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# Robust Solutions for the DWDM Routing and Provisioning Problem: Models and Algorithms

by

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## Abstract

The dense wavelength division multiplexing routing and provisioning problem with uncertain demands and a fixed budget is modeled as a multicriteria optimization problem. To obtain a robust design for this problem, the primary objective is to minimize a regret function that models the total amount of over and/or under provisioning in the network resulting from uncertainty in a demand forecast. Point-to-point demands are given by a set of scenarios each with a known probability, and regret is modeled as a quadratic function. The secondary objective is to minimize the equipment cost that achieves the optimal value for regret. We propose a two-phase robust optimization strategy that uses a pair of integer linear programs having a large number of continuous variables, but only two integer variables for each link. In an empirical study, the two-phase robust optimization strategy is compared to alternative techniques using a mean-value model, a worst-case model, and a two-stage stochastic integer program with recourse model. Both the worst-case model and the stochastic programming model exhibited a bias toward low-cost designs (well below the budget) at the expense of high expected over/under provisioning. For a tight budget, the mean-value model fails miserably yielding no design for comparison. The two-phase robust strategy produces the optimal design for a given budget that is the best compromise between expected regret and equipment cost.

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# 1 Introduction

The main components of a dense wavelength division multiplexing (DWDM) network are fiber, terminal equipment (TE), optical amplifiers (As), and regenerators (Rs). The term TE refers to the wavelength transponders that perform the optical-electrical-optical conversion. For this investigation, it is assumed that dark fiber is available for use at no additional cost and that it can be lit by the installation of optical amplifiers and regenerators, at appropriate intervals along the link, and terminal equipment at both incident nodes. We assume that other equipment needed to operate a DWDM network, such as switches and translators, has relatively small cost compared to the terminal equipment, amplifiers, and regenerators. The cost of this other equipment would be added to a final design. Links containing these components as illustrated in Figure 1 are the basic building blocks of a DWDM network. The optical amplifiers are associated with an individual fiber, while the regenerators are associated with a particular channel or wavelength ($\lambda$) within a given fiber. Each link can be composed of multiple fibers each with multiple channels per fiber. The size of the basic building blocks varies and determines the pipe size linking a pair of cities or stations. Throughout this paper we use the term *pipe* to represent a link with the associated equipment that determines the capacity of a link.

Using these building blocks of various sizes, the *DWDM routing and provisioning problem* can be stated as follows:

> *Given a network topology and an estimate of the point-to-point demand traffic, determine the routing for each demand and the least-cost DWDM equipment configuration required to support the routes.*

For a given demand forecast, this problem can be modeled as an integer linear program (ILP). Modern modeling languages such as AMPL [8, 23], GAMS [7, 26], and OPL [21, 27] can be used to create very detailed design models of a problem instance with only a moderate amount of effort. Modern solvers such as CPLEX [24, 25] can be called by the modeling languages to obtain near optimal solutions to real-world problem instances. The key data that drives these models is the demand forecast. Since the quality of the demand forecasts in this domain is often found to be lacking, network designers and their clients are often concerned that an optimal design based on an erroneous forecast may prove to be a poor investment. If the forecast is too low, the network will not have enough capacity to meet all the demand. Conversely, a network designed for a forecast that predicts more demand than is realized will be over provisioned with expensive, underutilized equipment. As illustrated in Figure 2, some of the pipes may be under provisioned while others may be over provisioned.

In the operations research literature, design problems of this type with uncertain demand use a set of forecasts, each with a given probability of occurrence. Such sets generally include optimistic and pessimistic forecasts as well as some intermediate values. It would then be possible to construct a design based on the *mean value* or *worst-case value* for the set of potential traffic demands. However, large error bounds may result from such procedures (see Birge [6]). Other strategies that appear in the literature include *sensitivity analysis* and *stochastic programming*.

While the stochastic programming approach has many disciples (the bibliography [20] lists more than 1,000 references), we chose to extend the *robust optimization methodology* as described in the influential paper of Mulvey, Vanderbei, and Zenios [14] to the DWDM routing and provisioning problem. The idea of robust optimization is to create a design that will be fairly good (i.e., robust) regardless of which demand scenario is realized. The robust methodology uses a regret function to capture this notion of robustness.

