Work Scheduling: The following table summarizes the Centerville Bank's requirements for tellers according to the time of day:

Shift	Time	Tellers Required
1	10.00 A.M. - 12.00 P.M.	10
2	12.00 P.M. - 2.00 P.M.	16
3	2.00 P.M. - 4.00 P.M.	14

Tellers report for duty at the start of each of the above three shifts. Tellers reporting at the start of shifts 1 and 2 work for 4 consecutive hours. Tellers reporting at the start of shift 3 work for 2 consecutive hours.

Formulate a MCNFP that Centerville can use to minimize the number of tellers it must hire.

Solution Let y_i be the number of nurses who start duty at shift i.

Start	Shifts Covered
y_1	1 and 2
y_2	2 and 3
y_3	3

$$
\begin{array}{llllllll}
\text{minimize} & y_1 & + & y_2 & + & y_3 & & \\
\text{s.t.} & y_1 & & & & & \geq & 10 \\
& y_1 & + & y_2 & & & \geq & 16 \\
& & & y_2 & + & y_3 & \geq & 14 \\
& y_1, & & y_2, & & y_3 & \geq & 0
\end{array}
$$

Add slack variables

$$
\begin{array}{lcccccccccccl}
\text{minimize} & y_1 & + & y_2 & + & y_3 & & & & & & & \\
\text{s.t.} & y_1 & & & & & - & s_1 & & & & & = 10 \\
 & y_1 & + & y_2 & & & & & - & s_2 & & & = 16 \\
 & & & y_2 & + & y_3 & & & & & - & s_3 & = 14 \\
 & y_1, & & y_2, & & y_3, & & s_1, & & s_2, & & s_3 & \geq 0
\end{array}
$$

Add a redundant constraint

$$
\begin{array}{lcccccccccccl}
\min & y_1 & + & y_2 & + & y_3 & & & & & & & \\
\text{s.t.} & & & & & & & & & & & & \\
(1) & y_1 & & & & & - & s_1 & & & & & = 10 \\
(2) & y_1 & + & y_2 & & & & & - & s_2 & & & = 16 \\
(3) & & & y_2 & + & y_3 & & & & & - & s_3 & = 14 \\
(4) & & - & y_2 & - & y_3 & & & & & + & s_3 & = -14 \\
 & y_1, & & y_2, & & y_3, & & s_1, & & s_2, & & s_3 & \geq 0
\end{array}
$$

Replace (2) with $(2)' = (2) - (1)$, and (3) with $(3)' = (3) - (2)$

$$
\begin{array}{llllllllllllll}
\min & y_1 & + & y_2 & + & y_3 & & & & & & & & \\
\text{s.t.} & & & & & & & & & & & & & \\
(1) & y_1 & & & & & - & s_1 & & & & & = & 10 \\
(2)' & & + & y_2 & & & + & s_1 & - & s_2 & & & = & 6 \\
(3)' & -y_1 & & & + & y_3 & & & + & s_2 & - & s_3 & = & -2 \\
(4) & & - & y_2 & - & y_3 & & & & & + & s_3 & = & -14 \\
 & y_1, & & y_2, & & y_3, & & s_1, & & s_2, & & s_3 & \geq & 0
\end{array}
$$

Does this look familiar?

Node-Arc Incidence Matrix

$$
\begin{array}{ll}
\text{Nodes} & \begin{array}{cccccc} x_{1,3} & x_{2,4} & x_{3,4} & x_{2,1} & x_{3,2} & x_{4,3} \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} &
\begin{bmatrix}
+1 & 0 & 0 & -1 & 0 & 0 \\
0 & +1 & 0 & +1 & -1 & 0 \\
-1 & 0 & +1 & 0 & +1 & -1 \\
0 & -1 & -1 & 0 & 0 & +1
\end{bmatrix}
\end{array}
$$