

Software Reliability and Safety

CSE 8317 — Fall 2007

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SRE.3: Reliability Models

- Reliability functions and definitions
- Software Reliability Growth Models
- Combinatorial and Other Models
- Model Assumptions/Limitations/Usage

Reliability Models

- Reliability modeling
 - ▷ Reliability-fault relations
 - ▷ Exposure assumptions
 - ▷ Lyu book: Chapter 3; Tian/AIC paper

- Time domain SRGMs
 - ▷ Reliability-fault relation over time
 - ▷ Stochastic process for failure arrivals
 - ▷ Reliability growth due to fault removal

- Combinatorial & other models
 - ▷ Reliability-fault relation over input
 - ▷ Fault seeding (FS) models
 - ▷ Input domain (ID) models
 - ▷ Cleanroom and coverage-based models

Develop and Use Models

1. Preparation:

- ▷ study failure data and environment
- ▷ choose reliability model(s)
(reliability expressed as math functions)
- ▷ influence of past experience

2. Modeling (function with parameters):

- ▷ estimate model parameters
- ▷ obtain fitted model
- ▷ goodness-of-fit test
- ▷ obtain performance measures

3. Followup and decision making:

(assessment/prediction/control aspects)

Environment and Choice of Models

- Environment and data
 - ▷ Modeling goals under the environment
 - ▷ Environmental constraints:
 - project/process environment
 - data availability/cost
 - ▷ Preliminary choice of models

- Model choice: goal driven
 - ▷ Goal: assessment/prediction/control?
 - ▷ Proper definition of reliability
 - time/input/stage/coverage?
 - ▷ Current or future reliability?
 - ▷ Reliability goals as exit criteria
 - ▷ Management and improvement

Choice of Models

- Choice based on experience
 - ▷ Previous choices and experience
 - models fitted obs. well?
 - other results: positive/negative?
 - overall feedback from development?
 - ▷ Both local and non-local experience
 - ▷ Baseline for comparison
 - ▷ Adaptation and refinement for now

- Other factors
 - ▷ Match model assumptions with reality
 - implications/limitations later
 - ▷ Tools and software support
 - SMERFS, CASRE, etc. (Lyu Book)
 - integration with other tools?

Basic Functions and Definitions

- Some basic functions/definitions:

- ▷ $F(t)$: cdf for failure over time
- ▷ $f(t)$: pdf, $f(t) = F'(t)$
- ▷ Reliability function $R(t) = 1 - F(t)$

$$R(t) = P(T \geq t) = P(\text{no failure by } t)$$

- ▷ Hazard function/rate/intensity

$$z(t)\Delta t = P\{t < T < t + \Delta t | T > t\}$$

- ▷ Mean function $m(t)$ in NHPP
- ▷ Failure rate/intensity, $\lambda(t) = m'(t)$
- ▷ Time domain definition:

$$R = \frac{s}{n} = \frac{n - f}{n} = 1 - \frac{f}{n} = 1 - r$$

- ▷ MTBF, MTTF, etc.

- Details/relations: Tian/SQE book Ch.22.

SRGM Classification

- Data used:
 - ▷ Time-between-failure (TBF) models
 - r.v.: failure interval
 - ▷ Failure-count (FC) models
 - r.v.: failure count for given interval
 - ▷ Most widely used (in this class)
 - ▷ Some models can use both TBF and FC data

- Other classifications possible
 - ▷ Time measurement:
 - calendar/wall-clock/execution/etc. time
 - ▷ Distribution/f-arrival function:
 - Poisson/binomial/etc.
 - ▷ Finite vs infinite failures
 - ▷ Musa Book, Chapter 9, Section 9.4.