While it is desirable to have a robust design, network designers are also quite concerned about network cost. In this investigation, the *DWDM routing and provisioning problem with uncertain demands* is viewed as a multicriteria design problem. The critical design criteria are equipment cost and regret as described in the robust optimization literature. A two-phase robust optimization strategy is proposed and demonstrated for this problem. In phase I, a minimum-regret solution that satisfies a budget constraint is obtained. In phase II, regret is fixed to the optimal value found in phase I and a minimum-cost solution is obtained.

There are several different concepts of *robustness* that appear in the literature (see e.g. Paraskevopoulos, Karakitsos, and Rustem [16], Mulvey, Vanderbei, and Zenios [14], Kouvelis and Yu [11], Ben-Tal and Nemirovski [4, 5], and Averbakh [2].) One of the most popular models is the minmax-regret model for combinatorial optimization problems. A state-of-the-art presentation of this strategy and important applications can be found in [11]. Another popular model is the min-regret model for linear programming models as described by Mulvey et al. [14], and demonstrated in Bai, Carpenter and Mulvey [3]. In our investigation, we adopt the concept and model presented in [3, 14]. However, we extend these ideas to the case of multiple criteria. Other applications of robust optimization for telecommunications design problems may be found in Laguna [12], Soteriou and Chase [18], and Gryseels, Sorbello, and Demeester [9]. Stochastic programming approaches for network capacity planning under uncertainty may be found in Sen, Doverspike and Cosares [17] and in Lisser, Ouorou, Vial, and Gondzio [13]. The issue of assigning specific wavelengths to lightpaths has been addressed in Kennington et al. [10].

The first contribution of this investigation is a detailed optimization model that determines the quantity and location of network terminal equipment, optical amplifiers, and regenerators required to satisfy a given demand forecast at minimum cost. Other models in the literature such as Sen et al. [17] and Lisser et al. [13] simply use link capacities as the decision variables. This fails to account for the fact that the hardware must be purchased in modular values and may lead to fairly large errors in the cost function. The second contribution is a new two-phase optimization strategy that accounts for the multiple goals of this design problem as well as the uncertainty in the demand forecast. The final contribution is a demonstration of this methodology and comparison with competing methods.

## 2 The Models

In this section, an ILP is presented for the basic provisioning problem. We use an arc-path model that determines the equipment required to route a set of point-to-point demands for a given scenario. In addition, a two-phase robust optimization procedure is presented that determines an optimal design when all scenarios are considered simultaneously. The robust models use a convex, piece-wise linear function to model regret and extend the strategy presented in Mulvey et al. [14].

## 2.1 Sets

The network topology is represented as a graph $G = [N, E]$, where $N$ denotes the set of nodes and $E \subseteq N \times N$ denotes the set of links. For each $n \in N$, $A_n$ denotes the set of links adjacent to node $n$. The origin/destination node pairs $o, d \in N$ corresponding to the point-to-point demands are given by $D \subseteq N \times N$. For each $(o, d) \in D$, $J_{od}$ denotes the set of possible paths from $o$ to $d$ that can be used to route this demand. For each $n \in N$ ($e \in E$), $P_n$ ($L_e$) denotes the set of paths containing node $n$ (link $e$). The set of scenarios for a problem having $\bar{s}$ scenarios is denoted $S = \{1, \ldots, \bar{s}\}$.

## 2.2 Constants

For this model we assume that a maximum of 192 DS3s can be carried on each wavelength ($\lambda$) and that a fiber has 80 channels (2 $\lambda$'s/channel). We assume that when signal regeneration is required, regenerators are installed on each bi-directional channel. When required by optical reach limitations, optical amplifiers are installed to boost the signals carried by an entire fiber. The constants used in our models along with the specific values used for the first of our two test problems, DA, may be found in Table 1. The equipment costs used in the study approximate current market prices, but do not represent specific prices offered by any particular vendor.

## 2.3 Decision Variables

The various types of decision variables used in the models are defined in Table 2. By requiring two of these variables to assume integer values, the number of optical amplifiers and number of regenerators will assume integer values. For our test cases, the number of TEs are always large and we simply round them up to the nearest integer.

