Question 1

(a) Full-Price service has the largest maximum payoff.

(b) Discount service has the largest minimum payoff.

(c) The regret table is shown below:

	Demand for Service		
Service	Strong	Weak	Max
Full Price	0	810	810
Discount	290	0	290

Discount service minimizes the maximum regret.

(e) & (d) Let p be the probability of Strong Demand. The EMV for each service as function of p is shown below:

$$\begin{aligned} \text{EMV(Full Price)} &= (p)(960) + (1-p)(-490) &&= 1450p - 490 \\ \text{EMV(Discount)} &= (p)(670) + (1-p)(320) &&= 350p + 320. \end{aligned}$$

When $p = 0.7$, EMV(Full Price) = 525 and EMV(Discount) = 565. Thus, Discount service is the optimal choice.

When $p = 0.8$, EMV(Full Price) = 670 and EMV(Discount) = 600. Thus, Full-Price service is the optimal choice.

Question 2

If we assume that any nine-digit string could potentially be a valid SSN and that when we randomly select an SSN we could select one that hasn't been assigned to anyone yet, then we can use the binomial distribution to answer these questions. Since all digits are equally likely the probability of a "success" in the first two problems is 0.1.

(a) Using the binomial tables in the book, we find that the probability of 2 successes in 3 trials when $p = 0.1$ is approximately 0.027.

(b) Using the binomial tables in the book, we find that the probability of 1 success in 2 trials when $p = 0.1$ is approximately 0.0.180.

(c) Since the digits are generated at random, the strings in each of three fields are independent. Thus, in this case we need exactly 1 success in 3 trials, then 1 success in another series 2 trials, and then 1 success in a third series of 4 trials. In this case the probability of a success is 0.5 since 5 of the 10 possible digits are larger than four.

$$P = \frac{3!}{1!2!}(0.5)^1(0.5)^2 \times \frac{2!}{1!1!}(0.5)^1(0.5)^1 \times \frac{4!}{1!3!}(0.5)^1(0.5)^3 = 0.375 \times 0.500 \times 0.250 = 0.047.$$

Question 3

(a) Let X_1 be the diameter of a randomly selected part produced by this process. Since X_1 has a normal distribution with mean $\mu_1 = 1$ and standard deviation $\sigma_1 = 0.1$, the percentage of parts meeting the specifications is given by

$$\begin{aligned}
P(0.99 \leq X \leq 1.03) &= P(\frac{0.99-1}{0.1} \leq Z \leq \frac{1.03-1}{0.1}) \\
&= P(-1 \leq Z \leq 0.3) \\
&= P(Z \leq 0.3) - P(Z \leq -1) \\
&= P(Z \leq 0.3 - (1 - P(Z \leq 1)) \\
&\approx 0.61791 - (1 - 0.84134) \\
&= 0.459256.
\end{aligned}$$

Approximately 46% of the parts will meet the specifications.

(b) Let the diameter of the rod be X_2. Note that X_2 is a normal random variable with mean $\mu_2 = 0.99$ and standard deviation $\sigma_2 = 0.1$. The rod will fit in the cylinder if $X_1 \geq X_2$ or $X_3 = X_1 - X_2 \geq 0$. From the hint it follows that X_3 is a normal random variable with mean $\mu_3 = 1 - 0.99 = 0.01$ and $\sigma_3 = \sqrt{\sigma_1^2 + \sigma_2^2} = 0.14$.

$$\begin{aligned}
P(X_3 \geq 0) &= P(Z \geq \frac{0-0.1}{0.14}) \\
&= P(Z \geq -0.07) \\
&= 1 - P(Z \leq -0.07) \\
&= P(Z \leq 0.07) \\
&\approx 0.528.
\end{aligned}$$