Hierarchical Cellular Network Design with Channel Allocation*

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Abstract

The design of a cellular network is a complex process that encompasses the selection and configuration of cell sites and the supporting network infrastructure. This investigation presents a net revenue maximizing model that can assist network designers in the design and configuration of a cellular system. The integer programming model takes as given a set of candidate cell locations with corresponding costs, the amount of available bandwidth, the maximum demand for service in each geographical area, and the revenue potential in each customer area. Based on these data, the model determines the size and location of cells, and the specific channels to be allocated to each cell. To solve problem instances, a maximal clique cut procedure is developed in order to efficiently generate tight upper bounds. A lower bound is constructed by solving the discrete optimization model with some of the discrete variables fixed. Computational experiments on seventy-two problem instances demonstrate the computational viability of our new procedure.

1 Introduction

Some 20 years after their commercial introduction, cellular mobile communication services are as popular as ever, with demand increasing at an exponential rate and network expansion following suit. Although not comparable to the wire-based communication infrastructure, investments in cellular systems are large and an economical use of resources is a necessity for companies operating in this competitive market.

All wireless systems are constrained in capacity by the available communication bandwidth. As demand for services has expanded in the cellular segment, several innovations have been made in order to increase the utilization of bandwidth. The cellular concept itself is one of the most prominent examples of such innovation. The cellular design increases bandwidth utilization by limiting the reach of a radio tower so that a frequency channel can be re-used by a tower that is located sufficiently far away from other towers utilizing the same channel. Given a service area and potential demand in this service area, an important decision is to determine the size of each cell in the service area.

Another proposed improvement is dynamic channel assignment. In traditional cellular systems designs, the allocation of frequency channels to cells is fixed. That is, once a frequency channel has been allocated

*This research was supported in part by the Office of Naval Research Award Number N00014-96-1-0315.

to a cell, it cannot be used by other cells that may interfere with this. Realizing that most of the time, all channels will not be in use in all cells, schemes have been designed so that channels may be 'borrowed' from other cells, given that the borrowing of the channel will not cause intereference in other cells. While the advantages of channel borrowing (and other dynamic cell assignment schemes) are obvious, they also have some drawbacks. In order to borrow a channel from another cell, the cell that borrows the channel must have sufficient capacity to handle the extra channel. This has implications for tower equipment allocation, mobile telephone switching office capacity, cellular network backbone design, and traffic management.

A third proposed method to improve resource utilization is the concept of *hierarchical (or multi-tier) network design.* In a hierarchical system, various cell sizes may be deployed. In the initial proposal, there were two types of cells; *macrocells* and *microcells.* The microcells are similar to regular cells in traditional designs. Superimposed on this structure are the macrocells, each of which covers the same geographical area as several microcells. There are two principal benefits of a two-tier design. First, if a macrocell covers an area, it is not necessary to cover the same area with a microcell. In rural areas with low overall demand and scattered high demand points, this means that fewer resources need to be spent on providing adequate service. Instead of covering the entire service area with microcells, a macrocell can be used in combination with microcells at high demand points. The cost savings from this design can be substantial compared to a traditional design. Second, if there are mobile phone users that are traveling at high speeds, these users can be serviced by the macrocell and thereby reduce the need for handoff of the calls. When a handoff is avoided, the risk of call dropping due to channel shortage in the handoff service area is eliminated. The two-tier model can be extended into a three-tier model where a smaller cell size *(picocell)* is introduced. Picocells can be used in very high density areas, such as urban commercial centers. Thus, a multi-tiered design combines high frequency reuse in high demand areas with low system investment in low demand areas. In this investigation, a new model is proposed that incorporates multi-tiered design with channel allocation based on demand for service in different geographical locations in the service area.

1.1 Previous Work

Hierarchical cellular networks have been the topic of several studies, with focus in particular on accommodation of users that travel at different velocities. Characteristics of hierarchical cellular networks, such as call blocking and handoff failure probabilities have been investigated extensively by Rappaport and Hu [11, 18] for entirely ground based networks as well as for hybrid networks that combine services from a ground based network with those of satellite based overlay networks. Extensions of this work have been concerned with resource management in hierarchical networks, with an emphasis on channel allocation between network levels (both static and dynamic allocation schemes), cell size determination, and user mobility management [6, 14, 23].

Several hierarchical cellular network design models have been proposed in the literature. Ganz et al. [8] propose a model that minimizes total system deployment cost subject to quality of service constraints for customers assigned to each tier. The quality of service measure used is the probability of call loss. While this formulation takes into account user mobility, it assumes that demand is uniformly distributed across the network service area. Moreover, the formulation is based on the assumption that all demand has to be serviced at some given quality of service level. In a more recent paper, Wu and Lin [24] develop a model that minimizes the cost of system development subject to quality-of-service (QoS) constraints. In this model, the authors use cell radius as an independent variable and find a solution that satisfies demand subject to QoS constraints. In their work, QoS measures include signal-to-noise ratios for each cell location and a requirement that the system meets average demand for service. Neither of the papers consider economic factors other than cost.

Sarnecki et al. [21] compare the characteristics of macrocells and microcells and conclude that the cost per subscriber of using microcells may be as much as 60-70% lower than the cost of using macrocells. However, this cost reduction is due to the use of low-power transmitters in microcells and the figures do not include the possible incremental cost of connecting the cell sites to switching offices. Using an engineering cost approach, Reed [19] estimates the costs of PCS networks and studies the economies of scope in PCS service provision. He finds that there is some minimum bandwidth that allows PCS operators to reach a point where all economies of scope are exhausted and concludes that this finding may have important regulatory implications. Finally, Gavish and Sridhar [9] use a cost and revenue model to find optimal configurations (in terms of cell size and channel allocation) of cellular systems. Their model assumes that demand is uniformly distributed and that each cell in the system will be configured exactly the same. Due to the symmetry arguments made, the channels available to the system can be divided *a priori* in equal numbers between cells without causing interference. The remaining channel allocation problem is to decide how many of the channels available to a cell should actually be used. Thus, interference considerations do not play a role in their model.

The problem considered in this work includes as part the assignment of frequency channels to cells in the network. The frequency assignment problem is closely related to the graph coloring problem and has been subject to intensive study in the literature (see Murphey et al. [17] for a comprehensive survey). The frequency assignment problem has been examined for a number of objective functions, mostly related to minimizing the number of channels used to satisfy demand or to minimizing the total system interference while satisfying demand for communication. In this context, methods for deriving strong lower bounds on the number of frequency channels utilized have been developed (see, e.g., Gamst [7] and Tcha et al. [22]). In common for most of the work on finding feasible solutions to the frequency assignment problem is that the solution methodology relies on specific geometric cell structures [3] or on the use of randomized local search procedures without known deviation from the optimal solution [2, 4, 10, 12, 15, 20], while integer programming based approaches have successfully solved only small problem instances [16].

The model presented in the next section differs from previous research in that it simultaneously incorporates base station location, cell size choice, and channel allocation in a multi-tier cellular network. Both cost and revenues are considiered in the design of the cellular network. The model does not, however, directly address queueing or mobility aspects of cellular network operations. It is assumed that the service demand parameters are adjusted for mobility and queueing effects and, thus, the proposed model is suited for planning rather than operational purposes.

