Slide 1

Working Capacity Allocation Problem

- Determine the least capacity needed to satisfy a set of given point-to-point demands in a telecommunications network.
- A *link* denotes the bi-directional connection between a pair of nodes.
  - For a modern DWDM telecommunications network, a link connecting nodes i and j consists of many pairs of fiber optic cable co-located in a single fiber optic duct.
  - One member of the pair is for traffic from i to j while the other is for traffic in the opposite direction.
  - In practice most large carriers use a separate cable for each direction of transmission.

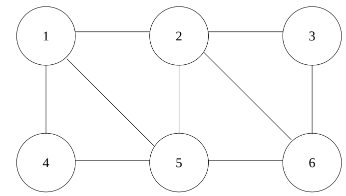
Slide 2

Notation

- The topology of the physical network is represented by a graph (N, L) where $N = \{1, \ldots, n\}$ denotes the set of nodes and L denotes unordered pairs of nodes corresponding to links.
- Let $A = \{(i, j), (j, i) : \{i, j\} \in L\}$ be a set of ordered pairs called *arcs* corresponding to the links.
- Flow on arc (i, j) implies that flow is from i to j.
- Flow in the opposite direction must be on arc (j, i).
- The directed graph (network) is given by $G = (N, A)$.

Slide 3

Graph for Example Problem



$N = \{1, 2, 3, 4, 5, 6\}$

$L = \{\{1, 2\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 5\}, \{2, 6\}, \{3, 6\}, \{4, 5\}, \{5, 6\}\}$

$A = \{(1, 2), (1, 4), (1, 5), (2, 3), (2, 5), (2, 6), (3, 6), (4, 5), (5, 6),$
$(2, 1), (4, 1), (5, 1), (5, 2), (6, 2), (6, 3), (5, 4), (6, 5)\}$

Slide 4

Demand Matrix

- Let d_{ij} denote the demand for traffic with origin node i and destination node j.
- Traffic demand need not be symmetrical (i.e., it's possible that $d_{ij} \neq d_{ji}$), and it's assumed that $d_{ii} = 0$ for $i = 1, \ldots, n$.
- The corresponding matrix is called the *demand* or *traffic matrix*.
- Since all the traffic prescribed by the demand matrix must share the same network represented by $G = [N, A]$, the problem is a member of the class of multicommodity network flow problems.
- An individual commodity can be expressed as either an (i, j) pair for each (i, j) such that $d_{ij} > 0$, or an aggregation of such (i, j)-pairs by their source (or destination) nodes.

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Demand Matrix for Example Problem

	1	2	3	4	5	6
1	-	10	0	10	10	0
2	0	-	10	10	10	10
3	0	0	-	0	10	10
4	0	0	0	-	10	0
5	0	0	0	0	-	10
6	0	0	0	0	0	-

11 commodities:

$d_{12} = 10$ $d_{14} = 10$ $d_{15} = 10$

$d_{23} = 10$ $d_{24} = 10$ $d_{25} = 10$

$d_{26} = 10$ $d_{35} = 10$ $d_{36} = 10$

$d_{45} = 10$ $d_{56} = 10$

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Demand Aggregation

- Produces smaller models (fewer constraints and variables) than treating each demand pair as separate commodity.
- Up to $|N| = n$ commodities, each having up to $n - 1$ destinations, corresponding to each of the other nodes in N rather than (as many as) $n^2 - n$ commodities corresponding to each demand pair in the traffic matrix.
- Sets up a multi-commodity flow problem where each commodity has a single "source" and multiple "sinks" corresponding to the nodes of a given commodity's demand matrix

Slide 7

Demand Notation

- Let the n-component requirement vector e^k for commodity $k \in N$ be given by

$$e_i^k = \begin{cases} \sum_{j \in N} d_{kj}, & \text{if } i = k \\ -d_{ki}, & \text{otherwise.} \end{cases}$$