## 2.4 The Basic Routing and Provisioning Model

For each scenario $s$, there is a basic provisioning model whose objective is to minimize the *total cost for provisioning* the network. The network has TE equipment located at each node and optical amplifiers and regenerators associated with the links as needed. The objective is as follows:

$$\text{minimize } \gamma^s \tag{1}$$

where

$$\gamma^s = \sum_{n \in N} C^{TE} \ell_n^s + \sum_{e \in E} (C^R r_e^s + C^A a_e^s) \tag{2}$$

There are six additional sets of constraints that define this model. The first set of constraints ensure *demand satisfaction* and are given as follows:

$$\sum_{p \in J_{od}} x_p^s = R_{od}^s, \qquad \forall (o,d) \in D \tag{3}$$

The second set of constraints *convert path capacity to link capacity* and are defined by

$$\sum_{p \in L_e} x_p^s = z_e^s, \qquad \forall e \in E \tag{4}$$

The third set of constraints *convert link capacity to TEs* and are as follows:

$$z_e^s \leq M^{TE} t_e^s, \qquad \forall e \in E \tag{5}$$

The fourth set of constraints *accumulate TEs on links to give the number of TEs at each node.* These are simply accounting constraints and can be substituted out of the model. They are

$$\sum_{e \in A_n} t_e^s = \ell_n^s, \qquad \forall n \in N \tag{6}$$

The fifth set of constraints *convert link capacity into fibers and channels and place bounds on fiber*

$$z_e^s \leq M^A f_e^s, \quad \forall e \in E \tag{7}$$

$$z_e^s \leq M^R c_e^s, \quad \forall e \in E \tag{8}$$

$$f_e^s \leq F_e, \qquad \forall e \in E \tag{9}$$

The sixth type of constraints *convert fiber and channels into amplifiers and regenerators.* They are defined as follows:

$$G_e^A f_e^s = a_e^s, \qquad \forall e \in E \tag{10}$$

$$G_e^R c_e^s = r_e^s, \qquad \forall e \in E \tag{11}$$

The basic routing and provisioning model for scenario $s$ is the ILP defined by (1)-(11). The only change for different scenarios is the right-hand-side for the demand-satisfaction constraints (3).

## 2.5 The Robust Models

We use the general modeling framework as described by Mulvey et al. [14] to construct our robust models. The key is the construction of a regret function to capture the trade-off between too little network capacity and excess capacity. For the DWDM routing and provisioning problem, the client experiences regret when either the network can not meet a substantial part of the demand or when the network has been over provisioned and most of the network only uses a small amount of its available capacity. Let $z_{ods}^+$ denote the amount of demand for pair $(o, d)$ that is not satisfied in scenario $s$. Likewise, $z_{ods}^-$ denotes the amount by which the demand for pair $(o, d)$ is exceeded in scenario $s$. The variables $z_{ods}^+$ and $z_{ods}^-$ may be viewed as representing the amount of under and over provisioning for pair $(o, d)$ in scenario $s$. Hence, the demand constraints can be modeled as follows:

$$\sum_{p \in J_{od}} x_p + z_{ods}^+ - z_{ods}^- = R_{od}^s, \qquad \forall (o, d) \in D, \forall s \in S \tag{12}$$

Using $P_s$ to denote the probability of scenario $s$, Mulvey et al. [14] recommend using some type of quadratic function of the form:

$$\sum_{s \in S} P_s \sum_{(o,d) \in D} [(z_{ods}^+)^2 + (z_{ods}^-)^2].$$

We believe that management will find it easier to deal with a little over provisioning as compared to a little under provisioning; and hence, we use a non-symmetrical quadratic function as illustrated in Figure 3. The relative weight of over provisioning versus under provisioning is subjective and should be determined by the client. To make our strategy computationally viable, we use a four-piece linear approximation for each side of the quadratic function.

Let $R_{\max}$ denote the largest demand value, then $0 \leq z^{+}_{ods} \leq R_{\max}$ and $0 \leq z^{-}_{ods} \leq R_{\max}$ for all $o$, $d$ and $s$. Let $c^{u}_{k}$ ($c^{o}_{k}$) denote the linear penalty cost for under (over) provisioning for linear piece $k$ of the regret function where we assume that $c^{u}_{k+1} > c^{u}_{k}$ ($c^{o}_{k+1} > c^{o}_{k}$) for $k = 1, 2, 3$. Under this assumption the regret function is convex. Then,

$$z^{+}_{ods} = \sum_{k=1,\ldots,4} \bar{z}^{+}_{odsk}, \qquad \forall (o,d) \in D, \forall s \in S \tag{13}$$

