

# Software Metrics and Quality Engineering

CSE 8314 — Fall 2013

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## **Module IV: Formal Models for Metrics Evaluation**

- Formal Models/Axioms for Metrics Evaluation
- Tian-Zelkowitz Approach
- Application and Validation;

## Overview: Measurement

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To achieve the goal of controlled software development, we need to:

- Develop an *engineering* discipline;
- Measure and evaluate the working product;
- Construct a *scientific* model for program measurement:
  - ▷ Techniques from other disciplines;
  - ▷ Develop new techniques if necessary;
  - ▷ Basic questions:
    - What to measure: goal & environ.
    - How to measure it: metrics & tools
    - Selection and validation

## Overview: Our Solution Strategy

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Need a *scientific* model of program complexity:

1. Develop a *theory* of program complexity to organize empirical knowledge;
2. Develop a *technique* for measure evaluation and selection to extrapolate measurement activities to new applications;
3. *Validate* the model using data from NASA Software Engineering Laboratory.

**Comments:** The theory is a systematic extension to earlier studies by Prather, Fenton and Whitty, and Weyuker.

## Previous Work: Prather

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- Prather's axioms:
  - ▷  $m(S_1; S_2; \dots; S_n) \geq \sum_i m(S_i)$
  - ▷  $2(m(S_1) + m(S_2))$   
 $\geq m(\text{if } P \text{ then } S_1 \text{ else } S_2)$   
 $> m(S_1) + m(S_2)$
  - ▷  $2m(S) \geq m(\text{while } P \text{ do } S) > m(S)$
- Observations/discussions:
  - ▷ earliest attempt on axiomatic model
  - ▷ some intuition captured:
    - e.g., interactions
  - ▷ limited scope
  - ▷ justification for some axioms?

## Previous Work: Fenton

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- Fenton's hierarchical complexity:

- ▷  $m(seq(F_1, \dots, F_n))$   
 $= g_n(m(F_1), \dots, m(F_n))$
- ▷  $m(F(F_1 \text{ on } x_1, \dots, F_n \text{ on } x_n))$   
 $= h_F(m(F), m(F_1), \dots, m(F_n))$

- Observations/discussions:

- ▷ general framework
  - too general?
- ▷ contrast with Prather's work
- ▷ relation to later work
  - add specifics
  - measurement theory based work

## Previous Work: Weyuker

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- Weyuker's Desirable Properties:

1.  $(\exists P, Q) (\mathcal{V}(P) \neq \mathcal{V}(Q))$
2.  $\{P, \mathcal{V}(P) = c\}$  is finite
3.  $(\exists P, Q) (P \neq Q) \wedge (\mathcal{V}(P) = \mathcal{V}(Q))$
4.  $(\exists P, Q) (\boxed{P} = \boxed{Q}) \wedge (\mathcal{V}(P) \neq \mathcal{V}(Q))$
5.  $(\forall P, Q)$   
 $(\mathcal{V}(P) \leq \mathcal{V}(P; Q) \wedge \mathcal{V}(Q) \leq \mathcal{V}(P; Q))$
6.  $(\exists P, Q, R)$   
 $(\mathcal{V}(P) = \mathcal{V}(Q) \wedge \mathcal{V}(P; R) \neq \mathcal{V}(Q; R))$
7.  $(\exists P, Q) (P = \text{perm}(Q) \wedge \mathcal{V}(P) \neq \mathcal{V}(Q))$
8.  $(\forall P) (\forall x, y) \mathcal{V}(P) = \mathcal{V}(P_y^x)$
9.  $(\exists P, Q) (\mathcal{V}(P) + \mathcal{V}(Q) < \mathcal{V}(P; Q))$

## Previous Work: Weyuker

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- About Weyuker's properties:
  - ▷ more systematic treatment
  - ▷ inspired/lead to many followup work
    - positive: refinement
    - negative: counter examples
    - other: development & alternatives
  