- For a given commodity k,
  - $e_i^k > 0$ means node i is a supply node
  - $e_i^k = 0$ means node i is a transshipment node
  - $e_i^k < 0$ means node i is a demand node

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Demand Matrix for Example Problem

	1	2	3	4	5	6
1	-	10	0	10	10	0
2	0	-	10	10	10	10
3	0	0	-	0	10	10
4	0	0	0	-	10	0
5	0	0	0	0	-	10
6	0	0	0	0	0	-

$e_1^1 = 30$	$e_2^1 = -10$	$e_3^1 = 0$	$e_4^1 = -10$	$e_5^1 = -10$	$e_6^1 = 0$
$e_1^2 = 0$	$e_2^2 = 40$	$e_3^2 = -10$	$e_4^2 = -10$	$e_5^2 = -10$	$e_6^2 = -10$
$e_1^3 = 0$	$e_2^3 = 0$	$e_3^3 = 20$	$e_4^3 = 0$	$e_5^3 = -10$	$e_6^3 = -10$
$e_1^4 = 0$	$e_2^4 = 0$	$e_3^4 = 0$	$e_4^4 = 10$	$e_5^4 = -10$	$e_6^4 = 0$
$e_1^5 = 0$	$e_2^5 = 0$	$e_3^5 = 0$	$e_4^5 = 0$	$e_5^5 = 10$	$e_6^5 = -10$

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Working Capacity Allocation: Decision Variables for Node-Arc Model

- For each arc (i, j),
  - Variable g_{ij}^k denotes the flow of commodity k on arc (i, j).
  - Variable c_{ij} denotes the capacity of link $\{i, j\}$.

$$g_{ij}^k \geq 0 \text{ and integer}, \forall k \in N, \forall (i,j) \in A$$
$$c_{ij} \geq 0, \forall \{i,j\} \in L$$

- This model assumes symmetrical capacity so that the capacity of c_{ij} on link $\{i, j\}$ can accommodate a simultaneous flow of c_{ij} in both directions.
- In the classical multicommodity network flow problem the capacities are constant and need not be identical in both directions.

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Working Capacity Allocation: Constraints for Node-Arc Model

minimize Total Working Capacity:

$$\sum_{\{i,j\} \in L} c_{ij}$$

subject to Flow Conservation:

$$\sum_{(i,j) \in A} g_{ij}^k - \sum_{(j,i) \in A} g_{ji}^k = e_i^k, \forall i \in N, \forall k \in N$$

subject to Capacity in Normal Direction:

$$\sum_{k \in N} g_{ij}^k \leq c_{ij}, \forall \{i,j\} \in L$$

subject to Capacity in Reverse Direction:

$$\sum_{k \in N} g_{ji}^k \leq c_{ij}, \forall \{i,j\} \in L$$

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Working Capacity Allocation: Optimal Solution for Node-Arc Model

$g_{12}^1 = 10 \quad g_{14}^1 = 10 \quad g_{15}^1 = 10$

$g_{14}^2 = 10 \quad g_{23}^2 = 10 \quad g_{25}^2 = 10 \quad g_{26}^2 = 10 \quad g_{21}^2 = 10$

$g_{25}^3 = 10 \quad g_{36}^3 = 10 \quad g_{32}^3 = 10$

$g_{45}^4 = 10$

$g_{56}^5 = 10$

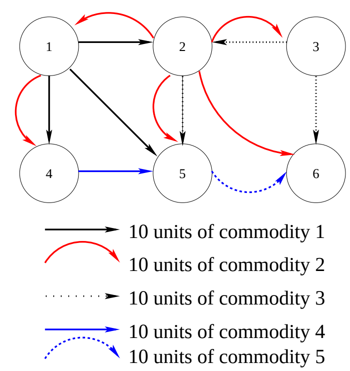
$c_{12} = 10 \quad c_{14} = 20 \quad c_{15} = 10$