1.2 Contributions

We claim four contributions from the investigation described in this manuscript. First, we present a new optimization model for the hierarchical network design problem. This is the first three-tier model that we have seen and it includes macrocell, microcell and picocell selection. It is a profit maximization model, as opposed to the usual cost minimization strategy, which better reflects the requirements of potential clients. Channels are assigned to cells in blocks, which reduces the problem size and improves computational tractability.

Our basic model is a large mixed-integer linear program whose problem instances are not solvable using standard commercial optimization software. However, there exists an underlying graph (called the interference graph) that can be used to create an almost unlimited number of valid inequalities for any large problem instance. While this class of valid inequalities is well-known for the frequency assignment problem [1], our second contribution is a set of algorithms to successively and selectively generate valid inequalities from the interference graph by repeatedly solving maximal clique problems to produce a problem with a much stronger continuous relaxation. Our third contribution is a computational study that provides convincing evidence that good solutions to realistically sized problems can be obtained with our procedures. The feasible solutions obtained, combined with upper bounds on the optimal solution value generated, provide a solution quality measure in terms of a known maximal deviation from the optimal solution value. Finally, we make our AMPL models and algorithms available on the World Wide Web for downloading free of charge for immediate use by industrial design groups who need solutions to this type of problem. Users only need supply their specific data to have a working design tool. Other research groups also have complete access to our models, algorithms, and test data for independent verification and comparison with their design tools.

2 The Model

In this section, we present a model for the hierarchical cellular network design problem. The model proposed in this investigation is applicable for cell planning for analog cellular systems (in particular, AMPS using FDMA), digital cellular (standard IS-54 using TDMA), and GSM. The minimum service requirements in the model conform to the guidelines for markets in the United States established by the Federal Communications Commission (FCC). In the remainder of this section, we present the integer programming formulation and discuss some properties of the mathematical model.

2.1 Sets Used in the Model

Let L denote the set of candidate locations for tower construction. In each tower location, the system can use one or more equipment types to generate coverage areas corresponding to different cell sizes. The set of equipment types, J, contains the three elements macro, micro and pico. There is a set of subscriber locations, M. This set may or may not be different from the set of candidate tower locations, L. The identification of suitable candidate tower locations is non-trivial and involves field measurements of radio signal propagation characteristics, resolution of right-of-way issues, etc. Our model takes the candidate locations emanating from this process as an input and makes a selection among these for construction. The set $T_m \subset L \times J$ is the tower-cell combinations that can service customers in location $m \in M$. For every $\ell \in L,\ j \in J$, $P_{\ell j} \subset M$ is the set of customer locations that can be serviced by tower-cell combination (ℓ, j). Finally, H is the pairwise set of tower-cell combinations (ℓ, j, ℓ', j') that cause interference with one another if the same frequency channel is assigned to both (ℓ, j) and (ℓ', j').

2.2 Constants Used in the Model

The demand for service in customer area $m \in M$ is denoted by d_m. This value is the number of frequency channels required to service the population in the area at an acceptable service level (call blocking rate). Let r denote the annual revenue (in \$) generated by each frequency channel allocated to a customer area. This number is not the revenue generated by a single customer in one year. Rather, it is the aggregate revenue collected as a result of providing the channel. For example, a customer is not connected 24 hours a day, so the same frequency channel can be used by multiple customers during a 24-hour period. Similarly, since cellular systems in general experience two busy hours in a 24-hour period [13], it is the system capacity during these two busy hours that define the number of subscribers the system can reasonably accommodate and, thus, it determines the revenue potential of a frequency channel. The relationship between demand for service, call blocking probability (one of the measures of quality of service) and system capacity in steady state can be modeled with the Erlang-B formula. Also note that it is difficult to assess the marginal revenue of a frequency channel as a function of the demand area to which it is assigned. Thus, using the same revenue parameter for all demand areas may be the best estimate available in the network planning process. If demand area specific data are available, the proposed model can be modified accordingly. The mandated minimum service requirement, given as a proportion of the number of customers in the market the service provider operates, is denoted by ρ. The cost (amortized annually) of building a tower in location $\ell \in L$ and connecting it to the backbone network is given by the parameter a_ℓ. The annual cost of operating and maintaining equipment of type $j \in J$ in location $\ell \in L$ (including ammortization of the equipment) is represented by $b_{\ell j}$. The annual cost of assigning a frequency channel to tower type $j \in J$ in location $\ell \in L$ is denoted by $c_{\ell j}$. This includes the cost of transmission power, marketing, accounting, customer aquisition and retention, and any other cost that is contingent upon channel allocation to a tower-cell combination. The number of frequency channels available to the service provider is given by N. Finally, we define for each tower-cell combination the parameter $N'_{\ell j} = \min\left(N, \sum_{m \in P_{\ell j}} d_m\right)$, which represents an upper bound on the average number of customers that can be serviced from this combination.

2.3 Decision Variables Used in the Model

The decision variables in this model includes both integer and continuous variables. The decision to build a tower in a candidate location is represented by variable z_ℓ, which is one if a tower is built in location $l \in L$ and zero, otherwise. The binary variable $y_{\ell j}$ represents the decision to install equipment type $j \in J$ in location $l \in L$ and is one if installation takes place and zero, otherwise. The model includes the number ($s_{m\ell j}$) of customers in location $m \in M$ that are serviced by equipment type $j \in J$ in tower location $l \in L$. The number of channels assigned to tower-cell combination (ℓ, j) is given by $tt_{\ell j}$, while $hh_{\ell j}$ ($ll_{\ell j}$) is the highest (lowest) channel number assigned to (ℓ, j). Thus, the model includes a specific, continuous range of channels to each of the tower-cell combinations that are installed. All variables of type tt, hh and ll are continuous.

We introduce five additional types of variables that ensure the proper assignment of frequency channels. The midpoint channel number assigned to tower-cell combination (ℓ, j) is given by $mm_{\ell j}$, and $dp_{\ell j \ell' j'}$ ($dm_{\ell j \ell' j'}$) is the positive (negative) distance between the midpoint channel number of tower-cell combinations (ℓ, j) and (ℓ', j').

An indicator variable which is one if the midpoint channel number of tower-cell combination (ℓ, j) is greater than that of (ℓ', j') is given by $bb_{\ell j \ell' j'}$. The integrality of the bb variables together with the objective function ensure that tt, hh and ll are all integer in the solution. Finally, q_m is an indicator variable that is one if customer location m can be serviced by at least one tower-cell combination and zero, otherwise. The variable q_m does not appear in the objective function and its value is closely tied to the channel assignment variable $tt_{\ell j}$. In other words, the service requirement represented by the constraints involving q_m will have an impact on the allocation of channels and, thus, indirectly on the infrastructure investment decisions.