- Tian/Zelkowitz as followup:
  - ▷ merit of Weyuker's properties
  - ▷ some universally satisfied
    - basis for universal axioms
  - ▷ some for certain types of metrics
    - classification?
  - ▷ a theory based on the above
  - ▷ an evaluation/selection process

## Overview: Tian/Zelkowitz

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- Tian/Zelkowitz Theory/Framework
- **Axioms:** Define program complexity and state common properties.
- **Dimensionality Analysis:**  
provide the basis for metrics classification
  - ▷ Aspects or dimensions:  
presentation, control, data;
  - ▷ Levels: lexical, syntactic, semantic.
- **Classification Scheme:** Define mutually exclusive and collectively exhaustive classes.



## Theory: Axiom Overview

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**Complexity:** Relationship between program-pairs;

**Comparability:** Programs with identical functionality are comparable (**A1**);  
Composite programs are comparable to their components (**A2**).

**Monotonicity:** Sufficiently large programs will become more complex (**A3**).

**Measurability:** Measures on programs must agree with underlying complexity (**A4**).

**Diversity:** Distribution of measured complexity must not form a single cluster (**A5**).

## Theory: Defining Complexity

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**Definition:** A *complexity ranking*  $\mathcal{R}$  is a binary relation on programs. Given programs  $P$  and  $Q$ , we interpret  $\mathcal{R}(P, Q)$  as  $P$  being no more complex than  $Q$ .

$$\mathcal{C}(P, Q) \text{ iff } \mathcal{R}(P, Q) \vee \mathcal{R}(Q, P).$$

### Comments:

- ▷ It is *internal* to the programs;
- ▷ Related empirical to external properties;
- ▷ Very broad definition, need further qualification and quantification.

## Theory: Comparability Axioms

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**Axiom A1:**  $(\forall P, Q) ( \boxed{P} = \boxed{Q} \Rightarrow \mathcal{C}(P, Q) )$

i.e., functionally equivalent programs are comparable.

**Axiom A2:**  $(\forall P, Q) ( IN(P, Q) \Rightarrow \mathcal{C}(P, Q) )$

i.e., a composite program is comparable with its components.

Comments:

- ▷ Hard problem due to undecidability;
- ▷  $\mathcal{R}$  is self-reflexive;
- ▷  $\mathcal{R}$  is not transitive.

## Theory: Monotonicity Axiom

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- **Axiom A3:**  $(\exists K \in \mathcal{N})(\forall P, Q)$   
 $( (IN(P, Q) \wedge (dist(P, Q) > K)) \Rightarrow R(P, Q) )$   
i.e., sufficiently large programs will not be ranked lower in complexity.
- **Comments:**
  - ▷ General trend must be followed;
  - ▷ Local deviations allowed.

## Theory: Measure Definition Axiom

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**Definition:** A *complexity measure*  $\mathcal{V}$  is a quantification of complexity ranking  $\mathcal{R}$ .  $\mathcal{V}$  maps programs into real numbers:

$$\mathcal{V} : \mathbf{U} \Rightarrow \mathbb{R}$$

**Axiom A4:**  $(\forall P, Q) (\mathcal{R}(P, Q) \Rightarrow \mathcal{V}(P) \leq \mathcal{V}(Q))$   
i.e., a measure must agree with the ranking it is approximating.

**Non-Axiom:** Commonly assumed by other complexity models:

$$(\forall P, Q) (\mathcal{V}(P) \leq \mathcal{V}(Q) \Rightarrow \mathcal{R}(P, Q))$$

- ▷ Incomparable cases;
- ▷ Non-transitive cases;

## Theory: Distribution Axiom

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**Requirement:** Measure values must not cluster around one single dominating point.