$$z^{-}_{ods} = \sum_{k=1,\ldots,4} \bar{z}^{-}_{odsk}, \qquad \forall (o,d) \in D, \forall s \in S \tag{14}$$

and regret is modeled as

$$\rho = \sum_{(o,d)\in D} \{ \sum_{s\in S} P_s [ \sum_{k=1,\ldots,4} (c^{u}_{k} \bar{z}^{+}_{odsk} + c^{o}_{k} \bar{z}^{-}_{odsk})]\} \tag{15}$$

where $0 \leq \bar{z}^{+}_{odsk}, \bar{z}^{-}_{odsk} \leq R_{\max}/4$. That is, each linear piece covers 25% of the demand interval.

The regret function is actually a feasibility penalty function. It penalizes violations of the demand constraints for the various scenarios. The regret function can also be viewed as expressing the pain that management will feel if there is a mismatch between the infrastructure created and the actual demand for services. An argument against using robust optimization is that it is difficult to determine a decision maker's regret function. However, similar arguments exist about the notion of utility which has been used extensively in the literature concerning decision making under uncertainty. This issue is also addressed in the the literature on game theory. Von Neumann and Morgenstern [15] address the notion of utility and argue that it can be quantitatively measured. For our investigation, we assume that a manager's regret function in the form given by (15) can be constructed.

For this problem, clients are also concerned about a budget restriction. The budget constraint is simply

*(budget constraint)*

$$\gamma = \sum_{n \in N} C^{TE} \ell_n + \sum_{e \in E} (C^R r_e + C^A a_e) \leq \text{Budget} \tag{16}$$

All other constraints are similar to those presented previously.

*(conversion of path flows to link flows)*

$$\sum_{p \in L_e} x_p = z_e, \qquad \forall e \in E \tag{17}$$

*(conversion of DS3s on links to TEs)*

$$z_e \leq M^{TE} t_e, \qquad \forall e \in E \tag{18}$$

*(conversion of TEs at nodes)*

$$\sum_{e \in A_n} t_e = \ell_n, \qquad \forall n \in N \tag{19}$$

*(conversion of DS3s on links to fibers and channels and bounds on fiber)*

$$z_e \leq M^A f_e, \qquad \forall e \in E \tag{20}$$

$$z_e \leq M^R c_e, \qquad \forall e \in E \tag{21}$$

$$f_e \leq Fe, \qquad \forall e \in E \tag{22}$$

*(conversion of fibers and channels to amplifiers and regenerators)*

$$G_e^A f_e = a_e, \qquad \forall e \in E \tag{23}$$

$$G_e^R c_e = r_e, \qquad \forall e \in E \tag{24}$$

*(bounds on individual pieces)*

$$0 \leq \bar{z}^+_{odsk} \leq R_{\max}/4, \qquad \forall (o,d) \in D, \forall s \in S, k = 1, \dots, 4 \tag{25}$$

$$0 \leq \bar{z}^-_{odsk} \leq R_{\max}/4, \qquad \forall (o,d) \in D, \forall s \in S, k = 1, \dots, 4 \tag{26}$$

All other variables have non-negativity restrictions and $f_e$ and $c_e$ are restricted to be integer.

The budget constraint (16) places an upper bound on the equipment cost, but does not guarantee that equipment cost is a minimum for a given regret value. Our primary objective is to produce a design that minimizes regret. The secondary goal is to achieve this regret value at a minimum equipment cost. This is accomplished by using a two-phase procedure. The phase I model is minimize $\rho$ subject to (12) - (26). Let $\bar{\rho}$ denote the optimal value for regret for phase I. In phase II, $\rho$ is fixed at the value of $\bar{\rho}$ and we minimize $\gamma$ subject to (12) - (26). The solution from Phase II is the robust design. No other design can achieve this regret with a lower cost, and any design with lower cost must have higher regret.

## 3 Alternative Techniques

Our unique implementation of the robust optimization strategy is only one of several methods that can be used to help design a network when the demand forecast is uncertain. Other possibilities include a *mean-value model*, a *stochastic programming model*, and a *worst-case model*. When applied to the DWDM routing and provisioning problem, these models differ only in the objective function to be optimized and possibly the formulation of the demand constraints.