TBF Models

- Model characteristics
 - ▷ Failure intervals as r.v.
 - T_i : r.v. for the time between $(i - 1)$ st and i th failures
 - ▷ Distribution $F_i(t)$ or $f_i(t)$
 - ▷ Directly define $z_i(t)$
 - ▷ Relate $z_i(t)$ to failures/faults

- Defining TBF models
 - ▷ Sequence of $z_i(t)$ over i
 - ▷ Initial value?
 - ▷ Physical interpretation
 - ▷ Cumulative or rate data plotting

TBF1: Jelinski-Moranda

- One of the earliest model using TBF (time-between-failure) measurement
- Failure rate (z_i or λ_i):
 - ▷ Proportional to defects remaining
 - ▷ Step function: $z_i = \phi(N - (i - 1))$
 - ▷ z_i : failure rate for the i -th failure
 - ▷ Two model parameters:
 - ϕ constant for failure exposure
 - N constant for total defects
- Relation to later models
 - ▷ Similar assumptions
 - ▷ Other failure rate: geometric etc.
 - ▷ Continuous version: Goel-Okumoto etc.

TBF2-3: Schick-Wolverton

- Variations of TBF1 model

- Schick-Wolverton linear model (TBF2):
 - ▷ Proportional to defects remaining
 - ▷ But it is time dependent
 - ▷ Slope function with renewal
 - ▷ $\lambda_i = \phi(N - (i - 1))t$
 - ▷ Assumptions/parameters similar to TBF1

- Schick-Wolverton parabolic model (TBF2.1):
 - ▷ Similar to TBF2
 - ▷ 2nd order function with renewal
 - ▷ $\lambda_i = \phi(N - (i - 1))(at^2 + bt + c)$
 - ▷ Assumptions/parameters similar to TBF1

TBF4: Geometric Models (Moranda)

- Similar to Jelinski-Moranda
- Failure rate
 - ▷ Step function but geometric step sizes
 - ▷ $\lambda_i = \lambda_0 \phi^{i-1}$
 - ▷ λ_i : failure rate for the i -th failure
 - ▷ Two model parameters:
 - ϕ : step reduction/curvature
 - λ_0 : initial failure rate
- Relation to later models
 - ▷ Close relation to Musa-Okumoto model (logarithmic Poisson)
 - ▷ Models defect discovery situations
 - ▷ Hybrid geometric Poisson

$$\lambda_i = \lambda_0 \phi^{i-1} + c$$

TBF5: Imperfect Debugging

- Goel-Okumoto

- Failure rate
 - ▷ Similar to Jelinski-Moranda
 - ▷ Step function
 - ▷ Allow for imperfect debugging
 - ▷ $\lambda_i = \phi(N - p(i - 1))$
 - ▷ p : prob(imperfect debugging)
 - ▷ Other parameters same

- Relation to later models
 - ▷ Close relation to Goel-Okumoto NHPP model
 - ▷ Models defect removal process

TBF6: Littlewood-Verrall

- Bayesian model

- ▷ t_i : i -th inter-failure interval

- ▷ Distribution (pdf) for t_i :

$$f(t_i|\lambda_i) = \lambda_i e^{-\lambda_i t_i}$$

- ▷ λ_i : failure rate parameter

- ▷ Distribution (pdf) for λ_i :

$$f(\lambda_i|\alpha, \psi(i)) = \frac{[\psi(i)]^\alpha \lambda_i^{\alpha-1} e^{-\psi(i)\lambda_i}}{\Gamma(\alpha)}$$

- ▷ $\psi(i)$: increasing function of i

- ▷ α : constant

- In SMERFS, LV model with $\psi(i)$:

- ▷ $\psi(i) = \beta_0 + \beta_1 i$, or

- ▷ $\psi(i) = \beta_0 + \beta_1 i^2$

FC Models

- Model characteristics
 - ▷ Failure count N_i as r.v.
 - ▷ Time interval: predefined
 - equal: Schneidewind model
 - different: other models
 - ▷ Distribution: failure arrival process
 - ▷ Directly define process parameters
 - ▷ NHPP most common

- Defining FC models
 - ▷ Time intervals
 - ▷ Underlying stochastic processes
 - ▷ Physical interpretation
 - ▷ Cumulative or rate data plotting

FC1: Goel-Okumoto

- Process assumption: NHPP
(Non-homogeneous Poisson Process)

- Model definition:

- ▷ Probability of n failures in $[0, t]$:

$$P(N(t) = n) = \frac{m(t)^n}{n!} e^{-m(t)}$$

- ▷ $m(t)$: mean function

$$m(t) = N(1 - e^{-bt})$$

- ▷ $\lambda(t) = m'(t)$: failure rate

$$\lambda(t) = Nbe^{-bt}$$

- ▷ N is the total estimated failures
- ▷ b captures failure exposure

- Data: period failure count (PFC model)
($N(t)$ is the random variable)