**Axiom A5:**  $(\forall k \in \mathbb{R})(\exists \delta > 0)$

$$(|U - \{P : \mathcal{V}(P) \in [k - \delta, k + \delta]\}| = |U|)$$

**Axiom A5':**  $(\forall k \in \mathbb{R})(\exists \delta > 0)$

$$\sum_{P, \mathcal{V}(P) \in [k - \delta, k + \delta]} \text{prob}(P) < 1$$

**Rationale:** A single dominating cluster is disallowed because it fails to achieve the goal of providing comparison for programs.

## Theory: Complexity Dimensions

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**Presentation:** Physical presentation for readers that has no effect on functionality.

**Control:** Instructions, control structures, and control dependencies.

**Data:** Data items, data structures, and data dependencies.

Comments:

- ▷  $\text{Control} + \text{Data} = \text{Abstract}$ ;
- ▷ Orthogonal dimensions.

## Theory: Measurement Levels

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**Lexical:** Token based measure computation;

**Syntactic:** Directly syntax based measure computation;

**Semantic:** Semantic analysis needed for measure computation.

Comments:

- ▷ 27 possible points in a 3-D space;
- ▷ Space proximity  $\approx$  Measure similarity;
- ▷ Dividing control and data dimensions: 1. count, 2. structure, 3. dependency.

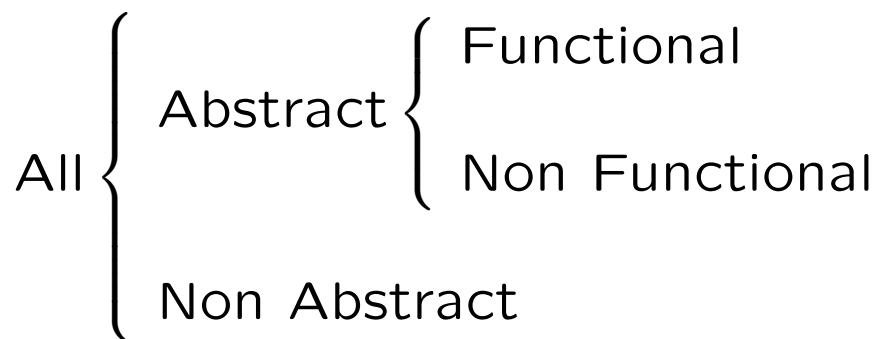


## Theory: Vertical Classification

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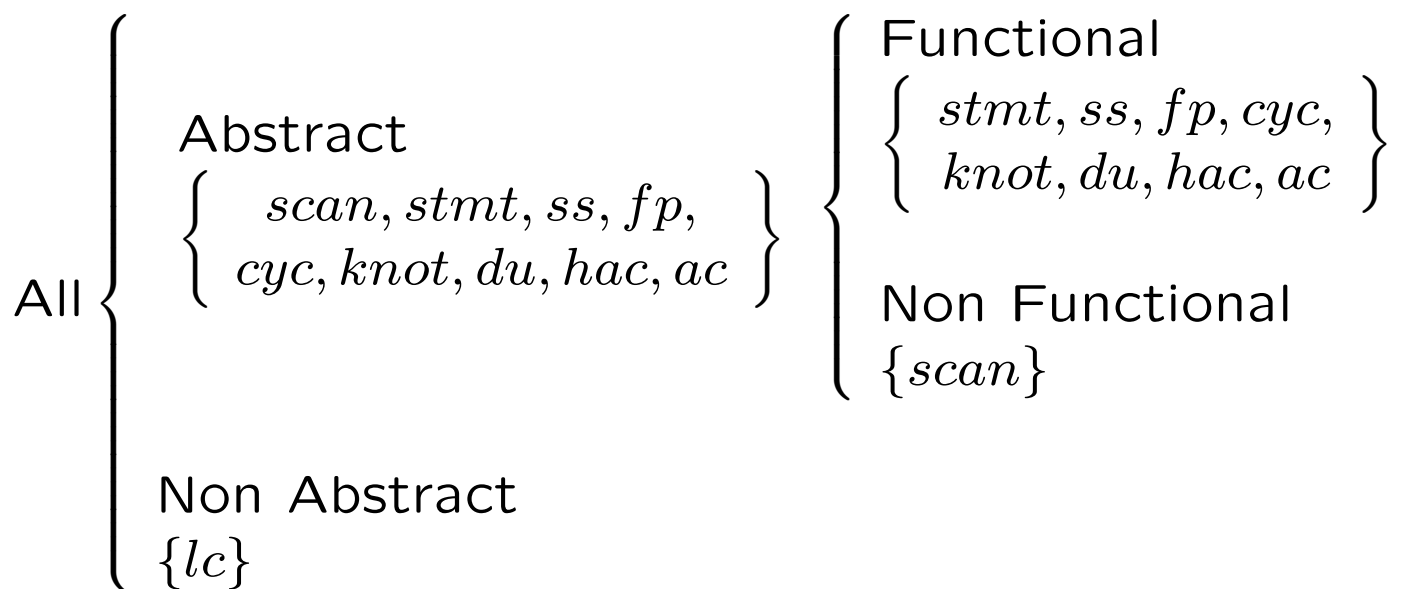
Classification based on computational models used:

- ▷ Depend only on syntax trees of programs?  
Yes, Abstract. No, non-abstract.
- ▷ Invariant to renaming?  
Yes, Functional. No, non-functional.



## Theory: Vertical Classification Example

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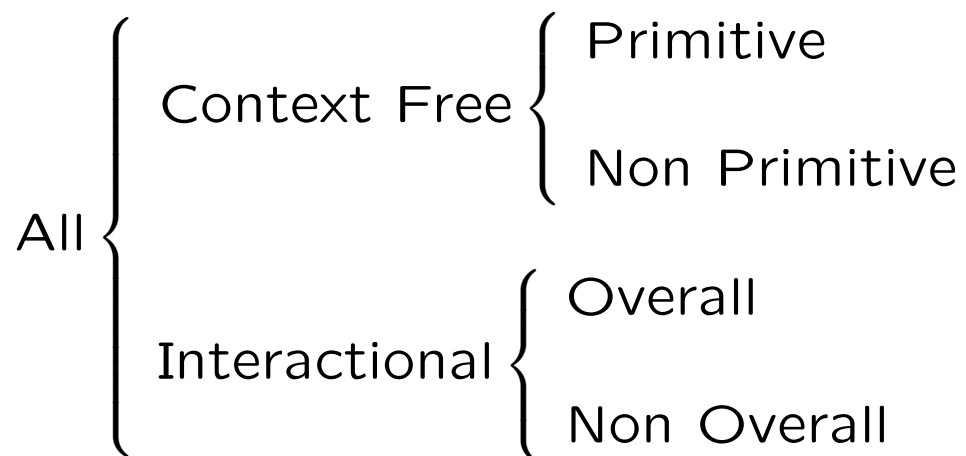