2.4 Mixed-Integer Linear Program

The objective of the model is to maximize the total annual revenue generated by the cellular network less the cost of building, maintaining and operating it. Mathematically we have

$$\text{maximize} \quad \underbrace{\sum_{m \in M} \sum_{(\ell,j) \in T_m} r s_{m\ell j}}_{\text{revenue}} \; - \; \underbrace{\sum_{\ell \in L} a_\ell z_\ell}_{\text{tower cost}} \; - \; \underbrace{\sum_{\ell \in L} \sum_{j \in J} b_{\ell j} y_{\ell j}}_{\text{cell cost}} \; - \; \underbrace{\sum_{\ell \in L} \sum_{j \in J} c_{\ell j} tt_{\ell j}}_{\text{channel cost}} . \tag{1}$$

There are 20 sets of constraints that define the model. The first three sets ensure that customers can be serviced only if there are towers and cell types that cover the demand area and there are frequency channels available at these tower-cell combinations. Please note that multiple types of equipment (cell sizes) can be used at any one tower that is constructed. This permits a designer to user the same tower to provide many channels in a micro- or picocell without causing widespread interference in neighboring candidate tower-cell locations, while at the same time allocating a few channels to a macrocell to reach a few periferal customers or to provide continuous coverage between cells without making an investment in an additional tower location. Also, it is possible to service customers in one demand area from several towers, provided that they do not use the same frequency channels.

$$\sum_{m \in P_{\ell j}} s_{m\ell j} \leq N'_{\ell j} y_{\ell j} \qquad \forall \ell \in L, j \in J \tag{2}$$

$$\sum_{j \in J} \sum_{m \in P_{\ell j}} s_{m\ell j} \leq N'_{\ell 1} z_{\ell} \qquad \forall \ell \in L \tag{3}$$

$$\sum_{m \in P_{\ell j}} s_{m\ell j} = tt_{\ell j} \qquad \forall \ell \in L, j \in J \tag{4}$$

The next set of constraints ensures that one cannot serve more customers in a location than there is demand for service (although it is possible to choose to serve only a portion of the customers in a demand area or none at all).

$$\sum_{(\ell,j) \in T_m} s_{m\ell j} \leq d_m \qquad \forall m \in M \tag{5}$$

The minimum service requirements are handled by three types of constraints. The first type states that customers cannot be serviced in location m if there are no frequency channels assigned to the tower-cell combinations that can reach demand area m. Second, if there is at least one channel that can reach area m, then customers in this location can be serviced. Finally, a constraint ensures that service is available in demand areas that have at least a proportion ρ of all customers in the operator's total service area. Note, however, that there does not have to be a sufficient assignment of channels to service all of the customers that can be reached by the cellular network. While these minimum service requirements may appear strange, they are reasonable in the sense that a service provider must provide a sufficient infrastructure to reach a certain proportion of the population in the market. When the investment in the infrastructure has been made, it is reasonable to assume that the service provider will offer at least some service in the covered demand areas.

$$q_m \leq \sum_{(\ell,j) \in T_m} tt_{\ell j} \qquad \forall m \in M \tag{6}$$

$$N q_m \geq \sum_{(\ell,j) \in T_m} tt_{\ell j} \qquad \forall m \in M \tag{7}$$

$$\sum_{m \in M} d_m q_m \geq \rho \sum_{m \in M} d_m \tag{8}$$

The next six constraint types assign frequency channels to tower-cell combinations. The first set defines the number of channels assigned to each (ℓ, j), while the second set calculates the midpoint of the assigned channel range. The third set defines the absolute value of the distance (in channels) between two pairs of channel midpoints. This distance can be either positive or negative. The fourth set of constraints forces the distance between a pair of channel assignments to be sufficiently far apart as not to overlap if the corresponding tower-cell type pairs are within interference distance of one another. Finally, the last two sets force either the positive or the negative distance between two pairs of channel assignments to be zero, thus serving as an absolute value operator. The value of the variable $bb_{\ell j \ell' j'}$ indicates whether the assigned channel range of tower-cell (ℓ, j) is to the left or to the right of the assigned channel range of tower-cell (ℓ', j').

$$tt_{\ell j} = hh_{\ell j} - ll_{\ell j} + 1 \qquad \forall \ell \in L, j \in J \tag{9}$$

$$2mm_{\ell j} = hh_{\ell j} + ll_{\ell j} \qquad \forall \ell \in L, j \in J \tag{10}$$

$$mm_{\ell j} - mm_{\ell' j'} = dp_{\ell j \ell' j'} - dm_{\ell j \ell' j'} \qquad \forall (\ell, j, \ell', j') \in H \tag{11}$$

$$2\left(dp_{\ell j \ell' j'} + dm_{\ell j \ell' j'}\right) \geq tt_{\ell j} + tt_{\ell' j'} \qquad \forall (\ell, j, \ell', j') \in H \tag{12}$$

$$dp_{\ell j \ell' j'} \leq N bb_{\ell j \ell' j'} \qquad \forall (\ell, j, \ell', j') \in H \tag{13}$$

$$dm_{\ell j \ell' j'} \leq N\left(1 - bb_{\ell j \ell' j'}\right) \qquad \forall (\ell, j, \ell', j') \in H \tag{14}$$

The last seven sets of constraints provide the domains for the variables. Note that if $ll_{\ell j} = h_{\ell j} + 1$, then no channels are assigned to tower-cell (ℓ, j). Thus, the range of variable $ll_{\ell j}$ which indicates the lowest channel number in the range assigned to tower-cell (ℓ, j) is one greater than that of variable $hh_{\ell j}$ which represents the highest channel number in the range, so that no channels can be assigned to tower-cell (ℓ, j).

$$1 \leq ll_{\ell j} \leq N + 1 \qquad \forall \ell \in L, j \in J \tag{15}$$

$$0 \leq hh_{\ell j}, mm_{\ell j}, tt_{\ell j} \leq N \qquad \forall \ell \in L, j \in J \tag{16}$$

$$0 \leq dm_{\ell j \ell' j'}, dp_{\ell j \ell' j'} \leq N \qquad \forall (\ell, j, \ell', j') \in H \tag{17}$$

$$0 \leq s_{m \ell j} \leq N \qquad \forall m \in M, \ell \in L, j \in J \tag{18}$$

$$bb_{\ell j \ell' j'} \in \{0, 1\} \qquad \forall (\ell, j, \ell', j') \in H \tag{19}$$

$$y_{\ell j} \in \{0, 1\} \qquad \forall \ell \in L, j \in J \tag{20}$$

$$z_\ell \in \{0, 1\} \qquad \forall \ell \in L \tag{21}$$

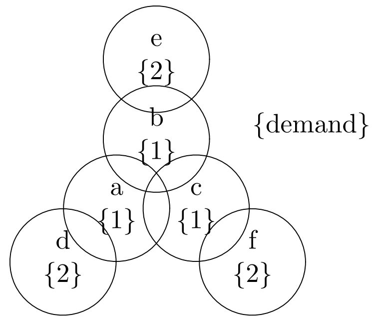
The model defined above is similar, but not equivalent to models that allow the independent assignment of single frequency channels to tower-cell combinations. Let I be the set of available frequency channels. Traditional frequency assignment models use a variable for each frequency assignment, $x_{\ell j i}$, which is one if frequency $i \in I$ is assigned to tower-cell (ℓ, j) and zero, otherwise. Let

$$tt_{\ell j} = \sum_{i \in I} x_{\ell j i} \qquad \forall \ell \in L, j \in J.$$

