Capacity Bounds of Half-Duplex Gaussian Cooperative Interference Channel

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July 2nd, 2009
Motivation

- In wireless ad hoc network, radio terminals can cooperate by relaying signals for each other to capture spatial diversity gain and increase transmission rate.
- For the Gaussian interference channel (IC), the capacity is known in a few special cases (e.g., strong interference case). In general, the capacity is known to within one bit.
- The capacity region of Gaussian IC with node cooperation is still an open problem.

We consider two user cooperation scenarios of the Gaussian IC: Case 1) TX coop. and case 2) RX coop.

We focus on the half-duplex transmission case, i.e., each of the nodes can be either in the transmit or receive mode.
Previous Works (full-duplex)

- TX coop. only, RX coop. only (A. Host-Madsen et. al.) and both TX & RX cooperation (C. Ng and A. Goldsmith et. al.) under full-duplex case has been intensively studied.

- **Assumption:** The TX’s and the RX’s are close together, respectively. **Cooperation strategies:** 1) TX’s use *decode and forward* (DF) and 2) RX’s use *compress and forward* (CF). These relaying strategies are shown to be near optimal under the given assumption (G. Kramer et. al.).

![Diagram](image)

**Figure:** (a) After the TX’s fully exchange their information, the network is equivalent to a BC where the TX has 2 antennas. (b) After each RX has a compressed version of the other’s signal, the network is equivalent to an IC where each RX has 2 antennas.
Previous Works (half-duplex)

- For the half-duplex case, limited results on TX coop. only are known.

- Slow fading channel: Outage behavior is studied where the TX’s employ either DF or amplify and forward. (J. Laneman, D. Tse and G. Wornell)

- Gaussian channel: Assume that the TX’s are close together, new achievable region is developed by introducing a 2-phase TX coop. scheme using DF and recycling DPC (RDPC). (both K. Shum and C. Sung, and N. Fawaz et al.)
Recycling DPC

Figure: System model of TX coop. by RDPC.

- **Advantage:** The TX’s can exchange their information w/o any interference.

- **Disadvantages**
  1) The TX pair cannot take advantage of joint encoding,
  2) the achievable region does not increase with the cooperation channel gain, $c_{12}$, when $c_{12}$ is much larger than the other channel gains.
Advantage 1) The TX’s can joint encode and use DPC to broadcast their signals to the RX’s, 2) The achievable region always increases with $c_{12}$ until it reaches the outerbound.

Assumptions: Channel gains: Deterministic; Power constraints: $P_1$ (node 1) and $P_2$ (node 2); Noises: Normalized to 1. The TX’s are close together, use DF to relay.
Achievable Rate

For half-duplex Gaussian IC where the TX’s can cooperate, all rate pairs \((R_{1}^{Tx}, R_{2}^{Tx})\) satisfying inequalities (1) and (2) are achievable:

\[
R_{1}^{Tx} \leq \min \left\{ R_{1,d}^{Tx} + R_{1,r_1}^{Tx}, R_{1,d}^{Tx} + R_{1,r_2}^{Tx} \right\} \tag{1}
\]

\[
R_{2}^{Tx} \leq \min \left\{ R_{2,d}^{Tx} + R_{2,r_1}^{Tx}, R_{2,d}^{Tx} + R_{2,r_2}^{Tx} \right\} \tag{2}
\]

- \(R_{1,d}^{Tx}\): The code rate for node 3 to decode \(W_d\).
- \(R_{1,r_1}^{Tx}\): The code rate for node 2 to decode \(W_1\).
- \(R_{1,r_2}^{Tx}\): The code rate for node 3 to decode \(W_1\).

The rates for user 2, \(R_{2,d}^{Tx}, R_{2,r_1}^{Tx}\) and \(R_{2,r_2}^{Tx}\), are similarly defined.
Computing $R_{1,r_2}^{Tx}$

**Encoding:** At node 1, generate $\exp(nR_{1,r_2}^{Tx})$ codewords $X(w_1)$.  
1) Divide them into $\exp(nR_{1,2}^{Tx})$ planes.  
2) Divide each plane into $\exp(nR_{1,3}^{Tx})$ bins,  
3) Each bin has $\exp(nR_{1,1}^{Tx})$ codewords. The total number of codewords is then $\exp(nR_{1,r_2}^{Tx}) = \exp(n(R_{1,1}^{Tx} + R_{1,2}^{Tx} + R_{1,3}^{Tx}))$. 

