Slide 1

What is an optimal solution to the LP below?

$$
\begin{array}{llcccccl}
\text{maximize} & z & & & & & & \\
\text{s.t.} & z & + & \frac{5s_1}{3} & + & \frac{s_3}{3} & = & \frac{22}{3} \\
 & x_1 & + & \frac{s_1}{3} & - & \frac{s_3}{3} & = & \frac{2}{3} \\
 & s_2 & + & s_1 & + & s_3 & = & 8 \\
 & x_2 & + & \frac{2s_1}{3} & + & \frac{s_3}{3} & = & \frac{10}{3} \\
 & x_1, & x_2, & s_1, & s_2, & s_3 & \geq & 0
\end{array}
$$

Slide 2

$$
\begin{array}{lrcrl}
\text{maximize} & x_1 & + & 2x_2 & \\
\text{s.t.} & x_1 & + & x_2 & \leq 4 \\
 & x_1 & - & 2x_2 & \leq 2 \\
 & -2x_1 & + & x_2 & \leq 2 \\
 & x_1, & & x_2 & \geq 0
\end{array}
$$

$$
\begin{array}{llcrcrcrl}
\text{maximize} & z & & & & & & & \\
\text{s.t.} & z & - & x_1 & - & 2x_2 & & & = 0 \\
 & & & x_1 & + & x_2 & + & s_1 & = 4 \\
 & & & x_1 & - & 2x_2 & + & s_2 & = 2 \\
 & & - & 2x_1 & + & x_2 & + & s_3 & = 2 \\
 & & & x_1, & x_2, & s_1, & s_2, & s_3 & \geq 0
\end{array}
$$

Slide 3

The vector-matrix form of the *structural* constraints:

$$
\begin{bmatrix} 1 & -1 & -2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} z_1 \\ x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 4 \\ 2 \\ 2 \end{bmatrix}
$$

z	x_1	x_2	s_1	s_2	s_3		
1	- 1	- 2	0	0	0	0	(Row 0)
0	1	1	1	0	0	4	(Row 1)
0	1	-2	0	1	0	2	(Row 2)
0	-2	1	0	0	1	2	(Row 3)

Slide 4

Using ero's to make x_1 a basic variable and s_1 non-basic

z	x_1	x_2	s_1	s_2	s_3		
1	- 1	- 2	0	0	0	0	(Row 0)
0	1	1	1	0	0	4	(Row 1)
0	1	-2	0	1	0	2	(Row 2)
0	-2	1	0	0	1	2	(Row 3)

z	x_1	x_2	s_1	s_2	s_3		
1	0	- 1	1	0	0	4	(Row 0)
0	1	1	1	0	0	4	(Row 1)
0	0	-3	-1	1	0	-2	(Row 2)
0	0	3	2	0	1	10	(Row 3)

Slide 5

$$\begin{aligned} z - 2x_2 + s_1 &= 4 \\ x_1 + x_2 + s_1 &= 4 \\ -3x_2 - s_1 + s_2 &= -2 \\ 3x_2 + 2s_1 + s_3 &= 10 \end{aligned}$$

Non-basic Variables:	x_2 and s_1
Basic Variables:	z, x_1, s_2, and s_3
Basic Solution:	$z = 4$, $x_1 = 4$, $s_2 = -2$, and $s_3 = 10$

Making x_1 basic and s_1 nonbasic improved the objective function value, but the solution is **infeasible**.

Slide 6

Using ero's to make x_1 a basic variable and s_2 non-basic

z	x_1	x_2	s_1	s_2	s_3		
1	- 1	- 2	0	0	0	0	(Row 0)
0	1	1	1	0	0	4	(Row 1)
0	1	-2	0	1	0	2	(Row 2)
0	-2	1	0	0	1	2	(Row 3)

z	x_1	x_2	s_1	s_2	s_3		
1	0	- 4	0	1	0	2	(Row 0)
0	0	3	1	-1	0	2	(Row 1)
0	1	-2	0	1	0	2	(Row 2)
0	0	-3	0	2	1	6	(Row 3)

Slide 7

$$\begin{aligned} z - 4x_2 + s_2 &= 2 \\ 3x_2 + s_1 - s_2 &= 2 \\ x_1 - 2x_2 + s_2 &= 2 \\ -3x_2 + s_2 + s_3 &= 6 \end{aligned}$$

Non-basic Variables:	x_2 and s_2
Basic Variables:	z, x_1, s_1, and s_3
Basic Solution:	$z = 2$, $x_1 = 2$, $s_1 = 2$, and $s_3 = 2$

Making x_1 basic and s_2 nonbasic gives a new, feasible solution with an objective function value of 2.