$c_{23} = 10 \quad c_{25} = 20 \quad c_{26} = 10 \quad c_{36} = 10$

$c_{45} = 10 \quad c_{56} = 10$

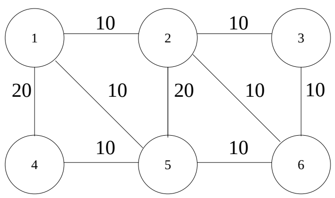
Slide 12

Working Capacity Allocation: Optimal Flow for Node-Arc Model



Slide 13

Working Capacity Allocation: Optimal Flow for Capacity Allocation



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Node-Arc Model: Advantages and Disadvantages

- The advantage of this model is that it requires very little input data and implicitly considers all possible paths for every demand pair.
- One slight disadvantage is that some additional analysis or post processing of the LP solution is required to find the paths and flow for a given demand pair. This can be easily accomplished with a procedure similar to that suggested by Dijkstra's algorithm for finding shortest paths.
- Also some of the paths in the optimal solution may use a large number of arcs. The number of arcs in a path is known as the *hop count* and this can not be restricted in the node-arc model.

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Arc-Path Formulation

- A *directed path* from node s to node t in the network $G = (N, A)$ is a sequence of nodes and arcs $p = \{i_i, (i_1, i_2), i_2, (i_2, i_3), i_3, \ldots, i_\ell, (i_\ell, i_{\ell+1}), i_{\ell+1}\}$, where $i_1 = s$, $i_{\ell+1} = t$, and each arc and node are distinct.
- Let D denote the set of demand pairs. That is, $(i, j) \in D$ implies that $d_{ij} > 0$.
- Let Q_{ij} denote the set of candidate directed paths from i to j in $G = (N, A)$ for all $(i, j) \in D$, and let $\mathcal{P} = \cup_{(i,j)\in D} Q_{ij}$.
- Let P_{ij} denote the set of paths that contain arc $(i, j) \in A$, and let y_p denote the flow on path p.

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Candidate Paths for Example Problem

$Q_{12} = \{1, 2, 41\}$	$Q_{13} = \{3, 4, 5\}$
$Q_{14} = \{6, 7, 42\}$	$Q_{15} = \{8, 9, 10\}$
$Q_{16} = \{11, 12, 13, 14, 15\}$	$Q_{23} = \{16, 17, 43\}$
$Q_{24} = \{18, 19\}$	$Q_{25} = \{20, 21, 22, 44, 45\}$
$Q_{26} = \{23, 24, 25, \}$	$Q_{34} = \{26, 27, 28, \}$
$Q_{35} = \{29, 30\}$	$Q_{36} = \{31, 32, 46\}$
$Q_{45} = \{33, 34, 47\}$	$Q_{46} = \{35, 36, 37, 48, 49, 50\}$
$Q_{56} = \{38, 39, 40\}$	$\mathcal{P} = \{1, 2, 3, \ldots, 50\}$

Path 1: $\{1, (1, 2), 2\}$
Path 2: $\{1, (1, 5), 5, (5, 2)\}$
Path 3: $\{1, (1, 4), 4, (4, 2)\}$
$P_{25} = \{14, 20, 27, 43, 47, 9, 19, 25, 29, 42, 46, 48\}$
$P_{52} = \{2, 39, 40, 41, 50\}$

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Working Capacity Allocation: Arc-Path Model

minimize Total Working Capacity: $\sum_{\{i,j\} \in L} c_{ij}$

subject to Demand:

$$\sum_{p \in Q_{ij}} y_p = d_{ij}, \forall (i,j) \in D \tag{1}$$

subject to Capacity in Normal and Reverse Directions:

$$\sum_{p \in P_{ij}} y_p \leq c_{ij}, \forall \{i,j\} \in L \tag{2}$$

$$\sum_{p \in P_{ji}} y_p \leq c_{ij}, \forall \{i,j\} \in L \tag{3}$$

subject to Nonnegativity and Integrality:

$$c_{ij} \geq 0, \forall \{i,j\} \in L \tag{4}$$

$$y_p \geq 0 \text{ and integer}, \forall p \in \mathcal{P} \tag{5}$$

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Example Constraints: Arc-Path Model