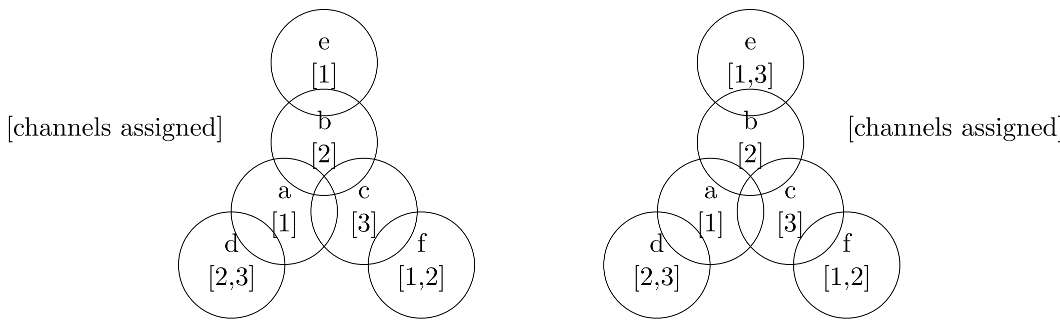
Constraints (9)–(14) in our model are replaced with

$$x_{\ell j i} + x_{\ell' j' i} \leq 1 \qquad \forall i \in I, (\ell, j, \ell', j') \in H.$$

Each of the $|I| = N$ channels generates a maximal independent set problem over the interference graph H.



a. Example with overlapping areas.



b. Maximum subscribers serviced using blocks of channels.

c. Maximum subscribers serviced without the block restriction.

Figure 1: Illustration where block assignment yields fewer subscribers.

Our model is more compact than the traditional models in the sense that it requires substantially fewer decision variables than models that assign individual channels to tower-cell combinations rather than an interval of frequency channels. From a computational point of view, the number of frequency assignment constraints in our model is $4|H|+2|L||J|$ while the traditional model formulation results in $N|H|$ constraints. Since the number of channels is 500 or more in this type of communication system, the difference in model size becomes subtantial. However, while a feasible solution to our formulation is feasible also for models using individual channel assignment, the converse is not true. Specifically, consider the example illustrated in Figure 1. Areas a, b and c have one possible subscriber while areas d, e and f each have two. The maximum number of subscribers that can be serviced using three channels and block assignment is eight, as illustrated in Figure 1b. By permitting channels 1 and 3 (non-consecutive channel assignment) to be assigned to area e, one additional subscriber can be accommodated, as shown in Figure 1c.

Theorem 1 *The hierarchical cellular network design with channel allocation problem is NP-complete.*

Proof *Consider the problem formulation given by (1)–(18). Restrict the problem by letting* $N = 1$, $\rho = 0$, $d_m = 1\ \forall m \in M$, $a_\ell = 0\ \forall \ell \in L$, $b_{\ell j} = 0\ \forall \ell \in L, j \in J$, $r - c_{\ell j} = 1\ \forall \ell \in L, j \in J$, $J = \{1\}$ *and* $P_{\ell j} = \ell\ \forall \ell \in L, j \in J$. *This eliminates index* j *and variables* q, y *and* z, *and all associated constraints. A pair of towers that interfere cannot be assigned an overlapping set of channels. Equation (4) now states that* $s_\ell = tt_\ell$ *where* $s_\ell \in \{0,1\}$. *Since only one channel is available, constraints (9)–(19) reduce to* $tt_\ell + tt_{\ell'} \leq 1$, $\forall (\ell, \ell') \in H$. *The restricted problem can now be written as* $\max \big\{ \sum_{\ell \in L} tt_\ell :\ tt_\ell + tt_{\ell'} \leq 1,\ \forall (\ell,\ell') \in H;\ \ tt_\ell \in \{0,1\},\ \forall \ell \in L \big\}$. *This is the maximal indpendent set problem, which is known to be NP-complete.* ■

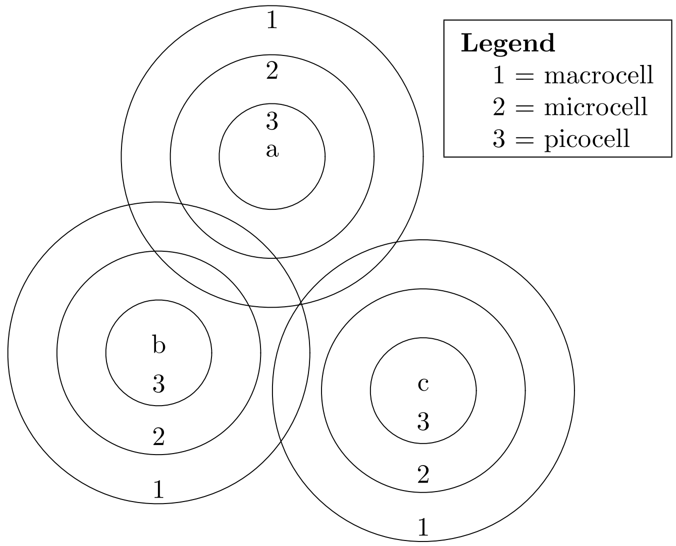


Figure 2: A three tower example.

3 Solution Procedure

The linear-programming (LP) relaxation of the ILP model presented in Subection 2.4 is weak; i.e., the gap between the LP upper bound and the ILP optimum can be relatively large. One difficulty is that the LP relaxation allows for channels to be shared among tower-cell combinations that are close enough to interfere with each other. Our experience with this model indicates that for even a toy problem, an ILP solver may have to branch an enormous number of times in order to resolve these conflicts. We developed a procedure to find an upper bound for the ILP by relaxing the integrality constraints on the bb variables and, then, successively adding valid inequalities to reduce the upper bound. Adding these valid inequalities to the model itself is not feasible since the number of these constraints increases exponentially with problem size, thus making the problem defined in this fashion impossible to solve with a commercial ILP solver. Instead, these valid inequalities are added iteratively as the procedure discovers that they are violated. A lower bound procedure was developed that makes a valid channel assignment to the best tower-cell combinations produced by the upper bound procedure. These two procedures produce a feasible solution along with a guaranteed deviation from optimality.

3.1 Upper Bound Procedure

We relax the integrality condition on variables $bb_{\ell j \ell' j'}$, $\forall(\ell, j, \ell', j') \in H$. That is, (19) is replaced with

$$0 \leq bb_{\ell j \ell' j'} \leq 1, \qquad \forall(\ell, j, \ell', j') \in H. \tag{22}$$

In turn, this implies that the same channel may be assigned to two interfering towers. In this subsection, we describe procedures for generating classes of valid inequalities which we add to the formulation to counter

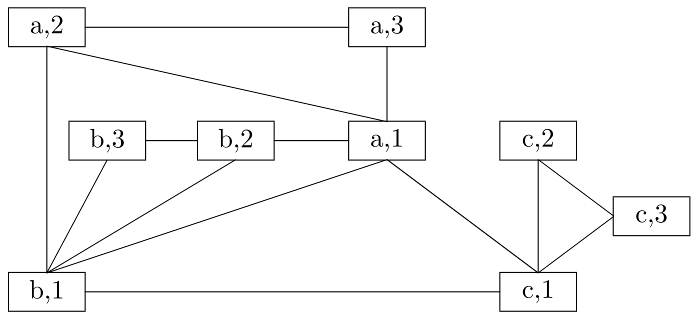


Figure 3: Interference graph fore the three tower example of Figure 2.