![Diagram showing the computation of $R_{1,r_2}^{Tx}$ with exponential quantities representing planes and bins.]
Computing $R_{1,r_2}^{T_X}$

Decoding: **Phase 1:** let node 1 sends a codeword $x(W_1)$. Assume node 2 can decode $x(W_1)$. **Phase 2:** node 2 sends the index of the plane, $W_2$ (with rate $R_{1,2}^{T_X}$), where $x(W_1)$ is located. By decoding $W_2$, node 3 finds the plane index of $x(W_1)$.
Computing $R^{TX}_{1,r_2}$

Decoding: **Phase 3:** Nodes 1 and 2 jointly send the message $W_3$ (with rate $R^{TX}_{1,3}$) corresponding to the index of the bin in plane $W_2$, where $x(W_1)$ belongs. Decoding $W_3$ at node 3 locates the bin index of $x(W_1)$.
Computing $R_{T X}^{1, r_2}$

Decoding: Finally, node 3 goes back to the signal it received in phase 1 and finds $x(W_1)$ in plane $W_2$ bin $W_3$ if each bin has $\leq \exp(nR_{T X}^{1, 1})$ codewords. ($R_{T X}^{1, 1}$ is the rate between nodes 1 and 3 in phase 1)
Computing $R_{1,r_2}^{T_X}$

Phase 2: Node 3 can decode $W_2$ if the rate of $W_2$ satisfies

$$R_{1,2}^{T_X} \leq \begin{cases} 
\lambda_2 C \left( \frac{c_{23}^2 P_{w_2}}{1 + c_{23}^2 P_{v_1}} \right), & \text{if } c_{24} > c_{23} \\
\lambda_2 C \left( c_{23}^2 P_{w_2} \right), & \text{otherwise}
\end{cases}$$

Phase 3: Node 3 can decode $W_3$ if $(g_1 = [c_{13} \ c_{23}])$

$$R_{1,3}^{T_X} \leq \begin{cases} 
\lambda_3 C \left( g_1 P_{w_3} g_1^T / (1 + c_{13}^2 P_{w_d} + c_{23}^2 P_{v_d}) \right), & \text{if } c_{13} + c_{23} > c_{14} + c_{24} \\
\lambda_3 C \left( g_1 P_{v_3} g_1^T / (1 + g_1 P_{v_3} g_1^T + c_{13}^2 P_{w_d} + c_{23}^2 P_{v_d}) \right), & \text{otherwise}
\end{cases}$$

Phase 1: Node 3 can go back to phase 1 and decode $W_1$ if

$$R_{1,1}^{T_X} \leq \begin{cases} 
\lambda_1 C \left( c_{13}^2 P_{w_1} \right), & \text{if } c_{13} > c_{14} \\
\lambda_1 C \left( c_{13}^2 P_{w_1} / (1 + c_{13}^2 P_{v_2}) \right), & \text{otherwise}
\end{cases}$$
Computing $R_{1,r_1}^{Tx}$ and $R_{1,d}^{Tx}$

Phase 1: Node 2 can decode $W_1$ if the rate of $W_1$ satisfies

$$R_{1,r_1}^{Tx} \leq \lambda_1 C \left( c_{12}^2 P_{w_1} \right) \quad (3)$$

Node 2 has to decode $W_1$ to enable transmitter cooperation. Thus, the code rate of $W_1$ is $\min(R_{1,r_1}^{Tx}, R_{1,r_2}^{Tx})$.

Phase 3: Node 3 can decode $W_d$ if the rate of $W_d$ satisfies

$$R_{1,d}^{Tx} \leq \begin{cases} 
\lambda_3 C \left( c_{13}^2 P_{w_d} / (1 + c_{23}^2 P_{v_d}) \right), & \text{if } c_{13} + c_{23} > c_{14} + c_{24} \\
\lambda_3 C \left( \frac{c_{13}^2 P_{w_d}}{1 + g_1 P'_{v_3} g_T^T + c_{23}^2 P_{v_d}} \right), & \text{otherwise}
\end{cases} \quad (4)$$
Outer Bound (TX coop.)

- Single user rate outer bound: Clearly, the achievable rate for each user cannot exceed the half-duplex relay channel max-flow-min-cut bound (A. Host-Madsen and J. Zhang):

\[
R_1^+ \leq \max_{0 \leq \rho_1 \leq 1} \min \left\{ \alpha_1 C \left( c_{12}^2 + c_{13}^2 P_1 \right) + \alpha_2 C \left( (1 - \rho_1) c_{13}^2 P_1 \right) \right\},
\]

\[
\alpha_1 C \left( c_{13}^2 P_1 \right) + \alpha_2 C \left( c_{13}^2 P_1 + c_{23}^2 P_2 + 2 \sqrt{\rho_1 c_{13}^2 c_{23}^2 P_1 P_2} \right) \right\}.
\]

The single user rate outer bound of user 2, \( R_2^+ \), can be similarly computed.

- Sum rate outer bound: When \( c_{12} = \infty \), the system becomes a two user 2-transmit-1-receive antenna MIMO BC, thus,

\[
R_1^+ + R_2^+ \leq \bigcup_{\forall P_1 + P_2 < P} C(g_1^T P_1 g_1 + g_2^T P_2 g_2)
\]

where \( C(x) = \log |I + x| \), \( g_2 = [c_{14} \ c_{24}] \).
Numerical Example (TX coop.)