Demand constraint (1) for demand between 1 and 2:

$$y_1 + y_2 + y_{41} = 10$$

Capacity constraint (2) for link $\{2,5\}$:

$$y_{14} + y_{20} + y_{27} + y_{43} + y_{47} + y_9 + y_{19} + y_{25} + y_{29} + y_{42} + y_{46} + y_{48} \leq c_{25}$$

Capacity constraint (3) for link $\{2,5\}$:

$$y_2 + y_{39} + y_{40} + y_{41} + y_{50} \quad \leq \quad c_{25}$$

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Arc-Path Model: Optimal Solution for Example Problem

$y_1 = 10$	$y_6 = 10$	$y_8 = 10$	$y_{16} = 10$
$y_{18} = 10$	$y_{20} = 10$	$y_{23} = 10$	$y_{30} = 10$
$y_{31} = 10$	$y_{33} = 10$	$y_{38} = 10$	
$c_{12} = 10$	$c_{14} = 20$	$c_{15} = 10$	
$c_{23} = 10$	$c_{25} = 10$	$c_{26} = 10$	
$c_{36} = 20$	$c_{45} = 10$	$c_{56} = 10$	

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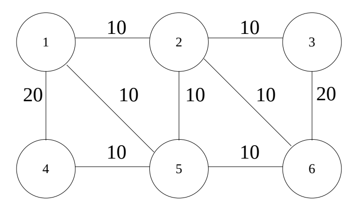
Working Capacity Allocation: Optimal Solution for Arc-Path Model



- Total working capacity = 110
- In this case, the total capacity installed is the same was with the solution to the node-arc model. However, the allocation to individual links is different.

Slide 21

Arc-Path Model: Advantages and Disadvantages

- One advantage of this model is that the hop count for all paths can be restricted.
- Paths that exceed the hop count do not appear in the sets Q_{ij}.
- A disadvantage is that the cardinality of the sets Q_{ij} can be very large.
  - For most applications Q_{ij} is replaced with $\bar{Q}_{ij} \subset Q_{ij}$ where only a few of the shortest paths from i to j appear in $\bar{Q}_{ij}$.
  - When this substitution is made, however, there is no guarantee that the arc-path model will give as good a solution as the node-arc model.

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Spare Capacity Allocation Problem

- The simplest idea for protecting the links in a path is to provision a node-disjoint backup path.
- This is also called *1:1* protection since each working path (the path(s) a demand normally takes when all links are functioning) has a backup path in reserve that will be used whenever, and only when, a link in the working path fails.
- Required for some applications, but generally the most expensive of the various protection strategies.
- In shared protection schemes, the spare capacity on a link is not dedicated to any given demand pair and may be used in the restoration of various demand pairs.
- Shared protection schemes come in two varieties: *link restoration* and *path restoration*. Models for each follow.

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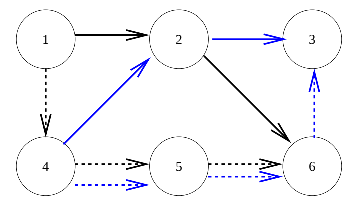
Shared vs. Dedidicated Capacity Example



Legend
Path 1: 1−2−6 (working path)
Path 2: 1−4−5−6 (backup path)
Path 3: 4−2−3 (working path)
Path 4: 3−6−5−4 (backup path)

Demands: (1,6) for 4 OC48s and (4,3) for 6 OC48s

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Table 1: Spare Capacity Used For Various Link Failures

Failed Link	Path Used		Spare Capacity Used			
	Demand 1-6	Demand 4-3	(1,4)	(3,6)	(4,5)	(5,6)
(1,2)	backup	working	4	0	4	4
(2,6)	backup	working	4	0	4	4
(2,3)	working	backup	0	6	6	6
(2,4)	working	backup	0	6	6	6
Spare Capacity Required Under Shared Protection			4	6	6	6
Spare Capacity Required Under Dedicated Protection			4	6	10	10