## Theory: Hierarchical Classification

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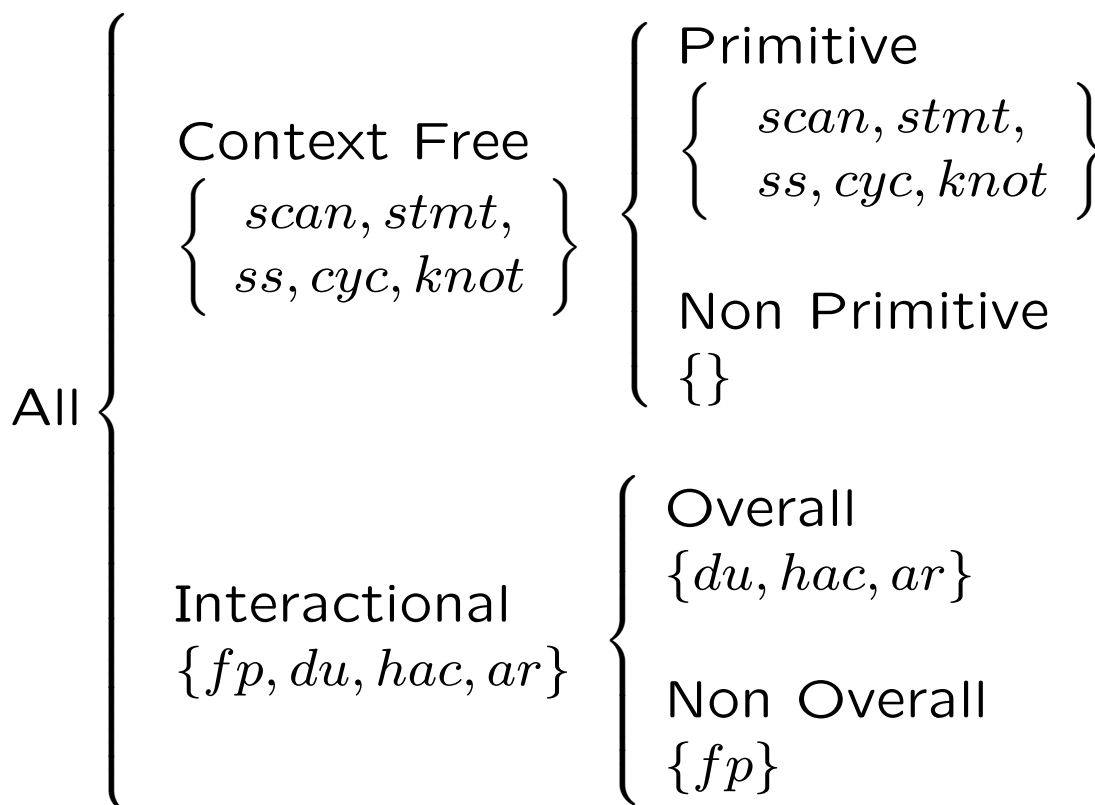
Classification based on complexity relations of component-composite programs:

- ▷ Sensitive to context?  
Yes, interactional. No, context free.
- ▷ Depend only on building element but not organization?  
Yes, Primitive. No, non-primitive.
- ▷ Capture both interface and internal?  
Yes, Overall. No, non-overall.



## Theory: Hier. Classification Example

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## Evaluation: Problems and Solutions

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**Problem:** Evaluation of complexity measures;

**Assumption:** Measures satisfy Axiom A4;

**View:** Measures as points in a measure space;

**Solution Strategy:**

- ▷ Define feasible region by using axioms and classification as boundary conditions;
- ▷ Derive scales for measures within the feasible region;
- ▷ Aggregate evaluations and select the optimal measure.

## Evaluation: Boundary Conditions

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Axioms as testable predicates:

**BC<sub>1</sub>**. Axiom A1: ( $\mathcal{D}(\mathcal{V})$ )— domain of  $\mathcal{V}$ )

$$(\forall P, Q)(\boxed{P} = \boxed{Q} \wedge P \in \mathcal{D}(\mathcal{V}) ) \Rightarrow Q \in \mathcal{D}(\mathcal{V})$$

**BC<sub>2</sub>**. Axiom A2:

$$(\forall P, Q)( (\text{IN}(P, Q) \vee \text{IN}(Q, P)) \wedge P \in \mathcal{D}(\mathcal{V}) )$$

**BC<sub>3</sub>**. Modified Axiom A3:

$$(\exists K)(\forall P, Q) ( \text{dist}(P, Q) > K ) \Rightarrow \mathcal{V}(P) \leq \mathcal{V}(Q)$$

**BC<sub>4</sub>**. Assumed true.

**BC<sub>5</sub>**. Modified Axiom A5:

$$(\forall k \in \mathbb{R})(\exists \delta > 0) \text{prob}(\mathcal{V}(P) \in [k - \delta, k + \delta]) < 1$$

## Evaluation: Screening Using Axioms

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**Example Measure:**  $\mathcal{V}(P) = 1 - \frac{1}{s(P)}$ , where  $s(P)$  is the statement count of  $P$ .

