Software Reliability and Safety

CSE 8317 — Fall 2006

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SRE.3: Reliability Models

- Reliability functions and definitions
- Software Reliability Growth Models
- Combinatorial and Other Models
- Model Assumptions/Limitations/Usage

Reliability Models

- Reliability modeling
  - ▷ Reliability-fault relations
  - ▷ Exposure assumptions
  - ▷ Lyu book: Chapter 3; Tian/AIC paper
- Time domain SRGMs
  - ▷ Reliability-fault relation over time
  - ▷ Stochastic process for failure arrivals
  - ▷ Reliability growth due to fault removal
- Combinatorial & other models
  - ▷ Reliability-fault relation over input
  - ▷ Fault seeding (FS) models
  - ▷ Input domain (ID) models
  - ▷ Cleanroom and coverage-based models

Develop and Use Models

1. Preparation:
   - ▷ study failure data and environment
   - ▷ choose reliability model(s) (reliability expressed as math functions)
   - ▷ influence of past experience
2. Modeling (function with parameters):
   - ▷ estimate model parameters
   - ▷ obtain fitted model
   - ▷ goodness-of-fit test
   - ▷ obtain performance measures
3. Followup and decision making: (assessment/prediction/control aspects)

Environment and Choice of Models

- Environment and data
  - ▷ Modeling goals under the environment
  - ▷ Environmental constraints:
    - – project/process environment
    - – data availability/cost
  - ▷ Preliminary choice of models

- Model choice: goal driven
  - ▷ Goal: assessment/prediction/control?
  - ▷ Proper definition of reliability
    - – time/input/stage/coverage?
  - ▷ Current or future reliability?
  - ▷ Reliability goals as exit criteria
  - ▷ Management and improvement

Choice of Models

- Choice based on experience
  - ▷ Previous choices and experience
    - – models fitted obs. well?
    - – other results: positive/negative?
    - – overall feedback from development?
  - ▷ Both local and non-local experience
  - ▷ Baseline for comparison
  - ▷ Adaptation and refinement for now

- Other factors
  - ▷ Match model assumptions with reality
    - – implications/limitations later
  - ▷ Tools and software support
    - – SMERFS, CASRE, etc. (Lyu Book)
    - – integration with other tools?

Basic Functions and Definitions

- Some basic functions/definitions:
  - ▷ $F(t)$: cdf for failure over time
  - ▷ $f(t)$: pdf, $f(t) = F'(t)$
  - ▷ Reliability function $R(t) = 1 - F(t)$

$$R(t) = P(T \geq t) = P(\text{no failure by } t)$$

  - ▷ Hazard function/rate/intensity

$$z(t)\Delta t = P\{t < T < t + \Delta t | T > t\}$$

  - ▷ Mean function $m(t)$ in NHPP
  - ▷ Failure rate/intensity, $\lambda(t) = m'(t)$
  - ▷ Time domain definition:

$$R = \frac{s}{n} = \frac{n-f}{n} = 1 - \frac{f}{n} = 1 - r$$

  - ▷ MTBF, MTTF, etc.

- Details/relations: Tian/SQE book Ch.22.

SRGM Classification

- Data used:
  - ▷ Time-between-failure (TBF) models
    – r.v.: failure interval
  - ▷ Failure-count (FC) models
    – r.v.: failure count for given interval
  - ▷ Most widely used (in this class)
  - ▷ Some models can use both TBF and FC data

- Other classifications possible
  - ▷ Time measurement:
    – calendar/wall-clock/execution/etc. time
  - ▷ Distribution/f-arrival function:
    – Poisson/binomial/etc.
  - ▷ Finite vs infinite failures
  - ▷ Musa Book, Chapter 9, Section 9.4.

