy is more problematic. Registration accuracy may be defined as the difference between the location of the image of a VCSEL and the location of its corresponding detector. Registration must be maintained at a level less than the size of the detector (\( \approx 50 \mu \text{m} \)) across the entire multichip plane (\( \approx 10 \text{ cm wide} \)). Distortion in the optical system will cause poor registration performance in the system. It is well known that holosymmetric systems (systems with rotational symmetry about their optical axis and symmetry along their optical axis about their aperture) cancel distortion.\(^14\)–\(^16\) Multichip optical interconnection systems, such as the one depicted in Fig. 1, do not have one aperture, \emph{per se}; instead, they have a collection of effective apertures for each lens pair defined by the location of the beam when its chief ray crosses the line connecting the centers of the transmitting and receiving lenses. Although the interconnection system depicted in Fig. 1 appears to be symmetric, the effective aperture of the system is not at the midpoint between the transmitting and the receiving lens planes. As depicted in Fig. 2(a), this asymmetry results from the normal orientation of the VCSEL beams—parallel to the optical axis. To cancel distortion, one must move the effective aperture to the midpoint between the transmitting lens and the receiving lens. Unfortunately, placing a physical aperture at this location would cause the narrow VCSEL beams to miss the aperture entirely or be severely vignetted. A hybrid micro/macro-optical approach to avoiding this vignetting was proposed\(^17\) and validated in initial experiments.\(^18\) In this approach the VCSEL beams are individually deflected at angles that cause
them to propagate through the new central effective aperture as shown in Fig. 2(b). This is possible because the VCSELs have narrow beam divergence. Since the VCSELs have been steered through the central effective aperture, no physical aperture is needed at this location.

The proposed method for implementing the beam deflection is depicted in Fig. 2(c). A linear diffraction aperture, no physical aperture is needed at this location. SELs have been steered through the central effective aperture as shown in Fig. 2(b). This is possible because the VCSEL planes are on the left, and the detector planes are on the right. (a) Telecentric interconnection system, (b) symmetric interconnection system, (c) symmetric interconnect system with auxiliary microbeam-deflection elements.

The five third-order aberrations (spherical, astigmatism, coma, distortion, and field curvature) are appropriately applied only to rotationally symmetric systems. These systems are, by definition, on axis. The global optical interconnection module depicted in Fig. 1 utilizes off-axis lens pairs to effect the global interconnection pattern. Such an offset lens imaging system does not contain a rotational axis of symmetry and, as such, cannot be adequately described by the five aberrations listed above. In place of an axis of rotational symmetry, each lens pair of the global connection module has a single plane of symmetry that contains the optical axes of both the transmitting and the receiving lenses. 

Figure 3 shows the plane of symmetry for an off-axis imaging system. Systems with single planes of symmetry have been analyzed. In such systems there are ten third-order aberrations that must be analyzed. Additionally, second-order aberrations (which are not present in a rotationally symmetric system) may arise. The framework of analysis presented in Ref. 20 will be followed with the additional constraints of small angular divergence sources and beam steering. This analysis will derive the residual aberration function of the global optical interconnection module and will develop a merit function for minimizing all aberrations in this off-axis configuration.

3. EIKONAL ANALYSIS FOR RAY-INTERCEPT-ERROR DETERMINATION

The eikonal of an optical system is a function that describes the optical path length that a given ray follows from the input to the output of an optical system. It maps a given ray in the input space onto a ray in the output space and determines the path length connecting these two rays. A ray is completely specified by its location in a given plane and its direction at that point. The direction of a ray is customarily given by its directional cosines \( L \) and \( M \), which are the dot products of a unit vector in the direction of the ray with the unit vectors in the \( x \) and \( y \) directions, respectively. A third directional cosine, \( N \), is given by the dot product of the ray with the unit vector of the optical axis (\( z \)); however, since the magnitude of the direction vector is defined to be unity, \( N \) is given by \( (1 - L^2 - M^2)^{1/2} \), and therefore all that is needed to specify a ray is the \( x \) and \( y \) coordinates and the \( L \) and \( M \) projections of the ray at a given plane. The optical system transforms an input ray \((x, y, L, M)\) in the input space into an output ray \((x', y', L', M')\) in the output space. Although this would seem to indicate that four functions of four variables would be required to describe an arbitrary optical system, in fact, Fermat’s principal greatly constrains the optical system so that only a single function of a properly chosen set of four variables, two of the input and two of the output, is required.