### Screening:

- ▷ **BC**<sub>1</sub> is satisfied because  $\mathcal{D}(\mathcal{V}) = \mathbf{U}$ ;
- ▷ **BC**<sub>2</sub> same as above;
- ▷ **BC**<sub>3</sub> is satisfied because:

$$(\forall P, Q) \quad (0 < s(P) < s(Q)) \Rightarrow \left( 1 - \frac{1}{s(P)} < 1 - \frac{1}{s(Q)} \right)$$

- ▷ **BC**<sub>5</sub> is not satisfied because:

$$\text{prob}(\mathcal{V}(P) \in [1 - \delta, 1 + \delta]) = 1$$

**Result:** Reject  $\mathcal{V}$  due to violation of **BC**<sub>5</sub>.

## Evaluation: Screening Using Classes

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**BC<sub>6</sub>**: Appropriate class depends on goals.

**Goal 1.** Documentation vs. comprehension.

Target: Non-abstract class.

Reject: Abstract class.

**Goal 2.** Object code size assessment.

Target: Abstract class.

- ▷ Total line count might be acceptable;
- ▷ Blank line count is rejected.

**Goal 3.** Programming effort prediction.

Target: Both abstract & non-abstract classes.

Reason: Both contribute to effort.



## Evaluation: Monotonicity Scale

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**Assumption:** Prefer measures that better approximates monotonicity;

**Need** to capture the *extent* and *frequency* of non-monotonic deviations;

**Scale  $S_1$ :** The monotonicity scale is  $\langle T, p_m \rangle$ , where  $T$  is the period of monotonicity:

$$T = \min_K ( \text{dist}(P, Q) > K \Rightarrow \mathcal{V}(P) \leq \mathcal{V}(Q) )$$

and  $p_m$  is the conditional probability of non-monotonic component-composite pairs:

$$p_m = \text{prob}(\mathcal{V}(P) > \mathcal{V}(Q) \mid \text{IN}(P, Q))$$

## Evaluation: Distribution Scale

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**Assumption:** Uniform distribution desirable.

**Need** to capture:

- ▷ Significant points on the scale;
- ▷ Uniformity of these points.

**Uniformity Scale  $\mathbf{S}_3$ :** For measure  $\mathcal{V}$ ,  $\epsilon > 0$ ,  $\delta > 0$ , and  $p_k = \text{prob}(k\delta \leq \mathcal{V}(P) < (k+1)\delta)$ ,  $\mathbf{S}_3 = \langle n, d \rangle$ , where  $n$  and  $d$  are the cardinality and the normalized *s.d.* of  $\{p_k \mid p_k > \epsilon\}$ .

$$d = \begin{cases} 0 & \text{if } n = 0 \\ \sqrt{\frac{\sum_k (1 - np_k)^2}{n}} & \text{otherwise} \end{cases}$$

## Evaluation: Scale & Dominance Relation

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**Global Scaling Vector**  $\mathcal{G}$  is defined on relevant scales  $\{\mathbf{S}_j\}$  with  $\mathcal{G}(\mathcal{V})[i]$  defined successively as:

$$\mathcal{G}(\mathcal{V})[i] = \begin{cases} \mathbf{S}_j(\mathcal{V})[k] & \text{if opt} = \max \\ -\mathbf{S}_j(\mathcal{V})[k] & \text{if opt} = \min \end{cases}$$

until all individual scaling dimensions  $\mathbf{S}_j(\mathcal{V})[k]$  are exhausted.