TBF Models

- Model characteristics
  - ▷ Failure intervals as r.v.
    - T_i: r.v. for the time between $(i-1)$st and ith failures
  - ▷ Distribution $F_i(t)$ or $f_i(t)$
  - ▷ Directly define $z_i(t)$
  - ▷ Relate $z_i(t)$ to failures/faults

- Defining TBF models
  - ▷ Sequence of $z_i(t)$ over i
  - ▷ Initial value?
  - ▷ Physical interpretation
  - ▷ Cumulative or rate data plotting

TBF1: Jelinski-Moranda

- One of the earliest model using TBF (time-between-failure) measurement

- Failure rate (z_i or λ_i):
  - ▷ Proportional to defects remaining
  - ▷ Step function: $z_i = \phi(N - (i - 1))$
  - ▷ z_i: failure rate for the i-th failure
  - ▷ Two model parameters:
    - – ϕ constant for failure exposure
    - – N constant for total defects

- Relation to later models
  - ▷ Similar assumptions
  - ▷ Other failure rate: geometric etc.
  - ▷ Continuous version: Goel-Okumoto etc.

TBF2-3: Schick-Wolverton

- Variations of TBF1 model
- Schick-Wolverton linear model (TBF2):
  - ▷ Proportional to defects remaining
  - ▷ But it is time dependent
  - ▷ Slope function with renewal
  - ▷ $\lambda_i = \phi(N - (i - 1))t$
  - ▷ Assumptions/parameters similar to TBF1
- Schick-Wolverton parabolic model (TBF2.1):
  - ▷ Similar to TBF2
  - ▷ 2nd order function with renewal
  - ▷ $\lambda_i = \phi(N - (i - 1))(at^2 + bt + c)$
  - ▷ Assumptions/parameters similar to TBF1

TBF4: Geometric Models (Moranda)

- Similar to Jelinski-Moranda

- Failure rate
  - ▷ Step function but geometric step sizes
  - ▷ $\lambda_i = \lambda_0 \phi^{i-1}$
  - ▷ λ_i: failure rate for the i-th failure
  - ▷ Two model parameters:
    - – ϕ: step reduction/curvature
    - – λ_0: initial failure rate

- Relation to later models
  - ▷ Close relation to Musa-Okumoto model (logarithmic Poisson)
  - ▷ Models defect discovery situations
  - ▷ Hybrid geometric Poisson

$$\lambda_i = \lambda_0 \phi^{i-1} + c$$

TBF5: Imperfect Debugging

- Goel-Okumoto
- Failure rate
  - ▷ Similar to Jelinski-Moranda
  - ▷ Step function
  - ▷ Allow for imperfect debugging
  - ▷ $\lambda_i = \phi(N - p(i - 1))$
  - ▷ p: prob(imperfect debugging)
  - ▷ Other parameters same
- Relation to later models
  - ▷ Close relation to Goel-Okumoto NHPP model
  - ▷ Models defect removal process

TBF6: Littlewood-Verrall

- Bayesian model
  - ▷ t_i: i-th inter-failure interval
  - ▷ Distribution (pdf) for t_i:

$$f(t_i|\lambda_i) = \lambda_i e^{-\lambda_i t_i}$$

  - ▷ λ_i: failure rate parameter
  - ▷ Distribution (pdf) for λ_i:

$$f(\lambda_i|\alpha, \psi(i)) = \frac{[\psi(i)]^\alpha \lambda_i^{\alpha-1} e^{-\psi(i)\lambda_i}}{\Gamma(\alpha)}$$

  - ▷ $\psi(i)$: increasing function of i
  - ▷ α: constant

- In SMERFS, LV model with $\psi(i)$:
  - ▷ $\psi(i) = \beta_0 + \beta_1 i$, or
  - ▷ $\psi(i) = \beta_0 + \beta_1 i^2$

FC Models

- Model characteristics
  - ▷ Failure count N_i as r.v.
  - ▷ Time interval: predefined
    - – equal: Schneidewind model
    - – different: other models
  - ▷ Distribution: failure arrival process
  - ▷ Directly define process parameters
  - ▷ NHPP most common

- Defining FC models
  - ▷ Time intervals
  - ▷ Underlying stochastic processes
  - ▷ Physical interpretation
  - ▷ Cumulative or rate data plotting

FC1: Goel-Okumoto

- Process assumption: NHPP
  (Non-homogeneous Poisson Process)

- Model definition:
  - ▷ Probability of n failures in $[0, t]$:

    $$P(N(t) = n) = \frac{m(t)^n}{n!} e^{-m(t)}$$

  - ▷ $m(t)$: mean function

    $$m(t) = N(1 - e^{-bt})$$

  - ▷ $\lambda(t) = m'(t)$: failure rate

    $$\lambda(t) = Nbe^{-bt}$$

  - ▷ N is the total estimated failures
  - ▷ b captures failure exposure

- Data: period failure count (PFC model)
  ($N(t)$ is the random variable)