Table I: Cliques and valid inequalities for the three-tower problem illustrated in Figures 2 and 3.

Clique Number	**Nodes in Clique**	**Valid Inequality**
1	(a,1), (b,1), (c,1)	$tt_{a1} + tt_{b1} + tt_{c1} \leq N$
2	(a,1), (a,2), (a,3)	$tt_{a1} + tt_{a2} + tt_{a3} \leq N$
3	(a,1), (a,2), (b,1)	$tt_{a1} + tt_{a2} + tt_{b1} \leq N$
4	(a,1), (b,1), (b,2)	$tt_{a1} + tt_{b1} + tt_{b2} \leq N$
5	(b,1), (b,2), (b,3)	$tt_{b1} + tt_{b2} + tt_{b3} \leq N$
6	(c,1), (c,2), (c,3)	$tt_{c1} + tt_{c2} + tt_{c3} \leq N$

this effect.

For a given problem instance, we construct an *interference graph* which has a vertex $v_{\ell j}$ corresponding to the tower/equipment combination (ℓ, j) and an edge $(v_{\ell j}, v_{\ell' j'})$ if and only if tower/equipment combination (ℓ, j) interferes with tower/equipment combination (ℓ', j'). Our valid inequalities are derived by finding cliques in the interference graph and limiting the total number of channels assigned to each clique to N.

The interference graph for the three-tower problem illustrated in Figure 2 is given in Figure 3. Each clique in this graph can be used to produce a valid inequality for (1)–(18). For this example, the six largest cliques and corresponding inequalities are given in Table I. Finding all cliques of maximal size for a problem with even a modest number of towers, say 20, is not computationally feasible. Our strategy is to create a subset of these cliques and valid inequalities that appear to be most beneficial.

3.1.1 Preprocessing Algorithms for Clique Generation

Let $IG = [E, V]$ denote the interference graph with edge set E and vertex set V. A clique is a subgraph of IG, say $[\overline{E}, \overline{V}]$, where $\overline{E} = \overline{V} \times \overline{V}$. A recursive algorithm can be applied to create a clique. Given a link $(n_1, n_2) \in E$, $\left\{\overline{E} = \{(n_1, n_2)\}, \overline{V} = \{n_1, n_2\}\right\}$ is a clique. If there exists another node $n_3 \in V$ such that $(n_1, n_3) \in E$ and $(n_2, n_3) \in E$, then $\left\{\overline{E} = \{(n_1, n_2), (n_1, n_3), (n_2, n_3)\}, \overline{V} = \{n_1, n_2, n_3\}\right\}$ is also a clique. Hence, a clique with $|\overline{V}| = m$ can be constructed from a clique with $|\overline{V}| = m - 1$ by appending one new vertex and $m - 1$ new edges. By varying the rules for selecting new vertices, a large set of cliques can be generated.

From the interference graph, we see that edges can be classified into the following six groups: macro-macro, macro-micro, macro-pico, micro-micro, micro-pico, and pico-pico. We developed six algorithms based on the above classifications by varying the rules for selecting the first link and additional vertices to be appended to a current clique. For example, macro-macro, micro-micro, and pico-pico cliques always begin with a link connecting like cell types. These six procedures create a set of valid inequalities that are appended to (1)–(18) prior to obtaining the first upper bound. These additional constraints are global and are retained for all upper and lower bound calculations. These constraints take the form

$$\sum_{(\ell,j)\in E_c} tt_{\ell j} \leq N, \qquad \forall c \in C. \tag{23}$$

The set $C = \{1, ..., \overline{c}\}$ denotes the set of cliques that have been appended to the problem, and is data dependent.

3.1.2 Online Algorithms for Clique Generation

Let $\hat{tt}_{\ell j}$ denote an optimal solution to (1)–(17), (20)–(23). Since the bb variables are continuous, there may exist cliques in the interference graph for which more than N channels are assigned. Such cliques can be found by solving a simple binary linear program. Let $\gamma_{\ell j}$ be one if vertex (ℓ, j) is in the clique and zero, otherwise. Then we seek a set of $\gamma_{\ell j}$ such that

$$\gamma_{\ell j} + \gamma_{\ell' j'} \leq 1 \qquad \forall (\ell, j, \ell', j') \notin H \tag{24}$$

$$cw = \sum_{\ell\in L}\sum_{j\in J} \hat{tt}_{\ell j}\gamma_{\ell j} \geq N + 1 \tag{25}$$

$$\gamma_{\ell j} \in \{0, 1\} \qquad \forall \ell \in L, j \in J \tag{26}$$

Possible objective functions include maximizing the weight of a clique

$$\text{maximize} \quad cw \tag{27}$$

and maximizing the size of a clique

$$\text{maximize} \quad \sum_{\ell\in L}\sum_{j\in J} \gamma_{\ell j} \tag{28}$$

We actually solve both problems and append the corresponding cuts to the valid inequality constraints (23). These cuts will force some modification to the channel allocation and this process is repeated. Each solution to (1)–(17), (20)–(23) is a valid upper bound and this loop continues until (24)–(27) has no feasible solution (i.e., (23) is satisfied for all cliques in the interference graph).

3.2 Lower Bound Procedure

Let $\hat{z}_\ell$, $\hat{y}_{\ell j}$, and $\hat{tt}_{\ell j}$ denote the optimal solution obtained from the upper bound procedure. Let $H_0 = \{(\ell, j, \ell', j') \in H : \hat{tt}_{\ell j} = 0\}$ and $H_1 = \{(\ell, j, \ell', j') \in H : \hat{tt}_{\ell j} > 0 \wedge \hat{tt}_{\ell' j'} = 0\}$. Then the following constraints fix bb variables that were not active in the upper bound problem:

$$bb_{\ell j \ell' j'} = 0 \qquad \forall (\ell, j, \ell', j') \in H_0 \tag{29}$$

$$bb_{\ell j \ell' j'} = 1 \qquad \forall (\ell, j, \ell', j') \in H_1 \tag{30}$$

The constraints below fix the tower selection and cell selection variables to their current state:

$$z_\ell = \hat{z}_\ell \qquad \forall \ell \in L \tag{31}$$

$$y_{\ell j} = \hat{y}_{\ell j} \qquad \forall \ell \in L, j \in J \tag{32}$$

The lower bound is simply (1)–(18), (23), (29)–(32). Since all variables must be integer, a solution is feasible for the hierarchical cellular network design problem and, hence, is a valid lower bound. Also note that there is always a feasible solution to the lower bound procedure since no customers are required to be served.