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Link Restoration

- In link restoration, it is assumed that each node has the capability of detecting link failures and implementing a rerouting algorithm around the defective link.
- If link $\{s,t\}$ fails, then restoration requires that all working traffic that uses link $\{s,t\}$ be rerouted on the reduced graph $[N, L \setminus \{s,t\}]$.
- Link failure refers to a cut that destroys all fiber in a duct.
- Examples (c_{ij} found with the arc-path model)
  - 10 units of spare capacity on links in one or more paths from node 1 to node 2 must be available to protect link $\{1,2\}$.
  - 20 units of spare capacity on links in one or more paths from node 1 to node 4 must be available to protect link $\{1,4\}$.
  - Spare capacity used to protect $\{1,2\}$ is available to procted $\{1,4\}$, too.

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The Node-Arc Model for Link Restoration

Let c_{ij} for all $\{i,j\} \in L$ denote the known volume of working traffic on link $\{i,j\}$. Suppose link $\{s,t\}$ fails. Then c_{st} units of flow must be rerouted from node s to node t and vice versa. In the node-arc model for link restoration, the requirement at node i is given by

$$r_i^{st} = \begin{cases} c_{st}, & \text{if } i = s \\ -c_{st}, & \text{if } i = t \\ 0, & \text{otherwise.} \end{cases}$$

Let the variable h_{ij} denote the spare capacity assigned to link $\{i,j\}$ and the variable f_{ij}^{st} denote the restoration flow on arc (i,j) when $\{s,t\}$ fails.

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The r_i^{st} Values for Working Capacity Determined by Arc-Path Model

$$\begin{array}{lllll}
\text{link } \{1,2\}: & r_1^{12} = 10 & r_2^{12} = -10 \\
\text{link } \{1,4\}: & r_1^{14} = 20 & r_4^{14} = -20 \\
\text{link } \{1,5\}: & r_1^{15} = 10 & r_5^{15} = -10 \\
\text{link } \{2,3\}: & r_2^{23} = 10 & r_3^{23} = -10 \\
\text{link } \{2,5\}: & r_2^{25} = 10 & r_5^{25} = -10 \\
\text{link } \{2,6\}: & r_2^{26} = 10 & r_6^{26} = -10 \\
\text{link } \{3,6\}: & r_3^{36} = 20 & r_6^{36} = -20 \\
\text{link } \{4,5\}: & r_4^{45} = 10 & r_5^{45} = -10 \\
\text{link } \{5,6\}: & r_5^{56} = 10 & r_6^{56} = -10
\end{array}$$

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The Node-Arc Model for Link Restoration

minimize Spare Capacity:

$$\sum_{\{i,j\} \in L} h_{ij}$$

subject to Flow Conservation:

$$\sum_{(k,j) \in A} f_{kj}^{st} - \sum_{(i,k) \in A} f_{ik}^{st} = r_i^{st}, \forall k \in N, \forall \{s,t\} \in L$$

subject to Capacity in Normal Direction:

$$f_{ij}^{st} \le h_{ij}, \forall \{i,j\} \in L, \forall \{s,t\} \in L$$

subject to Capacity in Reverse Direction:

$$f_{ji}^{st} \le h_{ij}, \forall \{i,j\} \in L, \forall \{s,t\} \in L$$

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The Node-Arc Model for Link Restoration

subject to Link Failures:

$$f_{st}^{st} + f_{ts}^{st} = 0, \forall \{s,t\} \in L$$

subject to Nonnegativity:

$$h_{ij} \geq 0, \forall \{i,j\} \in L$$
$$f_{ij}^{st} \geq 0 \text{ and integer}, \forall \{s,t\} \in L, \forall (i,j) \in A$$

where c_{ij} for all $\{i,j\} \in L$ are fixed (usually to values determined by solving a working capacity allocation model).