FC Models: Other NHPP

- Similar to Goel-Okumoto model

$$P(N(t) = n) = \frac{m(t)^n}{n!} e^{-m(t)}$$

- S-shaped SRGM (2 variations)
  - ▷ $m(t) = N(1 - (1 + bt)e^{-bt})$
  - ▷ $m(t) = N(1 - e^{-bt})(1 + ce^{-bt})$
  - ▷ Allow for slow start

- Modified Goel-Okumoto
  - ▷ $m(t) = N(1 - e^{-bt^c})$
  - ▷ Similar to modified Jelinski-Moranda

- Logarithmic Poisson (Musa-Okumoto)

$$m(\tau) = \frac{1}{\theta} \log(\lambda_0 \theta \tau + 1)$$

FC Models: Generalized Poisson

- Differences with previous NHPP:
  - ▷ Segmented rather that global NHPP
  - ▷ Each segment has own parameters
  - ▷ Sequence follows some function

- Schneidewind & Generalized Poisson:
  - ▷ NHPP overall
  - ▷ Each segment a Poisson process:

$$d_i(t) = \lambda_i(t) = \alpha e^{-\beta i}$$

  - ▷ Generalized Poisson

$$m_i(t) = \phi(N - M_{i-1}) g_i(x_1, x_2, \ldots, x_i)$$

Can treat many models as special cases of this model

FC Models: Brooks-Motley

- Process assumption:
  - ▷ Poisson variation
  - ▷ Binomial variation

- Model definition:
  - ▷ Each period a binomial/Poisson process
  - ▷ Progression:
    - – n_{ij} failures for ith session, jth module
    - – with length K_{ij} or t_{ij}
    - – binomial $q_{ij} = 1 - (1 - q)^{K_{ij}}$

$$P(X = n_{ij}) = \binom{N_{ij}}{n_{ij}} q_{ij}^{n_{ij}} (1 - q_{ij})^{N_{ij} - n_{ij}}$$

    - – Poisson $\phi_{ij} = 1 - (1 - \phi)^{t_{ij}}$

$$P(X = n_{ij}) = \frac{\left(N_{ij}\phi_{ij}\right)^{n_{ij}} e^{-N_{ij}\phi_{ij}}}{n_{ij}!}$$

    - – q_{ij}, ϕ_{ij}: binomial/Poisson constant

FC Models: Musa

- Variations of Musa models
  - ▷ Prescriptive: derived from product/process characteristics
  - ▷ Descriptive: fitted, similar to prev. SRGMs
  - ▷ Execution time: used in modeling
  - ▷ Calendar time: used in management
  - ▷ Conversion between the two times

- Musa models (descriptive):
  - ▷ Basic Musa: resembles Jelinski-Moranda
  - ▷ (Musa-Okumoto) logarithmic Poisson (a variation of NHPP model)

$$m(\tau) = \frac{1}{\theta} \log(\lambda_0 \theta \tau + 1)$$

  - ▷ Execution time used in both above

FC Models: Musa

- Practicality of Musa models
  - ▷ Software usage: operational profile and execution time
  - ▷ Predictions (prescriptive) based on process and product characteristics
  - ▷ Practical issues dealt in Musa book
  - ▷ Practicality vs. theoretical focus

- Applications of Musa models
  - ▷ AT&T projects: 10-20%
  - ▷ Best practice at AT&T
  - ▷ Adoption in other environments
  - ▷ Tool and other support:
    - – AT&T's SRE ToolKit
    - – training and benchmarking
  - ▷ Most publicized success stories

Choice of SRGMs

- Issues discussed before:
  - ▷ Goal/environment/experience
  - ▷ Tool/data availability

- Other model choice issues:
  - ▷ Time measurement and model fit.
  - ▷ Single vs. multiple models.
  - ▷ Composite models possible/meaningful?
  - ▷ Existing vs. new models.
  - ▷ Assumptions/limitations/applicability.
  - ▷ (to be examined further next...)