Let the mean of the demand scenarios be given by

$$\bar{R}_{od} = \sum_{s \in S} P_s R^s_{od}, \qquad \forall (o,d) \in D.$$

The *mean-value model* will determine the least-cost design that satisfies the mean of the demand. Mathematically, this model is

$$\begin{aligned} & \text{minimize } \gamma \\ & \text{subject to} \\ & \sum_{p \in J_{od}} x_p = \bar{R}_{od}, \qquad \forall (o,d) \in D \\ & \text{and } (16) - (24) \end{aligned}$$

A *two-stage stochastic integer program with recourse* uses a penalty, say $d$, to represent the cost of infeasibility (see [22]). Mathematically this model may be stated as follows:

$$\text{minimize } \gamma + \sum_{s \in S} P_s [\sum_{(o,d) \in D} d(z^+_{ods} + z^-_{ods})]$$

subject to

(13) – (26)

Note that an identical penalty is used for both over and under provisioning. The *worst-case model* selects a design that minimizes the largest possible value of a combination of equipment and infeasibility cost. Mathematically, the model is

$$\text{minimize } \{\max_{s \in S}[\gamma + \sum_{(o,d) \in D} d(z^{+}_{ods} + z^{-}_{ods})]\}$$

subject to

and (13) – (26)

Note that the probabilities for the various scenarios do not appear in this model.

# 4 Demonstration of the Robust Design Methodology

In simplest terms, the *DWDM routing and provisioning problem with uncertain demands* is to determine the proper size for a set of pipes linking nodes in the network. Consider the network illustrated in Figure 4 along with the three scenarios for the uncertain demand. Scenario 1 is a pessimistic forecast with low demand, scenario 3 is an optimistic forecast with high demand, and scenario 2 is the most likely forecast. The solutions for the individual scenarios and the robust solution are illustrated in Figure 5 where the thickness of a link is proportional to the corresponding pipe size. Note that the robust solution is clearly unique. It uses a pipe linking Los Angeles and San Francisco whereas the scenario 2 solution does not use this link. Also the robust solution uses a fairly large pipe linking Chicago and Dallas while scenario 3 solution does not provision this link at all.

To illustrate the practical application of our robust design methodology for this problem, two test problems have been solved using the various models presented in this manuscript. All models have been implemented using the AMPL modeling language [8, 23] with a direct link to the solver in CPLEX [24, 25] 6.6.0. All test runs were made on a Compaq AlphaServer DS20E with dual EV 6.7 (21264A) 667 MHz processors and 4096 MB of RAM. AMPL data files for the DA and KL test problems are available on line at `http://www.engr.smu.edu/~olinick/papers/`. The DA problem is a US network and was provided to us by Dr. J. D. Allen [1]. The KL problem corresponds to a European network [19]. Characteristics of the DA problem may be found in Tables 1 and 3. The model includes four candidate paths for each of the 200 demand pairs. These paths are the four shortest loopless paths and the demand values were randomly generated from a uniform distribution with the ranges specified in Table 4. For this application, five demand scenarios is considered large.

The runs for the individual scenarios are summarized in Table 5, where the numbers under the column titles TEs, Rs, and As denote the total quantity of transponders, regenerators, and optical amplifiers required for the optimal network design for that scenario. Total equipment cost varied from \$1.82 B to \$5.63 B. We used an optimality gap of 2% for CPLEX and each of the five runs required less than a minute of CPU time. Note the dramatic improvement in equipment cost resulting from the robust phase II calculation. The budget was large and did not affect the solution to phase I. This clearly demonstrates the merit of the two-phase approach as opposed to the single-phase model presented by Mulvey et al [14].

Both AMPL and CPLEX use preprocessors that attempt to reduce the size of the problem instance prior to application of the integer optimizer. The resulting problem sizes after preprocessing for the various models are reported in Table 6. The individual scenario models have the smallest number of nonzeros while the robust models have the largest. The number of integer variables corresponding to the number of fibers on each link and the number of channels on each link are approximately the same for all models $(107)(2) = 214$. The column increases over the individual scenarios are primarily additional continuous variables. The robust model uses the four-piece approximation for each $z^+_{ods}$ and each $z^-_{ods}$ resulting in $(200)(5)(2)(4) = 8000$ additional columns prior to preprocessing.