There are four forms of eikonals: point \([S(x, y, x', y')]\), point angle \([V(x, y, L', M')]\), angle...
point \([V'(L, M, x', y')]\), and angle \([W(L, M, L', M')]\). The name of the eikonal describes the type of input and output variables used by the function. For example, the point eikonal specifies the point (location) of the ray in the input and output spaces, whereas the point-angle eikonal specifies the point (location) of the ray in the input space and the angle of the ray in the output space. One must determine an eikonal for which the four variables are independent and is therefore useful for the system. For example, consider an imaging optical system with magnification \(M\). The position of the output ray in the image space is equal to \((Mx, My)\) for an object point located at \((x, y)\). Clearly, the point eikonal \([S(x, y, x', y')]\) would not provide the necessary independent variables, and its use would be degenerate and prohibited.22 The basic components of the multiclip interconnection module studied in this paper comprise a two-lens imaging system, so the point eikonal is prohibited, but any other eikonal will suffice. Since the focus of this paper is on analyzing the effects of beam steering on the system, the natural choice for the eikonal is the angle–angle eikonal \([W(L, M, L', M')]\).

Velzel20 analyzed the ray-intercept error (aberration function) of systems with various degrees of symmetry. The ray-intercept error is derived to be

\[
\epsilon_x = -\gamma_x \frac{\partial W}{\partial L} - \tau_x \frac{\partial W}{\partial M} - \frac{\partial W}{\partial L'},
\]

\[
\epsilon_y = -\gamma_y \frac{\partial W}{\partial M} - \tau_y \frac{\partial W}{\partial L} - \frac{\partial W}{\partial M'},
\]

where \(\gamma_y\) is the magnification of the system along the \(i\) direction and \(\tau_y\) is defined to be the shearing angle of the system. In the optical system analyzed here, the magnification in both the \(x\) and \(y\) directions is \(-1\), and the shearing angles are zero due to the single plane of symmetry in the system.20 Therefore the ray-intercept errors are given by

\[
\epsilon_x = \frac{\partial W}{\partial L} - \frac{\partial W}{\partial L'},
\]

\[
\epsilon_y = \frac{\partial W}{\partial M} - \frac{\partial W}{\partial M'},
\]

where \(\epsilon_x\) and \(\epsilon_y\) are the ray-intercept errors in the \(x\) and \(y\) directions, respectively. These ray-intercept-error functions will be the basis for analyzing the aberrations of the global multiclip optical interconnection system.

4. OFF-AXIS ANALYSIS

The analysis of the off-axis plane-symmetric system must be conducted to determine both second-order and third-order aberrations, as this more general optical system may contain both. To analyze the ray-intercept error for the global multiclip optical system, we used a general description of the path length. This general form of the path length is then constrained by the symmetries evident in the global multiclip optical system. As Eqs. (1) and (2) indicate, the ray-intercept error is related directly to the first derivative of the eikonal (path-length function). Therefore, in the analysis of the second-order ray-intercept errors, a general expansion of the third-order terms for the eikonal is used. Similarly, to analyze third-order aberrations, a general expansion of the fourth-order terms of the path length from object to image is required.

A. Second-Order Aberration Analysis

The general expansion of third-order terms of the angle-angle eikonal (path length) is given by

\[
W_{(3)}(L, M, L', M') = \sum_{i+j+k+l=3} a_{ijkl} L^i M^j L'^k M'^l,
\]

where \(a_{ijkl}\) is the coefficient of the term with powers \(L^i M^j L'^k M'^l\). The sum of the powers of the directional cosines \((L, M, L', M')\) is 3, so that, when the derivatives are taken for the aberration function, the resultant errors are of second order in \(L, M, L'\), and \(M'\).

In general, there are 20 coefficients of second order. However, the symmetry of the system can be used to relate and reduce them. The first type of symmetry that is used to relate them is the reversibility of the system. The global optical interconnection system is physically symmetric about the plane of the mirror; therefore the path length of a ray traveling forward through the system must be identical to that of the same ray traveling backward through the system, thereby reversing the input and output angles. This constraint dictates that

\[
W(L, M, L', M') = W(L', M', L, M).
\]

Equation (6) constrains the coefficients of the eikonal function \(W\) such that the following coefficients are equated:

\[
a_{1110} = a_{1011},
\]

\[
a_{0102} = a_{0201},
\]

\[
a_{0300} = a_{0003},
\]

\[
a_{2001} = a_{1020},
\]

\[
a_{3000} = a_{0030},
\]

\[
a_{1020} = a_{2010},
\]

\[
a_{2100} = a_{0120},
\]

\[
a_{1002} = a_{0210},
\]

\[
a_{0111} = a_{1101},
\]

\[
a_{1200} = a_{0012}.
\]