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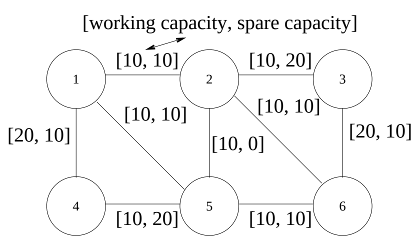
Spare Capacity Allocation with Link Restoration: Node-Arc Solution



This solution uses 100 units of spare capacity are required to protect 110 units of working capacity. This is typical for this type of problem.

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The Arc-Path Model for Link Restoration

- The arc-path model for link restoration uses Z^{st} for all $\{s,t\} \in L$ to denote the set of candidate directed paths from node s to node t excluding the direct arc (s,t).
- Therefore, the set of all potential backup paths is given by

$$\mathcal{P} = \cup_{\{s,t\} \in L} Z^{st}.$$

- The variable h_{ij} for all $\{i,j\} \in L$ denotes the spare capacity on link $\{i,j\}$ and the variable w_p^{st} denotes the restoration flow on path p when link $\{s,t\}$ fails.

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The Arc-Path Model for Link Restoration

minimize Total Working Plus Spare Capacity:

$$\sum_{\{i,j\} \in L} h_{ij}$$

subject to Lost Working Capacity:

$$\sum_{p \in Z^{st}} w_p^{st} = c_{st}, \forall \{s,t\} \in L$$

subject to Link Capacities Normal Direction:

$$\sum_{p \in A_{ij}} w_p^{st} \leq h_{ij}, \forall \{s,t\} \in L, \forall \{i,j\} \in L \setminus \{\{s,t\}\}$$

subject to Link Capacities Reverse Direction:

$$\sum_{p \in A_{ji}} w_p^{st} \leq h_{ij}, \forall \{s,t\} \in L, \forall \{i,j\} \in L \setminus \{\{s,t\}\}$$

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The Arc-Path Model for Link Restoration

subject to Nonnegativity:

$$h_{ij} \geq 0, \forall \{i, j\} \in L$$
$$w_p^{st} \geq 0, \forall \{s, t\} \in L, \forall p \in \mathcal{P}$$

where c_{ij} for all $\{i, j\} \in L$ are fixed.

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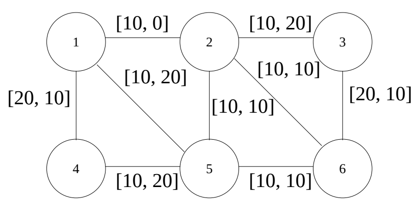
Spare Capacity Allocation with Link Restoration: Arc-Path Solution



In this solution, the total spare capacity is 110 as opposed to 100 in the solution from the node-arc model. This is due to the fact that not all paths are included in Z^{st}.

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Path Restoration

- Path restoration requires allocation of spare capacity to a set of paths that do not use the failed link.
- The distinction between path restoration and link restoration is that path restoration uses alternative routes from the origins to the destinations of the demand pairs affected by the failed link rather than simply taking a "detour" around it. Thus it is a distinction between "global" and "local" rerouting.
- The set V_{ij}^{st} is the set of paths available for restoration from i to j when link $\{s, t\}$ fails (i.e., V_{ij}^{st} is the set of directed paths from i to j that do *not* use link $\{s, t\}$.
- Let $\bar{V}_{ij}^{st}$ be the set of directed paths from i to j that are *not* available for working traffic when link $\{s, t\}$ fails.