FC Models: Other NHPP

- Similar to Goel-Okumoto model

$$P(N(t) = n) = \frac{m(t)^n}{n!} e^{-m(t)}$$

- S-shaped SRGM (2 variations)

- ▷ $m(t) = N(1 - (1 + bt)e^{-bt})$

- ▷ $m(t) = N(1 - e^{-bt})(1 + ce^{-bt})$

- ▷ Allow for slow start

- Modified Goel-Okumoto

- ▷ $m(t) = N(1 - e^{-bt^c})$

- ▷ Similar to modified Jelinski-Moranda

- Logarithmic Poisson (Musa-Okumoto)

$$m(\tau) = \frac{1}{\theta} \log(\lambda_0 \theta \tau + 1)$$

FC Models: Generalized Poisson

- Differences with previous NHPP:
 - ▷ Segmented rather than global NHPP
 - ▷ Each segment has own parameters
 - ▷ Sequence follows some function
- Schneidewind & Generalized Poisson:
 - ▷ NHPP overall
 - ▷ Each segment a Poisson process:

$$d_i(t) = \lambda_i(t) = \alpha e^{-\beta i}$$

- ▷ Generalized Poisson

$$m_i(t) = \phi(N - M_{i-1})g_i(x_1, x_2, \dots, x_i)$$

Can treat many models as special cases of this model

FC Models: Brooks-Motley

- Process assumption:
 - ▷ Poisson variation
 - ▷ Binomial variation

- Model definition:
 - ▷ Each period a binomial/Poisson process
 - ▷ Progression:
 - n_{ij} failures for i th session, j th module
 - with length K_{ij} or t_{ij}
 - binomial $q_{ij} = 1 - (1 - q)^{K_{ij}}$
$$P(X = n_{ij}) = \binom{N_{ij}}{n_{ij}} q_{ij}^{n_{ij}} (1 - q_{ij})^{N_{ij} - n_{ij}}$$
 - Poisson $\phi_{ij} = 1 - (1 - \phi)^{t_{ij}}$
$$P(X = n_{ij}) = \frac{(N_{ij} \phi_{ij})^{n_{ij}} e^{-N_{ij} \phi_{ij}}}{n_{ij}!}$$
 - q_{ij}, ϕ_{ij} : binomial/Poisson constant

FC Models: Musa

- Variations of Musa models
 - ▷ Prescriptive: derived from product/process characteristics
 - ▷ Descriptive: fitted, similar to prev. SRGMs
 - ▷ Execution time: used in modeling
 - ▷ Calendar time: used in management
 - ▷ Conversion between the two times

- Musa models (descriptive):
 - ▷ Basic Musa: resembles Jelinski-Moranda
 - ▷ (Musa-Okumoto) logarithmic Poisson (a variation of NHPP model)
$$m(\tau) = \frac{1}{\theta} \log(\lambda_0 \theta \tau + 1)$$
 - ▷ Execution time used in both above

FC Models: Musa

- Practicality of Musa models
 - ▷ Software usage: operational profile and execution time
 - ▷ Predictions (prescriptive) based on process and product characteristics
 - ▷ Practical issues dealt in Musa book
 - ▷ Practicality vs. theoretical focus

- Applications of Musa models
 - ▷ AT&T projects: 10-20%
 - ▷ Best practice at AT&T
 - ▷ Adoption in other environments
 - ▷ Tool and other support:
 - AT&T's SRE ToolKit
 - training and benchmarking
 - ▷ Most publicized success stories

Choice of SRGMs

- Issues discussed before:
 - ▷ Goal/environment/experience
 - ▷ Tool/data availability

- Other model choice issues:
 - ▷ Time measurement and model fit.
 - ▷ Single vs. multiple models.
 - ▷ Composite models possible/meaningful?
 - ▷ Existing vs. new models.
 - ▷ Assumptions/limitations/applicability.
 - ▷ (to be examined further next...)