**Dominance Relation:** A measure  $\mathcal{V}_i$  is said to dominate another measure  $\mathcal{V}_j$  if

$$(\mathcal{G}(\mathcal{V}_i) \geq \mathcal{G}(\mathcal{V}_j)) \wedge (\exists k)(\mathcal{G}(\mathcal{V}_i)[k] > \mathcal{G}(\mathcal{V}_j)[k])$$

**Elimination:** All dominated measures are eliminated.

## Evaluation: Objective Function

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Assess the importance and trade-off among  $\mathbf{S}_j$  to form weight vector  $\mathcal{W}$ .

$$(\forall i, j, k) \mathcal{W}(\mathcal{V}_i)[k] = \mathcal{W}(\mathcal{V}_j)[k] = \mathcal{W}[k]$$

Example: the weight for  $\mathbf{S}_1$  could be  $d \times p_c$ .

The selection problem reduces to the constrained optimization problem:

$$\max_i \left( f_i = \sum_j \mathcal{G}(\mathcal{V}_i)[j] * \mathcal{W}[j] \right)$$

such that:

$\mathcal{V}_i$  satisfies all boundary conditions.

## Application and Model Validation

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1. Application domain: risk identification for projects in NASA/SEL;
2. Pilot experiment: apply the scientific model to select complexity measures;
3. Data collection: run multiple applications and collect results;
4. Analyze resulting data-points to validate the scientific model.

## Application: Risk Identification

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**Risk** in software decisions:

- ▷ Multiple alternatives;
- ▷ Uncertainty about future development;
- ▷ Large investment;
- ▷ Significant consequences.

**Risk Identification** via CTA (Selby&Porter):

- ▷ Risk: likelihood of high cost or effort;
- ▷ High cost: highest quartile (80:20 rule);
- ▷ Basis: historical data;
- ▷ Methodology: classification trees.

## Application: CTA Prediction Example

Predictions are made based on:

- ▷ Classification tree;
- ▷ Sample module measurement data:

	Modules				
	$m_1$	$m_2$	$m_3$	$m_4$	$m_5$
cyclomatic complexity	3	8	13	30	45
function plus module call	8	40	7	3	12
operators count	30	18	10	33	58
module calls	3	4	3	0	5
prediction	—	?	—	—	+
actual	—	—	+	—	+

## Application: CTA Cost & Performance

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### Cost factors:

- ▷ Tree generation: measure pool size **S**;
- ▷ Tree usage: tree-complexity/node-count.

### Performance Measures:

- ▷ Coverage: Predictions made;
- ▷ Accuracy: Correct predictions;
- ▷ Completeness: Correct predictions of  
actual high cost modules;
- ▷ Consistency: Correct high cost predictions.



## Application: CTA Performance

Compare predicted and actual data:

$$\text{Coverage} = \frac{P}{P + N}$$

$$\text{Accuracy} = \frac{M_{11} + M_{22}}{P}$$

$$\text{Completeness} = \frac{M_{11}}{A_+}$$

$$\text{Consistency} = \frac{M_{11}}{P_+}$$

		Actual		
		+	−	+/−
Predicted	+	$M_{11}$	$M_{12}$	$P_+$
	−	$M_{21}$	$M_{22}$	$P_-$
	+/−	$A_+$	$A_-$	$P$
not identified		$N$		

## Pilot: Problem & Screening

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**Environment:** NASA/SEL;

**Goal:** Identify high cost modules using complexity measures.

Consequence: Eliminate non-complexity measures, reducing **S** from 74 to 40 .

**Screening** of measures:

- ▷ **BC**<sub>1</sub> and **BC**<sub>2</sub> true because  $\mathcal{D}(\mathcal{V}) = \mathbf{U}$ ;
- ▷ **BC**<sub>3</sub> eliminates right half of Table 2;
- ▷ **BC**<sub>5</sub> true from observing data;
- ▷ **BC**<sub>6</sub> true because all aspects contribute to total effort;
- ▷ **Result:** Measure pool **S** reduced from 40 to 18.