4 Empirical Analysis

The upper and lower bound algorithms have been implemented using A Mathematical Programming Language (AMPL) and CPLEX (http://www.ampl.com and http://www.ilog.com/products.cplex). Each iteration of the upper bound procedure requires the solution of three problems, UB, C1, and C2. Problem UB is (1)–(17), (20)–(23), problem C1 is (24)–(27), and C2 is (24)–(26), (28). Exact solutions are obtained for C1 and C2 and an optimality gap of 1% is used for UB. That is, CPLEX terminates as soon as a solution is found that is guaranteed to be within 1% of an optimum. Optimality gaps greater than zero are used to speed convergence. In each iteration, C1 and C2 each generate a new valid inequality for UB that cuts off the current solution. This process continues until C1 has no feasible solution. The solution available at this time is taken as the best upper bound.

The lower bound procedure involves solving two problems LB1 and LB2. The problem LB1 is (1)–(18), (23), and (29)–(32). The problem LB2 is obtained from the solution to LB1 by fixing all bb variables and dropping constraints (31) and (32). Each problem used an optimality gap of 10% and a time limit of 16 hours.

All test runs were made on a Compaq Alpha Server DS20E with dual EV6.7 (21264A) 667 MHz processors and 4,096 MB of RAM. The algorithm in AMPL format and selected test cases may be found at http://www.engr.smu.edu/~jlk/publications/01-EMIS-08/.

Previous work on cellular system design has been based on certain assumptions about system cost and demand structure. Consistent with the studies by Gavish and Sridhar [9] and Wu and Lin [24], we assume that mobility of users can be ignored. This is a reasonable assumption in a system planning model. Instead

Table II: Constants for test problems.

Constant	Value or Range	Description
d_m	$U[50, 250]$	Demand in each service area is different, but is drawn from the same uniform distribution with an expected offered load of 150 customers per service area.
ρ_m	0.45	This is under the assumption that there are two service providers in the licensed area and that they have to cover areas with at least 90% of all demand combined.
r	\$64,800	Each channel is used 518,400 minutes per year while each subscriber uses 12,000 minutes per year at a charge of \$1,500 per year, resulting in 43.2 customers per channel paying \$64,800 per channel per year.
N	500	In GSM systems, two service providers typically share 1,000 channels in each direction.
a_ℓ	\$144,500	This parameter includes the annualized costs of land acquisition, tower construction and connection of the tower to an MTSO.
$b_{\ell j}$	\$1,734, \$1,445, \$1,156	For macrocells, microcells and picocells, respectively; this parameter includes the cost of installation and maintenance of tower transmission equipment, which will vary depending on the desired cell size.
$c_{\ell j}$	\$21,980	This annualized cost includes per-channel equipment cost, power, licensing, administration, accounting, marketing and other costs associated with the per subscriber operation of the system.

Table III: Cell range and interference data.

Cell Type	**Range** (mi.)	**Interference Distance to Cell Type** (mi.)		
		macro	micro	pico
macro	1.00	3.000	2.625	2.250
micro	0.75	2.625	2.250	1.875
pico	0.50	2.250	1.875	1.500

of formally modeling the call arrival and duration processes (which can be translated with the Erlang B formula into an equivalent channel requirement for a given call rejection rate), we use the equivalent channel requirement to represent demand for service. The revenue per minute per channel is stated in channel requirement equivalents. Our test problems are based on the data in Tables II and III.

The candidate tower locations and demand area locations are assumed to be the same. We use the four sets of candidate tower locations first introduced by Farmehr [5]. These are labeled the Southwest, Southeast, Northwest and Northeast market clusters, respectively. For each market cluster, we generated three random sets of demand for service, resulting in twelve market cluster/demand combinations. From each such combination, we created smaller subsets of 15, 20, 25, 30, 35 and 40 candidate tower locations, for a total of 72 test problems. The computational results are summarized in Table IV. The solution for the 20-tower Northwest Market Cluster A problem is illustrated in Figures 4 and 5. The picocells are represented

Table IV: Empirical Results

Test Case	Number of Towers 15	20	25	30	35	40
Southwest Cluster – A						
Gap (%)	1.4	8.0	7.0	8.8	9.1	9.6
CPU Time (hh:mm:ss)	00:00:01	00:00:08	00:02:06	00:04:06	05:14:14	13:23:22
Southwest Cluster – B						
Gap (%)	1.6	2.7	6.7	6.0	8.7	7.7
CPU Time (hh:mm:ss)	00:00:01	00:00:26	00:01:13	00:07:06	00:52:03	06:09:49
Southwest Cluster – C						
Gap (%)	0.0	0.0	8.4	5.6	7.2	6.5
CPU Time (hh:mm:ss)	00:00:01	00:00:07	00:01:56	00:05:05	00:32:43	02:57:40
Southeast Cluster – A						
Gap (%)	6.5	2.9	8.2	9.7	7.8	9.7
CPU Time (hh:mm:ss)	00:00:04	00:00:12	00:12:31	04:18:53	00:31:46	02:32:40
Southeast Cluster – B						
Gap (%)	8.2	8.2	8.0	6.5	8.0	5.5
CPU Time (hh:mm:ss)	00:00:03	00:00:18	00:02:14	00:35:29	00:19:09	02:06:46
Southeast Cluster – C						
Gap (%)	0.7	5.6	7.3	10.0	5.7	6.9
CPU Time (hh:mm:ss)	00:00:02	00:00:07	00:01:04	00:12:10	00:18:54	01:27:22
Northwest Cluster – A						
Gap (%)	6.1	11.5	8.8	8.4	9.5	1.7
CPU Time (hh:mm:ss)	00:01:13	00:06:24	01:39:46	01:10:50	05:30:25	06:09:34
Northwest Cluster – B						
Gap (%)	5.9	5.9	9.6	7.3	9.6	13.1
CPU Time (hh:mm:ss)	00:01:29	00:04:51	16:40:30	05:55:21	21:10:56	38:02:24
Northwest Cluster – C						
Gap (%)	2.9	9.0	8.3	2.5	8.7	8.3
CPU Time (hh:mm:ss)	00:00:20	00:07:32	00:32:50	00:39:14	01:26:49	09:07:46
Northeast Cluster – A						
Gap (%)	6.0	8.0	7.8	12.4	19.4	9.3
CPU Time (hh:mm:ss)	00:00:37	00:04:43	02:53:13	18:34:48	19:30:46	06:51:57
Northeast Cluster – B						
Gap (%)	6.7	9.1	6.1	9.4	11.6	18.0
CPU Time (hh:mm:ss)	00:04:31	00:25:51	01:00:07	15:58:57	34:24:19	40:04:45
Northeast Cluster – C						
Gap (%)	4.5	8.4	8.5	5.7	8.8	7.9
CPU Time (hh:mm:ss)	00:00:19	00:02:10	02:07:36	01:15:46	03:57:11	12:48:56
Average						
Gap (%)	4.3	6.4	7.9	8.2	9.4	7.2
CPU Time (hh:mm:ss)	00:00:43	00:04:19	02:06:16	04:04:49	07:49:06	11:48:35

by the smallest circles, microcells by the intermediate-sized circles, and macrocells by the largest circles. Note that the circles in Figure 4 are smaller than those in Figure 5, reflecting the fact that the service area is smaller than the interference area. The numbers in Figure 4 give the total subscribers serviced by a tower and the numbers in Figure 5 give the channels assigned. The 374 subscribers assigned to the tower in the northeast corner use channels 127–344 on a microcell and channels 345–500 on a macrocell.