Choice of SRGMs

- Time measurement and model fit:
 - ▷ experience at AT&T (exec. time!)
 - ▷ IBM experience
 - ▷ bad fit \Rightarrow time appropriate?
 - ▷ (compare: bad fit \Rightarrow other model)

- Single vs. multiple models:
 - ▷ best fitted vs. optimistic (fast rel. growth) vs. pessimistic (slow ..)
 - ▷ band/range instead of single estimate
 - ▷ related: synthesized/composite models

- Existing vs. new models:
 - ▷ simplicity of existing models
 - ▷ validation of new models
 - ▷ caution against ad-hoc new models

Alternatives to SRGMs

- *Reliability*: Prob(failure-free operation)
 - ▷ Time: how to measure \Rightarrow SRGMs
 - ▷ Input: characterize/classify
 - ▷ Assumptions: failure/OP/time/distr
 - ▷ Applicability and limitations

- Alternatives to SRGMs:
 - ▷ Input domain/combinatorial
 - also fault seeding
 - ▷ Hybrid models: Cleanroom model
 - ▷ Coverage-based and predictive
 - ▷ TBRMs: tree-based reliability models
 - both time/input info. (SRE.2)

Mills Fault Seeding Model

- Assumptions (BIG!)
 - ▷ Random seeding, same distribution
 - ▷ Same probability for detection
 - ▷ Hyper-geometric distribution
- Seeding/tagging to estimate population
 - ▷ n_s seeded, x_s captured
 - ▷ n_o original, x_o captured
 - ▷ Prob(finding exactly x_s and x_o):

$$P = \frac{\binom{n_o}{x_o} \binom{n_s}{x_s}}{\binom{n_o + n_s}{x_o + x_s}}$$

- ▷ ML estimate of n_o given by \hat{n}_0

$$\hat{n}_0 = \frac{n_s x_o}{x_s}$$

Nelson's Input Domain Model

- Nelson Model:

- ▷ Running for a sample of n inputs.
- ▷ Randomly selected from set E :

$$E = \{E_i : i = 1, 2, \dots, N\}$$

- ▷ Sampling probability vector:

$$\{P_i : i = 1, 2, \dots, N\}$$

- ▷ $\{P_i\}$: Operational profile.
- ▷ Number of failures: f .
- ▷ Estimated reliability = success rate:

$$R = \frac{n - f}{n} = 1 - \frac{f}{n} = 1 - r$$

- ▷ r : failure rate.

- Repeated sampling without fixing.

Other Input Domain Models

- Brown-Lipow model:

- ▷ Explicit input state distribution.
- ▷ Known probability for sub-domains E_i
- ▷ f_i failures for n_i runs from subdomain E_i

$$R = 1 - \sum_{i=1}^N \frac{f_i}{n_i} P(E_i)$$

- Ramamoorthy-Bastani:

- ▷ Safety critical systems, $\hat{R} = 1$
- ▷ Confidence level for \hat{R}
- ▷ x_i specific set of inputs
- ▷ $P(\text{program correct} \mid \text{correct for } x_i\text{'s})$

$$P = e^{-\lambda V} \prod_{i=1}^{n-1} \frac{2}{1 + e^{-\lambda x_i}}$$

- ▷ λ source code complexity
- ▷ Recent development by Woit-Parnas

Ho's Input Domain Model

- Step 1: Symbolic execution tree
 - ▷ Execution tree generation
 - ▷ Path identification T_i
 - ▷ Path frequency assignment p_i

- Step 2: Path reliability R_i
 - ▷ Estimate vs. bound
 - ▷ Use Nelson models
 - ▷ Ramamoorthy-Bastani model

- Step 3: System reliability for m paths with probability p_i and reliability R_i

$$R = \sum_{i=1}^m p_i R_i$$

Cleanroom Reliability Model

- Hybrid model
 - ▷ Reliability growth over stages
 - ▷ Random sampling within stage

- Factors affecting reliability
 - ▷ Increment testing: reliability change
 - ▷ Mixture of untested and tested codes

- Certifying statistical quality
 - ▷ $MTTF = MR^c$
 - ▷ M: Initial MTTF
 - ▷ R: Effective ratio for change
 - ▷ c: software changes

Coverage and Coverage-Based Models

- Alternative: coverage analysis
 - ▷ Defect fixing effect
 - ▷ Infeasibility of exhaustive testing
 - ▷ Pure coverage vs. cov-based models
- Focus on input/internal state coverage:
 - ▷ Function/data/statement coverage.
 - ▷ Path and dependency coverage.
 - ▷ Assumption: coverage $\uparrow \Rightarrow$ reliability \uparrow
(qualitative relation, not quantified)
- Coverage-based modeling:
 - ▷ Analytical: Weyuker etc.
 - ▷ Empirical: Mathur etc.
 - ▷ Mixed: Chen/Lyu/Wong.