The next reduction in coefficients is due to the fact that there should be no odd powers of \(M\) in \(\epsilon_y\) in a system with a single plane of symmetry.20 Odd powers of \(M\) would be contrary to the existence of the single plane of symmetry. Applying this constraint, we determine relationships between coefficients to be

\[
a_{1110} = a_{2001} = a_{2100}.
\]

Similarly, there should be no even powers of \(M\) in \(\epsilon_y\). Therefore

\[
a_{0102} = a_{0300}.
\]
When the leftmost terms of Eqs. (7)–(18) are used to replace their equivalents, the ray-intercept errors from Eqs. (3) and (4) are determined to be

\[
\begin{align*}
\epsilon_x &= 3\alpha_{3000}L^2 + 3\alpha_{1002}M'^2 + 3\alpha_{1200}M^2 + 3\alpha_{1020}L'^2 \\
&\quad - 3\alpha_{3000}L'^2 - 3\alpha_{1020}L^2 - 3\alpha_{1002}M^2 \\
&\quad - 3\alpha_{1200}M'^2, \\
\epsilon_y &= L[6M(\alpha_{1200} - \alpha_{0111}) + 6M'(\alpha_{0111} - \alpha_{1002})] \\
&\quad + L'[6M(\alpha_{1002} - \alpha_{0111}) + 6M'(\alpha_{0111} - \alpha_{1200})].
\end{align*}
\]

(19)

If the proposed hybrid beam deflection is employed to steer the central ray of the VCSEL cone to an angle such that \( L = L' \) and \( M = M' \), then for this ray, which defines the distortion of the system, the ray-intercept errors, \( \epsilon_x \) and \( \epsilon_y \), are identically zero.

Since the ray-intercept error of the central ray of the VCSEL cone is identically zero, the system is distortion free, but what effect does beam steering have on the remaining rays in the steered cone? Figure 4 depicts the steered ray cone and sets up a geometry for analyzing the steered chief ray. Substituting expressions (21)–(24) into Eqs. (19) and (20) gives

\[
\begin{align*}
\epsilon_x &= N_0(12\alpha_{3000}L_0 \cos \theta - 12\alpha_{1002}M_0 \sin \theta) \\
&\quad + 24\alpha_{1002}L_0 \sin \theta + 12\alpha_{1200}M_0 \cos \theta, \\
\epsilon_y &= N_0(12\alpha_{1002}L_0 \sin \theta - 12\alpha_{1002}M_0 \cos \theta) \\
&\quad - 24\alpha_{0111}L_0 \sin \theta + 12\alpha_{1200}L_0 \sin \theta \\
&\quad + 12\alpha_{1200}M_0 \cos \theta).
\end{align*}
\]

(25)

(26)

Equations (25) and (26) represent the second-order ray-intercept error. For any given angle \( \theta \) of these ray-intercept errors are linear functions of \( \Delta \) with roots at \( \Delta = 0 \). Therefore, for a ray cone of fixed divergence, the lowest rms error (spot size) can be achieved by centering the cone about the steered chief ray \( (L_0, M_0) \) (i.e., \( \Delta = 0 \)). Therefore, it has been shown that there is no second-order ray-intercept error for the chief ray and hence no distortion, and the residual ray-intercept error of the remaining rays in the cone (i.e., the rms spot size) are minimized through beam steering. Next, the third-order aberrations must be similarly analyzed.

B. Third-Order Aberration Analysis

In a rotationally symmetric optical system, the third-order aberrations, or Seidel aberrations, are the typical figures of merit for the optical system. To analyze the third-order aberrations of the plane-symmetric global multichip optical interconnection system, one requires fourth-order terms. The general expansion of fourth-order terms of the angle-angle eikonal (path length) is given by

\[
W_{44}(L, M, L', M') = \sum_{i+j+k+l=4} a_{ijkl}L^iM^jL'^kM'^l,
\]

(27)

where \( a_{ijkl} \) is the coefficient of the term with powers \( L^iM^jL'^kM'^l \). The sum of the powers of the directional cosines \( (L, M, L', M') \) is 4, so that when the derivatives are taken for the aberration function the resultant errors are of third-order in \( L, M, L', \) and \( M' \).