Inspection of Figure 4 indicates that there is not coverage overlap between all selected cells. Instead, the 'coverage' graph has five components. A customer in the northeast corner will have his call dropped when

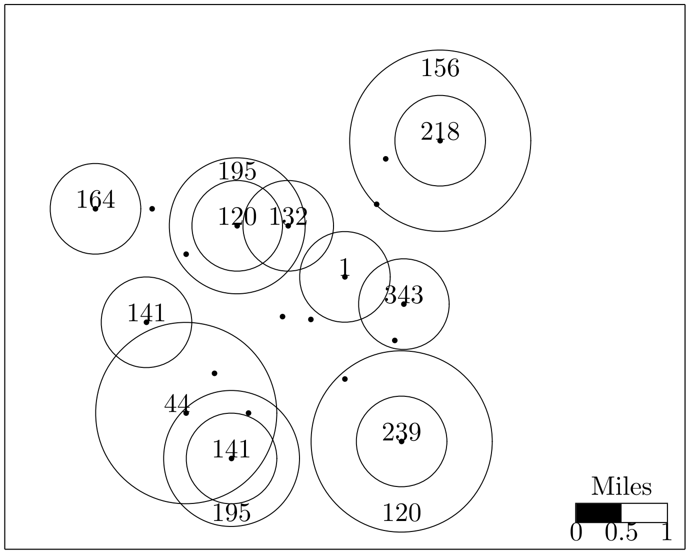


Figure 4: Subscribers serviced by the towers selected for the 20-node Northwest Market Cluster – A.

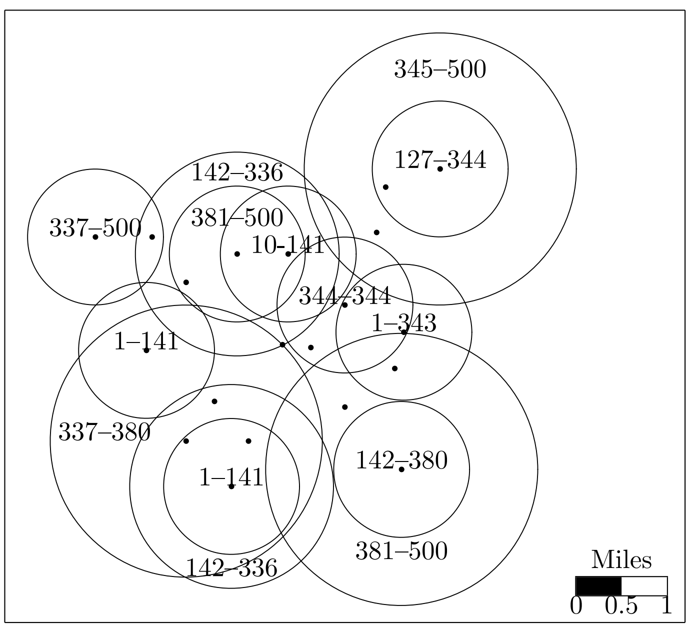


Figure 5: Channel assignment for the 20-tower Northwest Market Cluster – A.

traveling toward the southwest in the service area. From the figure, it is obvious that this problem can be resolved by adding a macrocell in the three locations with 164, 336 and 343 channels assigned, respectively. Thus, when the computational procedure terminates, these operational constraints can be added interactively by the network planner and the model can be solved again. In this case, we added the constraints

$$y_{1,1} = 1, \qquad tt_{1,1} \geq 1,$$

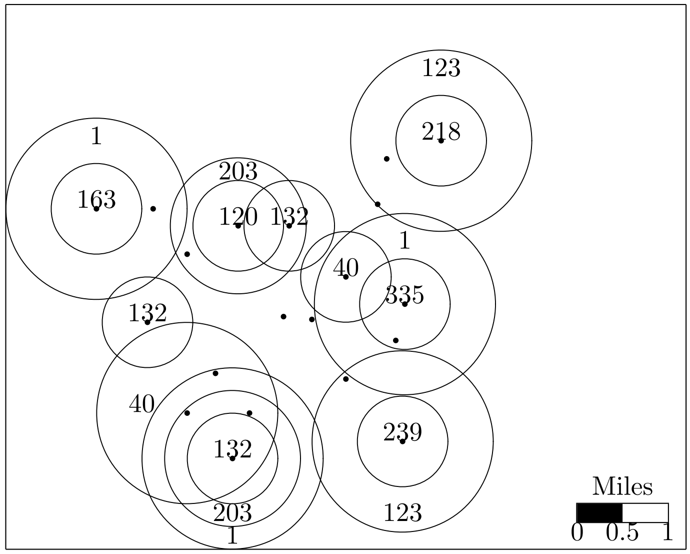


Figure 6: Revised coverage and service plan for the 20-node Northwest Market Cluster – A.

$$y_{7,1} = 1, \qquad tt_{7,1} \geq 1,$$
$$y_{17,1} = 1, \qquad tt_{17,1} \geq 1.$$

The additional computational time was 71 CPU seconds and the objective function value was reduced by 0.14% (or $131,928). The new solution is displayed in Figure 6 and we observe that coverage is contiguous. At the same time, the total number of channels assigned to towers (including frequency reuse) was reduced from 2,209 to 2,206, reflecting the additional interference introduced when assigning channels to the three new macrocells.

The decision where to provide additional coverage in spite of low customer demand for service is beyond the model proposed in this paper and can be done interactively in an ad-hoc fashion by the network planner. Considerations such as coverage on roads connecting heavy demand areas serviced by the network to provide consistent service to mobile customers may be added to the model in terms of hard constraints, similar to the ones added above, on cell sites constructed and channels provided to a specific low volume demand point. Note, however, that coverage in our solutions is *not* complete, but rather only in those areas of the market where demand for service makes it profitable. This is consistent with industry practice as evidenced by the actual coverage map (Figure 7) for the market we have obtained tower data for.

Our goal was to obtain solutions guaranteed to be within 10% of an optimum. That is, our lower bound problem LB1 terminated whenever such a feasible solution was found. We successfully found such a solution in 66 of the 72 problems attempted (or in 92% of the cases). Six of the 72 problems were terminated due to the 16-hour time limit without finding the guaranteed 10% solution. However, all were within a 20% guarantee and four provide guarantees no larger than 13.1% . The two most difficult problems were from the Northwest Cluster Groups A and B. The most difficult part of the hierarchical cellular network design problem is the allocation of channels. Even if the tower-cell combinations are fixed, an optimal channel

Figure 7: Actual coverage in the market from which the empirical study draws its candiate tower locations.

allocation to these cells is a challenging problem.

The results show that it is possible to find good feasible solutions as the problem size (in the number of candidate tower locations) increases. However, the computational time increases substantially with problem size. This is consistent with expectations since the lower bound procedure solves reduced-size versions of the original problem. As problem size increases, so does the reduced-size version of it and the associated computational time. Although the average computational times are relatively long, the solution procedure solves a planning problem rather than an operational problem and, consequently, computational times that support real-time decision-making are not a consideration. The cost of the computing resources required to implement the solution procedure are minor compared to the potential cost savings provided by the model's use.

