Problem 1

A complex-valued continuous-time signal $x_c(t)$ has the Fourier transform shown in the figure below, where $\Omega_2 - \Omega_1 = \Delta\Omega$. This signal is sampled to produce the sequence $x(n) = x_c(nT)$.

What is the lowest sampling frequency that can be used without incurring aliasing distortion, i.e., so that $x_c(t)$ can be recovered from $x(n)$?
Problem 2

Find the frequency response of the system defined by the following network:

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Problem 3

A discrete time causal LTI system has the system function

\[ H(z) = \frac{(1 + 0.2z^{-1})(1 - 9z^{-2})}{(1 + 0.81z^{-2})} \]

a) Is the system stable?

b) Find expressions for a minimum-phase system \( H_1(z) \) and an all-pass system \( H_{ap}(z) \) such that \( H(z) = H_1(z)H_{ap}(z) \).

Problem 4

We want to design a first-order discrete IIR system using the impulse invariance method. It turns out that the best continuous time filter that fits our needs has the system function

\[ H_c(s) = \frac{1}{s + 2} \]

As a reminder, if \( h_c(t) \) is the impulse response of the continuous-time system, the impulse invariance requires that \( h(n) = T_d h_c(nT_d) \), where \( T_d \) is the sampling period.

a. Design the discrete-time filter by specifying its system function \( H(z) \) with \( T_d = 2 \) sec.

b. Draw the signal flow graph of \( H(z) \).

(Note: A useful Laplace transform pair is \( \frac{1}{s - s_k} \leftrightarrow e^{s_k t} \).)
Problem 5 (ONLY for the EE 7372 students)

The impulse response of an FIR filter is \( h(n) = \begin{cases} 
\alpha^n & 0 \leq n \leq 5 \\
0 & \text{otherwise} 
\end{cases} \)

a. Draw the direct form implementation of this system.

b. Show that the corresponding system function is

\[
H(z) = \frac{1 - \alpha^6 z^{-6}}{1 - \alpha z^{-1}} \quad |z| > 0
\]

and use this to draw a flow graph that uses both an FIR system and an IIR system with the smallest number of storage elements (direct form II-like).