In general, there are 35 coefficients of third order. However, similar to the analysis of the second-order aberrations, the symmetry of the system can be used to relate and thereby reduce the number of them. The first type of symmetry that is used to relate them is the reversibility of the system. As described before, the global optical interconnection system is physically symmetric about the plane of the mirror; therefore the path length of a ray traveling forward through the system must be identical to that of the same ray traveling backward through the system, thereby reversing the input and output angles. Recalling from before (and repeated for clarity) that this requires that

\[
W(L, M, L', M') = W(L', M', L, M).
\]

(6)

Equation (6) constrains the third-order coefficients of the eikonal function \( E \) such that

\[
\begin{align*}
a_{0121} &= a_{2101}, \\
a_{1210} &= a_{1012}, \\
a_{0112} &= a_{1201}.
\end{align*}
\]

(28)

(29)

(30)
The next reduction in coefficients is due to the fact that there should be no odd powers of $M$ in $\varepsilon_\gamma$ in a system with a single plane of symmetry. Odd powers of $M$ would be contrary to the existence of the single plane of symmetry. Applying this constraint, we determine relationships between coefficients to be

$$
a_{0211} = a_{2110} = a_{0130} = a_{0061},
$$
\hspace{1cm} (42)$$

$$
a_{0211} = a_{0112},
$$
\hspace{1cm} (43)$$

$$
a_{0013} = a_{0310}.\hspace{1cm} (44)$$

Similarly, there should be no even powers of $M$ in $\varepsilon_\gamma$. Therefore

$$
a_{0211} = a_{0013}.\hspace{1cm} (45)$$

When the leftmost terms of Eqs. (28)–(45) are used to replace their equivalents, the ray-intercept errors that are due to the third-order terms from Eqs. (3) and (4) are determined to be

$$
\varepsilon_x = 4(L - L')(a_{0040}L^2 - a_{1030}L^2 - 3a_{0200}LL')
$$
$$
+ 2a_{1030}LL' + a_{0040}L'L - a_{1030}L'^2 + a_{0040}L^2)
$$
$$
+ 12M^2(\alpha_{0202}L - \alpha_{1210}L + \alpha_{0220}L + \alpha_{1210}L')
$$
$$
+ 24MM(\alpha_{1111}L - \alpha_{1111}L + \alpha_{0121}L - \alpha_{0121}L')
$$
$$
+ 12M(\alpha_{1210}L - \alpha_{0220}L + \alpha_{0220}L + \alpha_{1210}L'),
$$
\hspace{1cm} (46)$$

$$
\varepsilon_y = 4(M - M')(a_{0400}M^2 - a_{0103}M^2 - 3a_{0202}MM')
$$
$$
+ 2a_{0103}MM' + a_{0400}MM' - a_{0103}M'^2
$$
$$
+ a_{0400}M^2 + 12L^2(\alpha_{0022}M - \alpha_{0121}M)
$$
$$
+ a_{0220}M + \alpha_{0121}M' + 24LL'(\alpha_{1111}M'
$$
$$
- \alpha_{1111}M + \alpha_{1210}M - \alpha_{1210}M' + 12L^2(\alpha_{0121}M
$$
$$
- \alpha_{0220}M + \alpha_{0022}M + \alpha_{0121}M')).
$$
\hspace{1cm} (47)$$

Again, it has been shown that if beam steering is employed to steer the central ray of the VCSEL cone to an angle such that $L = L'$ and $M = M'$, then for this ray, which defines the distortion of the system, the ray-intercept errors $\varepsilon_x$ and $\varepsilon_y$ are identically zero.

Since the third-order ray-intercept error of the central ray of the VCSEL cone is identically zero, the system is free of third-order distortion. As before, rays that surround the cone can be analyzed. Recalling that Fig. 4 depicts the steered ray cone and sets up a geometry for analyzing other rays in the cone, a ray under inspection will exist in a cone of rays that deviates from the central chief ray by a fixed angular offset $\Delta$. This ray can be rotated about the chief ray through an angle $\theta.$ Owing to the infinite conjugate ratio of the transmitting and receiving lenses in the given off-axis lens pair and the symmetry of the steering angle of the ray cone in the object and image planes, the relationship between the directional cosines in the object space and the image space for such a ray is (repeated here for clarity)

$$
L \equiv L_0 + N_0\Delta \cos \theta,
$$
\hspace{1cm} (21)$$

$$
L' \equiv L_0 - N_0\Delta \cos \theta,
$$
\hspace{1cm} (22)$$

$$
M \equiv M_0 - N_0\Delta \sin \theta,
$$
\hspace{1cm} (23)$$

$$
M' \equiv M_0 + N_0\Delta \sin \theta,
$$
\hspace{1cm} (24)$$

where $L_0$, $M_0$, and $N_0$ are the direction cosines of the steered chief ray. Substituting expressions (21)–(24) into Eqs. (46) and (47) gives