Using the equipment cost from the Table 5 runs, three budgets were selected, the largest value (\$5.63 B) corresponds to a generous budget, the smallest value (\$1.82 B) corresponds to a very tight budget, and \$3.75 B corresponds to a mid-range budget. Results from runs using these three budgets with the various models may be found in Table 7. Two metrics were used to help compare the various models, the equipment cost and regret. The regret has been scaled for each budget so that the best value for regret is 1. For the generous budget, the regret for the mean-value model is 40% higher than for robust optimization. For the mid-range budget, the mean-value model was much closer to the robust strategy, but still inferior with respect to regret. For the tight budget, the mean-value model failed, but the stochastic programming model does fairly well. The worst-case model designs were poor for all three budgets. All runs used a 2% optimality gap that permits early termination of the CPLEX optimizer. The robust strategy takes more CPU time than any of the other models. However, this is a planning problem and CPU times is not critical.

Solutions for the individual scenarios for problem KL may be found in Table 8. Again we observe the dramatic improvement in equipment cost obtained from the second phase of the robust optimization strategy. Without this extension to the traditional robust optimization model, the robust strategy would not look nearly as attractive. Comparisons with the alternative methods may be found in Table 9. For the generous budget, the robust strategy reduced regret by provisioning a larger network. This occurs because under provisioning has a larger penalty than over provisioning. For a tight budget, the robust strategy clearly dominates the other competing methods.

## 5 Summary and Conclusions

This manuscript presents a two-phase robust optimization strategy for the DWDM routing and provisioning problem with uncertain demands. In phase I, an optimal design is obtained that minimizes regret. In phase II, regret is fixed at this optimal value and equipment cost is minimized. The resulting solution is optimal with respect to the primary and secondary goals of regret and equipment cost. Our strategy is unique and different from that suggested by Mulvey et al. [14] in three respects: our models are integer programming models rather than linear programming models, our models consider multiple criteria, and our models use a nonsymmetrical piece-wise linear approximation to a nonlinear regret function.

Alternative models for this problem include the mean-value model, the stochastic programming model, and the worst-case model. A major weakness of the mean-value model is that it does not guarantee a feasible solution for all problem instances. This occurred for both test cases under the tight budget restriction. For any given budget, the stochastic programming model, the worst-case model, and the robust optimization model always produce a design. Both the stochastic programming model and worst-case model require a unit cost for under and over provisioning that is not required for the robust optimization model. In addition, the worst-case model ignores the scenario probabilities which means that a very unlikely scenario could be the one that drives the solution for this model. For our test cases, regret was always highest for the worst-case model. Even though this investigation makes a compelling case for using the robust optimization strategy for this problem, the strategy presents challenges for both the analyst and the client. After using this methodology to create a robust design, we expect a manager to ask why our design is better than some other design. Our response is simply that if one believes that the regret function used in our problem instance is what one wishes to optimize, then the robust design does this as well as or better than any other design. Our design may not be the least cost, but it is definitely the least-regret design. Any design that costs less will yield a larger value on the regret function. Hence, one of the major difficulties is to define a regret function that accurately reflects the client's position regarding under and over provisioning. Also, instances of integer linear programs may require an excessive amount of computational time. However, neither of these issues should prohibit use of the robust optimization strategy.

Network restorability using a dedicated protection scheme (e.g. 1+1 protection) can be accommodated with the present model by replacing shortest paths with shortest cycles. The issue of a shared protection scheme will be addressed in a companion study.

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Table 1: Description of Constants in Problem DA

| Constant | Value or Range | Description |
|---|---|---|
| $R^s_{od}$ | 300-1500 | traffic demand for pair $(o, d)$ in scenario $s$ in units of DS3s |
| $M^{TE}$ | 192 | number of DS3s that each TE can accommodate |
| $M^R$ | 192 | number of DS3s that each regenerator can accommodate |
| $M^A$ | 15,360 | number of DS3s that each optical amplifier can accommodate |
| $C^{TE}$ | \$50,000 | unit cost for a TE |
| $C^R$ | \$80,000 | unit cost for a regenerator |
| $C^A$ | \$500,000 | unit cost for an optical amplifier |
| $F_e$ | 24 | max number of fibers on link $e$ |
| $R$ | 80km | max distance that a signal can traverse without amplification, also called the optical reach |
| $Q$ | 5 | max number of amplified spans above which signal regeneration is required |
| $B_e$ | 2km-1106km | the length of link $e$ |
| $G^A_e$ | 0-11 | number of amplifier sites on link $e$ |
| $G^R_e$ | 0-2 | number of regenerator sites on link $e$ |