Choice of SRGMs

- Time measurement and model fit:
  - ▷ experience at AT&T (exec. time!)
  - ▷ IBM experience
  - ▷ bad fit ⇒ time appropriate?
  - ▷ (compare: bad fit ⇒ other model)

- Single vs. multiple models:
  - ▷ best fitted vs. optimistic (fast rel. growth) vs. pessimistic (slow ..)
  - ▷ band/range instead of single estimate
  - ▷ related: synthesized/composite models

- Existing vs. new models:
  - ▷ simplicity of existing models
  - ▷ validation of new models
  - ▷ caution against ad-hoc new models

Alternatives to SRGMs

- *Reliability:* Prob(failure-free operation)
  - ▷ Time: how to measure ⇒ SRGMs
  - ▷ Input: characterize/classify
  - ▷ Assumptions: failure/OP/time/distr
  - ▷ Applicability and limitations

- Alternatives to SRGMs:
  - ▷ Input domain/combinatorial
    – also fault seeding
  - ▷ Hybrid models: Cleanroom model
  - ▷ Coverage-based and predictive
  - ▷ TBRMs: tree-based reliability models
    – both time/input info. (SRE.2)

Mills Fault Seeding Model

- Assumptions (BIG!)
  - ▷ Random seeding, same distribution
  - ▷ Same probability for detection
  - ▷ Hyper-geometric distribution

- Seeding/tagging to estimate population
  - ▷ n_s seeded, x_s captured
  - ▷ n_o original, x_o captured
  - ▷ Prob(finding exactly x_s and x_o):

$$P = \frac{\binom{n_o}{x_o}\binom{n_s}{x_s}}{\binom{n_o+n_s}{x_o+x_s}}$$

  - ▷ ML estimate of n_o given by $\widehat{n_0}$

$$\widehat{n_0} = \frac{n_s x_o}{x_s}$$

Nelson's Input Domain Model

- Nelson Model:
  - ▷ Running for a sample of n inputs.
  - ▷ Randomly selected from set E:

$$E = \{E_i : i = 1, 2, \ldots, N\}$$

  - ▷ Sampling probability vector:

$$\{P_i : i = 1, 2, \ldots, N\}$$

  - ▷ $\{P_i\}$: Operational profile.
  - ▷ Number of failures: f.
  - ▷ Estimated reliability = success rate:

$$R = \frac{n - f}{n} = 1 - \frac{f}{n} = 1 - r$$

  - ▷ r: failure rate.

- Repeated sampling without fixing.

Other Input Domain Models

- Brown-Lipow model:
  - ▷ Explicit input state distribution.
  - ▷ Known probability for sub-domains E_i
  - ▷ f_i failures for n_i runs from subdomain E_i

$$R = 1 - \sum_{i=1}^{N} \frac{f_i}{n_i} P(E_i)$$

- Ramamoorthy-Bastani:
  - ▷ Safety critical systems, $\hat{R} = 1$
  - ▷ Confidence level for $\hat{R}$
  - ▷ x_i specific set of inputs
  - ▷ P(program correct | correct for x_i's)

$$P = e^{-\lambda V} \prod_{i=1}^{n-1} \frac{2}{1 + e^{-\lambda x_i}}$$

  - ▷ λ source code complexity
  - ▷ Recent development by Woit-Parnas

Ho's Input Domain Model

- Step 1: Symbolic execution tree
  - ▷ Execution tree generation
  - ▷ Path identification T_i
  - ▷ Path frequency assignment p_i
- Step 2: Path reliability R_i
  - ▷ Estimate vs. bound
  - ▷ Use Nelson models
  - ▷ Ramamoorthy-Bastani model
- Step 3: System reliability for m paths with probability p_i and reliability R_i

$$R = \sum_{i=1}^{m} p_i R_i$$

Cleanroom Reliability Model

- Hybrid model
  - ▷ Reliability growth over stages
  - ▷ Random sampling within stage
- Factors affecting reliability
  - ▷ Increment testing: reliability change
  - ▷ Mixture of untested and tested codes
- Certifying statistical quality
  - ▷ MTTF = MR^c
  - ▷ M: Initial MTTF
  - ▷ R: Effective ratio for change
  - ▷ c: software changes