$$
\varepsilon_x = 8N_0^3\Delta^3 \cos \theta(-3\alpha_{0220}\sin^2 \theta - 3\alpha_{0022}\sin^2 \theta
$$
$$
+ 6\alpha_{1210}\sin^2 \theta - 6\alpha_{1111}\sin^2 \theta + 6\alpha_{0121}\sin^2 \theta
$$
$$
+ 3\alpha_{2020}\cos^2 \theta - 4\alpha_{1030}\cos^2 \theta + 6\alpha_{0400}\cos^2 \theta)
$$
$$
+ N_0\Delta[24M_0^2\sin \theta(\alpha_{0020} + 2\alpha_{0120} - 2\alpha_{1111}
$$
$$
+ \alpha_{0022} - 2\alpha_{1210}) + 48L_0M_0 \sin \theta(\alpha_{0020} - \alpha_{0220})
$$
$$
+ 24L_0^2 \cos \theta(\alpha_{0040} - \alpha_{2020})],
$$
\hspace{1cm} (48)$$

$$
\varepsilon_y = 8N_0^3\Delta^3 \sin \theta(-\alpha_{0400}\sin^2 \theta - 3\alpha_{0220}\sin^2 \theta
$$
$$
+ 4\alpha_{0103}\sin^2 \theta + 3\alpha_{0220}\cos^2 \theta - 6\alpha_{0121}\cos^2 \theta
$$
$$
- 6\alpha_{1210}\cos^2 \theta + 6\alpha_{1111}\cos^2 \theta + 6\alpha_{0220}\cos^2 \theta)
$$
$$
+ N_0\Delta[24M_0^2\sin \theta(\alpha_{0400} - \alpha_{0220})
$$
$$
+ 48L_0M_0 \cos \theta(\alpha_{0022} + \alpha_{0220})
$$
$$
+ 24L_0^2 \sin \theta(\alpha_{0022} + \alpha_{0220} - 2\alpha_{1111} - 2\alpha_{0121}
$$
$$
+ 2\alpha_{1210})].
$$
\hspace{1cm} (49)$$

Equations (48) and (49) represent the third-order ray-intercept error. If the coefficients of the $\Delta$ and $\Delta^3$ terms are of the same sign, then for any given angle ($\theta$) the ray-intercept errors are monotonic functions of $\Delta$ with only one root ($\Delta = 0$). As in the second-order analysis, the minimum rms spot size is obtained by centering the ray cone about the steered ray ($\Delta = 0$). If the coefficients of the $\Delta$ and $\Delta^3$ terms are of opposite sign, then Eqs. (48) and (49) each have two roots: one at $\Delta = 0$ and one with nonzero $\Delta$. Since it can be shown that the magnitude of the slope at the nonzero $\Delta$ root is twice that of the root at $\Delta = 0$, the lowest rms error can again be achieved by centering the cone about the steered chief ray ($L_0$, $M_0$, or $\Delta = 0$).
5. SUMMARY AND CONCLUSION

Through the application of eikonal analysis, it has been shown that if the VCSEL cones are individually deflected to create symmetry about a central aperture, then there is no second- or third-order ray-intercept error for the chief ray and hence no distortion. Furthermore, the residual second- and third-order ray-intercept errors of the remaining rays in the cone is minimized through this beam deflection. So it has been shown that steering the VCSEL ray cone not only cancels distortion in the system but also results in the minimum rms spot size as well. The net result is a hybrid micro/macro approach that achieves high registration accuracy across the multichip smart pixel with the use of only one surface to control distortion despite the wide field angles present in the system. As the required beam-deflection elements are simple gratings or microprisms, the absolute alignment to the VCSEL–detector array is not a critical parameter. Furthermore, since resolution requirements are relaxed by the fact that detectors (~50 μm) are somewhat larger than VCSELS (~10 μm), the design of the macro-optical lenses, responsible now only for minimizing spot size, will be less complex. Canceling distortion in a conventional manner (through the use of additional several surfaces) would be prohibitively costly for such a wide field angle system.

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Corresponding author Marc Christensen’s present address is Department of Electrical Engineering, School of Engineering, Southern Methodist University, P.O. Box 750338, Dallas, Texas 75275-0338. Telephone, 214-768-1407; fax, 214-768-3573; e-mail, mpc@engr.smu.edu.

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