Table 2: Decision Variables

| Variables | | Variable Type | Description |
|---|---|---|---|
| Scenario $s$ Model | Robust Models | | |
| $x_p^s$ | $x_p$ | continuous | number of DS3s assigned to path $p$ |
| $\ell_n^s$ | $\ell_n$ | continuous | number of TEs assigned to node $n$ |
| $t_e^s$ | $t_e$ | continuous | number of TEs assigned to link $e$ |
| $a_e^s$ | $a_e$ | continuous | number of optical amplifiers assigned to link $e$ |
| $r_e^s$ | $r_e$ | continuous | number of regens assigned to link $e$ |
| $f_e^s$ | $f_e$ | integer | number of fibers assigned to link $e$ |
| $c_e^s$ | $c_e$ | integer | number of channels assigned to link $e$ |
| $z_e^s$ | $z_e$ | continuous | number of DS3s assigned to link $e$ |
| | $z_{ods}^+$ | continuous | positive infeasibility for demand $(o, d)$ and scenario $s$ |
| | $z_{ods}^-$ | continuous | negative infeasibility for demand $(o, d)$ and scenario $s$ |
| | $\bar{z}_{odsk}^+$ | continuous | $k^{\text{th}}$ linear piece for $z_{ods}^+$ |
| | $\bar{z}_{odsk}^-$ | continuous | $k^{\text{th}}$ linear piece for $z_{ods}^-$ |
| $\gamma^s$ | $\gamma$ | continuous | total equipment cost |
| | $\rho$ | continuous | regret |

Table 3: Characteristics of Test Problems

| Name | DA | KL |
|---|---|---|
| Source | [1] | [19] |
| Total Nodes | 68 | 18 |
| Total Links | 107 | 35 |
| Total Demand Pairs | 200 | 100 |
| Number of Paths/Demand | 4 | 4 |
| Total Demand Scenarios | 5 | 5 |

Table 4: Demand Range in DS3s for Problem DA

| Scenario | Probability | Demand Range | Average Demand |
|---|---|---|---|
| 1 | 0.15 | 800-9600 | 5200 |
| 2 | 0.20 | 2400-10800 | 6600 |
| 3 | 0.30 | 4000-12000 | 8000 |
| 4 | 0.20 | 4400-16800 | 10,600 |
| 5 | 0.15 | 4800-21600 | 12,200 |

Table 5: Solution for Individual Scenarios and Robust Design for Problem DA

| Scenario | Prob. | TEs | Rs | As | Links Used | CPU Seconds | Scaled Equip. Cost | Regret |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.15 | 24,636 | 3,854 | 558 | 101 | 41 | 1.00 | 5.73 |
| 2 | 0.20 | 38,882 | 6,463 | 861 | 102 | 37 | 1.59 | 2.73 |
| 3 | 0.30 | 50,799 | 8,149 | 1,081 | 101 | 3 | 2.05 | 1.37 |
| 4 | 0.20 | 63,840 | 10,228 | 1,332 | 102 | 7 | 2.57 | 1.01 |
| 5 | 0.15 | 76,848 | 12,447 | 1,584 | 104 | 1 | 3.10 | 1.40 |
| Robust Phase I | | 63,887 | 10,636 | 7,824 | 104 | 1 | 4.37 | 1.00 |
| Robust Phase II | | 64,109 | 10,029 | 1,333 | 102 | 3 | 2.57 | 1.00 |

Table 6: Model Sizes after Preprocessing for Test Problems

| Problem | DA | | | KL | | |
|---|---|---|---|---|---|---|
| Model | Rows | Columns | Nonzeros | Rows | Columns | Nonzeros |
| Individual Scenarios | 353 | 953 | 6,122 | 270 | 570 | 3,566 |
| Mean Value | 512 | 1,112 | 7,062 | 365 | 665 | 3,855 |
| Stochastic Programming | 1,326 | 3,126 | 12,290 | 769 | 1,669 | 641 |
| Worst Case | 1,331 | 3,127 | 14,300 | 774 | 1,670 | 7,425 |
| Robust Optimization | 1,164 | 8,963 | 25,745 | 671 | 4,570 | 13,166 |