General Assumptions and Implications

- Times between failures are independent
 - ▷ Implies randomized testing
 - ▷ Practical scenarios:
 - defect fixing effect
 - structure/progression in testing

- Immediate defect removal
 - ▷ Duplicate defect counting
 - ▷ Related but not duplicate?
 - ▷ Infeasible for in-field defects

- No new fault injected
 - ▷ Reliability growth assured
 - ▷ Practical: injection < removal
 - ▷ Related: Decreasing failure rate

Assumptions and Implications

- Relating failure rate to number of faults
 - ▷ Variations to the assumption
 - proportionality between the two
 - functional relation between the two
 - time dependent relation
 - ▷ Implications of failure detection and detection sequences

- Operational profile
 - ▷ Ensures reasonable/meaningful reliability assessments and predictions
 - ▷ Limits applicability

- Time as a basis for failure rate
 - ▷ Equivalent time units
 - ▷ Requires proper time measurement

Assumptions and Applicability

- General considerations
 - ▷ Assumptions for different model types
 - ▷ Tian/AIC paper
 - ▷ Match them to application environment
 - models necessarily simple
 - impossible perfect match

- Applicability to different processes
 - ▷ Waterfall generally assumed
 - ▷ Testing phases
 - ▷ UBST (BBT also?): SRGMs and ID
 - ▷ WBT: coverage
 - ▷ Incremental development: cleanroom
 - ▷ Spiral model: iterations
 - ▷ Operational phases
 - difference in defect removal
 - data availability

Applicability to Different Phases

- Requirement and specification
 - ▷ Reliability goal from customer expectation and feasibility (also affordable?)
 - ▷ Operational profile construction
 - ▷ Prepare for random testing

- Design and coding
 - ▷ Fault detection and removal (QA)
 - ▷ Musa's prescriptive model
 - ▷ Other existing models not applicable
 - ▷ Alternative models may be needed:
 - fault and error based models
 - constructive information (white box)
 - predictive models relating to reliability

Applicability to Different Phases

- Unit testing
 - ▷ White-box deterministic testing
 - ▷ Tester = developer
 - ▷ Applicable: fault seeding, coverage-based, (Musa's prescriptive?)
 - ▷ Other models not applicable

- Integration and system testing
 - ▷ FVT, SVT, regression, integration
 - ▷ Focus: customer oriented operations
 - ▷ Less emphasis on coverage
 - ▷ Main phase for SRGMs
 - ▷ FC models more robust
 - ▷ Random testing conformance?
 - ▷ Use of other models

Applicability to Different Phases

- Acceptance testing
 - ▷ Gate: accept/release or not
(also plan for product support)
 - ▷ Basis: snapshot(s) or random sampling
 - ▷ Cleanroom-like model usage
 - ▷ Input domain model appropriate
 - ▷ Others, maybe?

- Operational phase:
 - ▷ Actual operations (post-release)
 - ▷ Beta or ECI programs (pre-release)
 - ▷ Difference in operational environments
 - ▷ Data availability and treatment
 - ▷ Reliability vs. availability
 - ▷ Defect fix and product refreshing
 - ▷ Business decisions

Applications and Examples

- Overall procedure
 - ▷ A lot of preparation
 - ▷ Generic: preparation/modeling/followup
 - ▷ Routine procedure once started
 - ▷ Often periodic activities
 - ▷ Evaluation/feedback/improvement

- Application examples
 - ▷ Data: telecommunications (Musa)
 - ▷ Wide applications of Goel-Okumoto, Musa, and other models
 - ▷ Shuttle: Schneidewind and Keller
 - ▷ Examples in IBM