Coverage and Coverage-Based Models

- Alternative: coverage analysis
  - ▷ Defect fixing effect
  - ▷ Infeasibility of exhaustive testing
  - ▷ Pure coverage vs. cov-based models

- Focus on input/internal state coverage:
  - ▷ Function/data/statement coverage.
  - ▷ Path and dependency coverage.
  - ▷ Assumption: coverage↑ $\Rightarrow$ reliability ↑ (qualitative relation, not quantified)

- Coverage-based modeling:
  - ▷ Analytical: Weyuker etc.
  - ▷ Empirical: Mathur etc.
  - ▷ Mixed: Chen/Lyu/Wong.

General Assumptions and Implications

- Times between failures are independent
  - ▷ Implies randomized testing
  - ▷ Practical scenarios:
    - – defect fixing effect
    - – structure/progression in testing

- Immediate defect removal
  - ▷ Duplicate defect counting
  - ▷ Related but not duplicate?
  - ▷ Infeasible for in-field defects

- No new fault injected
  - ▷ Reliability growth assured
  - ▷ Practical: injection $<$ removal
  - ▷ Related: Decreasing failure rate

Assumptions and Implications

- Relating failure rate to number of faults
  - ▷ Variations to the assumption
    - – proportionality between the two
    - – functional relation between the two
    - – time dependent relation
  - ▷ Implications of failure detection and detection sequences

- Operational profile
  - ▷ Ensures reasonable/meaningful reliability assessments and predictions
  - ▷ Limits applicability

- Time as a basis for failure rate
  - ▷ Equivalent time units
  - ▷ Requires proper time measurement

Assumptions and Applicability

- General considerations
  - ▷ Assumptions for different model types
  - ▷ Tian/AIC paper
  - ▷ Match them to application environment
    - – models necessarily simple
    - – impossible perfect match
- Applicability to different processes
  - ▷ Waterfall generally assumed
  - ▷ Testing phases
  - ▷ UBST (BBT also?): SRGMs and ID
  - ▷ WBT: coverage
  - ▷ Incremental development: cleanroom
  - ▷ Spiral model: iterations
  - ▷ Operational phases
    - – difference in defect removal
    - – data availability

Applicability to Different Phases

- Requirement and specification
  - ▷ Reliability goal from customer expectation and feasibility (also affordable?)
  - ▷ Operational profile construction
  - ▷ Prepare for random testing

- Design and coding
  - ▷ Fault detection and removal (QA)
  - ▷ Musa's prescriptive model
  - ▷ Other existing models not applicable
  - ▷ Alternative models may be needed:
    - – fault and error based models
    - – constructive information (white box)
    - – predictive models relating to reliability

Applicability to Different Phases

- Unit testing
  - ▷ White-box deterministic testing
  - ▷ Tester = developer
  - ▷ Applicable: fault seeding, coverage-based, (Musa's prescriptive?)
  - ▷ Other models not applicable
- Integration and system testing
  - ▷ FVT, SVT, regression, integration
  - ▷ Focus: customer oriented operations
  - ▷ Less emphasis on coverage
  - ▷ Main phase for SRGMs
  - ▷ FC models more robust
  - ▷ Random testing conformance?
  - ▷ Use of other models

Applicability to Different Phases

- Acceptance testing
  - ▷ Gate: accept/release or not (also plan for product support)
  - ▷ Basis: snapshot(s) or random sampling
  - ▷ Cleanroom-like model usage
  - ▷ Input domain model appropriate
  - ▷ Others, maybe?
- Operational phase:
  - ▷ Actual operations (post-release)
  - ▷ Beta or ECI programs (pre-release)
  - ▷ Difference in operational environments
  - ▷ Data availability and treatment
  - ▷ Reliability vs. availability
  - ▷ Defect fix and product refreshing
  - ▷ Business decisions

Applications and Examples

- Overall procedure
  - A lot of preparation
  - Generic: preparation/modeling/followup
  - Routine procedure once started
  - Often periodic activities
  - Evaluation/feedback/improvement

- Application examples
  - Data: telecommunications (Musa)
  - Wide applications of Goel-Okumoto, Musa, and other models
  - Shuttle: Schneidewind and Keller
  - Examples in IBM