Table 7: Comparisons for Various Methods for Problem DA

| Budget | Method | TEs | Rs | As | CPU | Equip. Cost | Scaled Regret |
|---|---|---|---|---|---|---|---|
| $5.63 B | Mean Value | 51,800 | 8,117 | 1,081 | 1 | 3.79 B | 1.40 |
| | Stoch. Prog. | 44,272 | 7,443 | 909 | 2 | 3.26 B | 1.85 |
| | Worst Case | 40,542 | 5,468 | 786 | 7 | 2.86 B | 3.48 |
| | Robust Opt. | 63,680 | 10,121 | 1,337 | 123 | 4.66 B | 1.00 |
| $3.75 B | Mean Value | 51,224 | 8,093 | 1,079 | 1 | 3.75 B | 1.09 |
| | Stoch. Prog. | 44,272 | 7,443 | 909 | 1 | 3.26 B | 1.44 |
| | Worst Case | 40,542 | 5,468 | 786 | 2 | 2.86 B | 2.71 |
| | Robust Opt. | 51,489 | 8,108 | 1,068 | 4,803 | 3.75 B | 1.00 |
| $1.82 B | Mean Value | No Feasible Solution | | | — | — | — |
| | Stoch. Prog. | 25,651 | 3,413 | 527 | 17 | 1.82 B | 1.14 |
| | Worst Case | 26,897 | 2,816 | 498 | 15 | 1.82 B | 1.50 |
| | Robust Opt. | 25,285 | 3,548 | 542 | 5,511 | 1.82 B | 1.00 |

Table 8: Solution for Individual Scenarios and Robust Design for Problem KL

| Scenario | Prob. | TEs | Rs | As | Links Used | CPU Seconds | Scaled Equip. Cost | Scaled Regret |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.15 | 12,767 | 7,275 | 638 | 52 | 9 | 1.00 | 5.47 |
| 2 | 0.20 | 18,064 | 11,320 | 917 | 62 | 24 | 1.47 | 2.93 |
| 3 | 0.30 | 23,645 | 15,416 | 1,188 | 66 | 9 | 1.96 | 1.37 |
| 4 | 0.20 | 29,295 | 19,196 | 1,455 | 66 | 1 | 2.42 | 1.01 |
| 5 | 0.15 | 35,008 | 23,231 | 1,667 | 64 | 3 | 2.89 | 1.46 |
| Robust Phase I | | 28,086 | 22,750 | 14,256 | 68 | 1 | 6.73 | 1.00 |
| Robust Phase II | | 28,707 | 19,544 | 1,420 | 70 | 3 | 2.41 | 1.00 |

Table 9: Comparisons for Various Methods for Problem KL

| Budget | Method | TEs | Rs | As | CPU | Equip. Cost | Scaled Regret |
|---|---|---|---|---|---|---|---|
| $4.44 B | Mean Value | 24,126 | 15,196 | 1,177 | 6 | 3.01 B | 1.41 |
| | Stoch. Prog. | 24,127 | 14,210 | 1,042 | 1 | 2.73 B | 1.75 |
| | Worst Case | 18,749 | 11,780 | 894 | 3 | 2.33 B | 3.63 |
| | Robust Opt. | 29,862 | 18,808 | 1,414 | 5 | 3.70 B | 1.00 |
| $3.00 B | Mean Value | 24,066 | 15,222 | 1,152 | 3 | 3.00 B | 1.08 |
| | Stoch. Prog. | 21,427 | 14,136 | 1,042 | 1 | 2.72 B | 1.35 |
| | Worst Case | 18,768 | 11,798 | 886 | 2 | 2.33 B | 2.76 |
| | Robust Opt. | 23,910 | 15,266 | 1,161 | 322 | 3.00 B | 1.00 |
| $1.54 B | Mean Value | No Feasible Solution | | | — | — | — |
| | Stoch. Prog. | 12,466 | 7,646 | 598 | 9 | 1.53 B | 1.13 |
| | Worst Case | 13,709 | 6,974 | 592 | 2 | 1.54 B | 1.68 |
| | Robust Opt. | 13,637 | 7,144 | 572 | 42 | 1.54 B | 1.00 |