## Pilot: Measure Selection

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**Criteria:** Conformance between  $\mathcal{V}$  and *total-effort* distribution. No need for  $\mathbf{S}_1$ .

**Derivation:** Mark a quartile “+” if  $p_i(\mathcal{V}) \geq 0.75$  and “−” if  $n_i(\mathcal{V}) \geq 0.75$ , where:

- ▷  $m_i(\mathcal{V}) = \# \text{modules in quartile } i$ ;
- ▷  $p_i(\mathcal{V}) = \text{prob}(m_4(\text{effort}) | m_i(\mathcal{V}))$ ;
- ▷  $n_i(\mathcal{V}) = 1 - p_i(\mathcal{V})$ .

$$\max_{\mathcal{V}, \mathcal{V} \in \mathbf{S}} \left\{ \sum_{i=1}^4 \{m_i(\mathcal{V})p_i(\mathcal{V}) + m_i(\mathcal{V})n_i(\mathcal{V})\} \right\}$$

$$\left\{ \begin{array}{l} p_i(\mathcal{V}) \geq 0.75 \\ \forall n_i(\mathcal{V}) \geq 0.75 \end{array} \right.$$

## Pilot: Prediction Result

ACTA:		Actual		
		+	–	total
Predicted	+	7	17	24
	–	4	143	147
	total	11	160	171

OCTA:		Actual		
		+	–	total
Predicted	+	7	32	39
	–	4	129	133
	total	11	161	172

performance measure	OCTA	ACTA
not identified	4	5
correctly identified	136	150
incorrectly identified	36	21
coverage	97%	97%
accuracy	79%	87%
completeness	63%	63%
consistency	17%	29%

## Validation: Problems and Environment

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**Goal:** Extrapolate pilot study result to validate our model.

### Embedded Environment: NASA/SEL:

- ▷ 16 ground support projects ;
- ▷ SLOC: 3K to 112K of Fortran code;
- ▷ Staffing: 4-23 (5-25M / 5-140 MM);
- ▷ Modules: 83-531/proj, 4700+ total;
- ▷ Measures: 74 collected.

### Direct Environment: CTA:

- ▷ Training set size: 1;
- ▷ Testing on immediate next project;
- ▷ 10 data points from 16 raw data;
- ▷ 5 data points from isolated data.

## Validation: Result Comparison

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Overall Comparison:

		OCTA	ACTA
cost	measure pool	40	18
	tree size (all)	12.5	9.1
	tree size (-1)	7.3	4.4
perform- ance	coverage	97.6%	97.0%
	accuracy	69.7%	74.5%
	consistency	38.4%	50.4%
	completeness	35.6%	36.0%

## Validation: Validation Result

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1. Comparing with original CTA, measure selection using our model is effective:
  - ▷ Cost: Measure pool size and classification tree complexity are reduced dramatically;
  - ▷ Performance: Coverage and completeness remain virtually the same; Accuracy and consistency are improved.
2. Comparing with random guessing, CTA based on either measure selection method made great improvement, well worth the cost.
3. The multiple data-points indicate the validity of our model.

## Validation: Baselines

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**Baseline 1:** Original CTA.

**Baseline 2:** Optimal Random Guessing:

coverage	=	100%
accuracy	=	62.5%
completeness	=	25%
consistency	=	25%

**Comment:** Other random guessing:

- ▷ consistency  $\equiv$  25%;
- ▷ max(accuracy) = 75%  
with 0 completeness;
- ▷ max(completeness) = 100%  
with 25% accuracy.



## Conclusion

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- Our model provides a scientific model of program complexity to understand and improve software process;
- Our theory of program complexity embodies the empirical research and extends formal models in this area;
- Our technique of measure evaluation demonstrates the usability of our theory in solving software engineering problems;
- Our model appears valid and effective as demonstrated by the multiple applications.