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Arc-Path Formulation of the Path Restoration Version of the Spare Capacity Allocation Model

minimize Total Spare Capacity:

$$\sum_{\{i,j\} \in L} h_{ij} \tag{6}$$

subject to Spare Demand 1:

$$\sum_{p \in V_{ij}^{st}} w_p^{st} = \sum_{p \in \bar{V}_{ij}^{st}} y_p, \qquad \forall \{s, t\} \in L, \forall (i, j) \in D \tag{7}$$

subject to Spare Demand 2:

$$\sum_{p \in V_{ji}^{st}} w_p^{st} = \sum_{p \in \bar{V}_{ji}^{st}} y_p, \qquad \forall \{s, t\} \in L, \forall (i, j) \in D \tag{8}$$

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Arc-Path Formulation of the Path Restoration Version of the Spare Capacity Allocation Model

subject to Spare Capacity Normal Direction:

$$\sum_{p \in A_{ij}} w_p^{st} \leq h_{ij}, \forall \{s,t\} \in L, \forall \{i,j\} \in L \setminus \{\{s,t\}\} \qquad (9)$$

subject to Spare Capacity Reverse Direction:

$$\sum_{p \in A_{ji}} w_p^{st} \leq h_{ij}, \forall \{s,t\} \in L, \forall \{i,j\} \in L \setminus \{\{s,t\}\} \qquad (10)$$

subject to Nonnegativity:

$$h_{ij} \geq 0, \forall \{i,j\} \in L \qquad (11)$$

$$w_p^{st} \geq 0, \forall \{s,t\} \in L, \forall p \in \mathcal{P} \qquad (12)$$

where c_{ij} and y_p are constants determined by solving the arc-path formulation of the working capacity allocation model.

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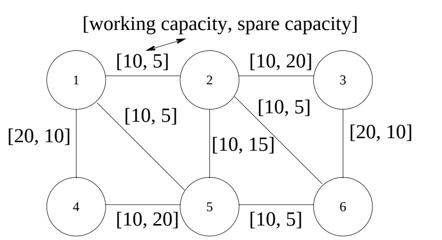
Arc-Path Formulation of the Path Restoration Version of the Spare Capacity Allocation Model: Optimal Solution

- When this model is applied to the example problem, the total spare capacity needed was only 95 compared to 110 for link restoration.
- A more sophisticated restoration procedure is needed to achieve these savings.



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Joint Capacity Planning Models

- A joint model is one in which both working and spare capacity can be determined in a single model.
- In the previous discussion, working capacity was determined with one model and then spare capacity was determined to protect the optimal working capacity against single link failures.
- Since the amount of working capacity on a link determines the amount of spare capacity needed elsewhere to provide for restoration, joint optimization should require less total capacity.

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Joint Capacity Planning Models

- The joint model is a combination of the *arc-path formulation of the working capacity allocation model* and the *arc-path formulation of the path restoration version of the spare capacity allocation model.*
- The model is stated mathematically as minimize $\sum_{\{i,j\} \in L}(c_{ij} + h_{ij})$ subject to (1)-(5) and (7)-(12).
- When applied to the example problem, the total capacity needed was 176 compared to 205 for the two-phase approach. This optimal solution is illustrated on the next slide.
- The routing for the traffic demands in the absence of failure is split across multiple paths, which results in a smaller overall spare capacity requirement than was the case with the two-stage approaches considered earlier.

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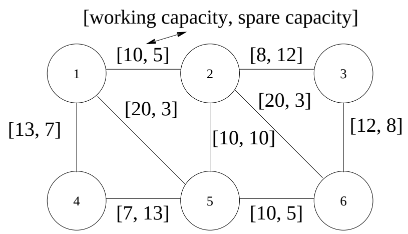
Solution to the Joint Model (Working Capacity = 110 and Spare Capacity = 66)



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Summary of Example Problem Results :

$|N| = 6, |L| = 9, |A| = 18, |D| = 11, |\mathcal{P}| = 110$

	Capacity		
Model	Working	Spare	Total
Working Capacity: Node-Arc Model	110	—	—
Working Capacity: Arc-Path Model	110	—	—
Spare Cap: Link Restoration with Node-Arc Model	110	100	210
Spare Cap: Link Restoration with Arc-Path Model	110	110	220
Spare Cap: Path Restoration with Arc-Path Model	110	95	205
Joint Working and Spare Capacity (Arc-Path Model)	110	66	176