Overall, the proposed solution procedure meets the stated goals including a) implementation based on commercially available linear integer programming software, b) reasonable resource commitments, c) ability to solve realistic problem instances, and d) satisfactory quality of obtained solutions.

5 Conclusion

This manuscript presents a formulation and solution procedure for the hierarchical network design problem with channel allocation. The proposed model includes the cost of system investments and operation, as well as revenues generated by the system based on customer demand for service. In contrast to previous work in this area, the model specifically includes heterogeneous demand for service. Simultaneously, the model also solves the channel allocation problem taking into consideration interference constraints. In a hierarchical design, which includes macrocells, microcells and picocells, the interference matrix is very dense. This results in a channel allocation problem that is very difficult to solve. The proposed model differs from previous work also in that a consecutive range of channels is allocated to each tower. This feature makes the solution more easily implemented in the field. We demonstrated that consecutive channel solution may differ in objective function value from models in which this requirement is not imposed. Finally, we show that the hierarchical cellular network design model with channel allocation is an NP-complete problem.

The solution procedure, which was implemented and tested on a Compaq Alpha Server, uses as a base the commercially available integer linear programming software package CPLEX. While CPLEX cannot solve even small instances of the problem, our solution procedure makes use of the problem structure to obtain a near-optimal solution. In particular, our model relaxes the integrality constraint of some of the model variables. The relaxed problem is solved using CPLEX. For each solution to the relaxed problem, we identify valid global cuts by solving two maximal clique problems. Based on the solutions from the maximal clique problems, we add a pair of cuts and resolve. This procedure is repeated until no further constraint violations can be identified with this procedure. The final solution is a tight upper bound. The feasible solution procedure creates a smaller-size version of the original problem based on the upper bound solution. This smaller-size problem is also solved by CPLEX until either a satisfactory solution is found (defined as being within 10% of the upper bound) or a 16-hour time limit is reached.

The computational experience with the proposed solution procedure is very encouraging. The procedure finds a solution within the specified tolerance in over 90% of all test problems. The computational times may be large, but are certainly acceptable given that the model is for planning purposes rather than operational use.

References

[1] K. I. Aardal, A. Hipolito, C. P. M. van Hoesel, and B. Jansen. A branch-and-cut algorithm for the frequency assignment problem. EUCLID CALMA Project, Delft and Eindhoven Universities of Technology, The Netherlands, 1995.

[2] J. Abril, F. Comellas, A. Cortés, J. Ozón, and M. Vaquer. A multiagent system for frequency assignment in cellular radio networks. *IEEE Transactions on Vehicular Technology*, 49(5):1558–1565, 2000.

[3] A. A. Bertossi, C. M. Pinotti, and R. B. Tan. Efficient use of radio spectrum in wireless networks with channel separation between close stations. In *Proceedings of the 4th International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications*, pages 18–27, Boston, August 2000. ACM.

[4] A. Capone and M. Trubian. Channel assignment problem in cellular systems: a new model and a tabu search algorithm. *IEEE Transactions on Vehicular Technology*, 48(4):1252–1260, 1999.

[5] A. Farmehr. Design strategies for both survivable and non-survivable personal communication networks. A Praxis Report, School of Engineering and Applied Science, Southern Methodist University, Dallas, TX, 2000.

[6] G. Frullone, P. Grazioso, and G. Falciasecca. Advanced planning criteria for cellular systems. *IEEE Personal Communications*, 3(6):10–15, 1996.

[7] A. Gamst. Some lower bounds for a class of frequency assignment problems. *IEEE Transactions on Vehicular Technology*, 35(1):8–16, 1986.

[8] A. Ganz, C. M. Krishna, D. Tang, and Z. J. Haas. On optimal design of multitier wireless cellular systems. *IEEE Communications Magazine*, 35(2):88–93, 1997.

[9] B. Gavish and S. Sridhar. Economic aspects of configuring cellular networks. *Wireless Networks*, 1(1):115–128, 1995.

[10] F. C. Gomes, P. Pardalos, C. S. Oliveira, and M. G. C. Resende. Reactive GRASP with path relinking for channel assignment in mobile phone networks. In *Proceedings of the 5th International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications*, pages 60–67, Rome, July 2001. ACM.

[11] L.-R. Hu and S. S. Rappaport. Personal communication systems using multiple hierarchical cellular overlays. *IEEE Journal on Selected Areas in Communications*, 13(2):406–415, 1995.

[12] S. Hurley, S. U. Thiel, and D. H. Smith. A comparison of local search algorithms for radio link frequency assignment problems. In *Proceedings of the 1996 ACM Symposium on Applied Computing*, pages 251–257. ACM, February 1996.

[13] N. Keon and G. Anandalingam. A new pricing model for competitive telecommunication services using congestion discounts. Working paper, Edwin L. Cox School of Business, Southern Methodist University, 2002.

[14] X. Lagrange. Multitier cell design. *IEEE Communications Magazine*, 35(8):60–64, 1997.

[15] R. Mathar and T. Niessen. Optimum positioning of base stations for cellular radio networks. *Wireless Networks*, 6:421–428, 2000.

[16] F. F. Mazzini and G. R. Mateus. A mixed-integer programming model for the cellular telecommunication network design. In *Proceedings of the 5th International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications*, pages 68–76, Rome, July 2001. ACM.

[17] R. A. Murphey, P. M. Pardalos, and G. C. Resende. Frequency assignment problems. In D.-Z. Zu and P. M. Pardalos, editors, *Handbook of Combinatorial Optimization*. Kluwer Academic Publishers, 1999.

[18] S. S. Rappaport and L.-R. Hu. Microcellular communication systems with hierarchical macrocell overlays: traffic performance models and analysis. *Proceedings of the IEEE*, 82(9):1383–1392, 1996.

[19] D. P. Reed. The cost structure of personal communication services. *IEEE Communications Magazine*, 31(4):102–108, 1993.

[20] A. R. Rouskas, M. G. Kazantzakis, and M. E. Anagnostou. Minimization of frequency assignment span in cellular networks. *IEEE Transactions on Vehicular Technology*, 48(3):873–882, 1999.

[21] J. Sarnecki, C. Vinodrai, A. Javed, P. O'Kelly, and K. Dick. Microcell design principles. *IEEE Communications Magazine*, 31(4):76–82, 1993.

[22] D.-W. Tcha, Y.-J. Chung, and T.-J. Choi. A new bound for the frequency assignment problem. *ACM/IEEE Transactions on Networking*, 5(1):34–39, 1997.

[23] F. Valois and V. Vèque. Using preemption policy to extend capacity of hierarchical cellular network. In B. Gavish, editor, *Proceedings of the 7th International Conference on Telecommunication Systems – Modeling and Analysis*, pages 185–192, 1999.

[24] S.-Y. Wu and Y.-S. Lin. Design and management of wireless communications networks. In E. Rolland and N. S. Umanath, editors, *Proceedings of the Fourth INFORMS Conference on Information Systems and Technology*, pages 284–306, 1999.