Figure: Achievable regions for transmitter cooperation and RDPC.
\( (c_{13} = c_{24} = 1, \ c_{14} = c_{23} = \sqrt{2} \text{ and } P_1 = P_2 = 5. ) \)
System Model (RX coop.)

- Power constraints: $P_3$ (node 3) and $P_4$ (node 4).
- Assume that the RX’s are close together, it is preferable that the RX’s use CF to relay signals to each other.
For half-duplex Gaussian IC where the RX’s can cooperate, all rate pairs \((R_{RX}^{1}, R_{RX}^{2})\) satisfying inequalities (5) and (6) are achievable

\[
R_{RX}^{1} \leq R_{1,d}^{RX} + R_{1,r_{1}}^{RX} + R_{1,r_{2}}^{RX} \quad (5)
\]

\[
R_{RX}^{2} \leq R_{2,d}^{RX} + R_{2,r_{1}}^{RX} + R_{2,r_{2}}^{RX} \quad (6)
\]

- \(R_{1,d}^{RX}\): The code rate for node 3 to decode \(W_{d}\).
- \(R_{1,r_{1}}^{RX}\): The code rate for node 3 to decode \(W_{1}\).
- \(R_{1,r_{2}}^{RX}\): The code rate for node 3 to decode \(W_{2}\).

The rates for user 2, \(R_{2,d}^{RX}\), \(R_{2,r_{1}}^{RX}\) and \(R_{2,r_{2}}^{RX}\), are similarly defined.
Computing $R_{RX}^{1,r_1}$

1) Since both nodes 3 and 4 have their own received signal and a noisy version of the received signal at the other receiver, the network is equivalent to an IC with 1 antenna at nodes 1 and 2 and 2 antennas at nodes 3 and 4.

2) The rate pair $(R_{RX}^{1,r_1}, R_{RX}^{2,r_1})$ for the RX’s to decode $W_1$ and $V_1$ is given by the IC achievable region, depending on the channel gains.
Computing $R_{1,r_2}^{R_X}$ and $R_{1,d}^{R_X}$

- Node 1 sends message $W_2$ to node 4 in phase 3. Node 4 decode-and-forward $W_2$ to node 3 in phase 2 of the next transmission block. Thus, the rate of $W_2$, $R_{1,r_2}^{R_X}$, is the smaller of the rates between nodes 1 and 4 and nodes 4 and 3, i.e.,

$$R_{1,r_2}^{R_X} \leq \min \left\{ \lambda_3 C \left( \frac{c_{14}^2 P_1, w_2}{1 + c_{24}^2 P_{v_d}} \right) \right\},$$

$$\lambda_2 C \left( \frac{c_{34}^2 P_{4, w_2}}{1 + c_{13}^2 P_{w_d} + c_{23}^2 P_{v_2} + c_{34}^2 P_{w_s}} \right). \quad (7)$$

- After decoding $W_s$, $W_2$ and $V_2$, node 3 can decode $W_d$ if the rate of $W_d$ satisfies

$$R_{1,d}^{R_X} \leq \lambda_2 C \left( c_{13}^2 P_{w_d} \right). \quad (8)$$
Outer Bound (RX coop.)

- The single user half-duplex relay channel rate upper bounds are also the upper bounds for RX coop..
- Further, by letting $c_{34} = \infty$, the system becomes a two user 1-transmit-2-receive antenna MIMO MAC. Thus, the achievable region is also bounded by this MIMO MAC capacity, which is given by

$$R_1^+ + R_2^+ \leq C(h_1^T P_1 h_1 + h_2^T P_2 h_2).$$

where $h_1 = [c_{13} \ c_{14}]$ and $h_2 = [c_{23} \ c_{24}]$. 
Numerical Example (RX coop.)

Figure: Achievable regions for receiver cooperation. ($c_{13} = c_{24} = 1$, $c_{14} = c_{23} = \sqrt{2}$ and $P_i = 5$, $i = 1, 2, 3, 4$.)
Numerical Example (TX coop. vs. RX coop.)

**Figure:** Comparison of the achievable regions with TX coop. and RX coop.. ($P_i = 5, \ i = 1, 2, 3, 4. \ P_{total}^{TX} = 10, \ P_{total}^{RX} = 20$)
Numerical Example (TX coop. vs. RX coop.)

Figure: Comparison of the achievable regions with TX coop. and RX coop. (\(P_i = 5, \ i = 1, 2, 3, 4. \ P_{total}^{TX} = 10, P_{total}^{RX} = 20\))
Conclusion

- By using our TX coop. scheme, there is significant capacity improvement compared to the previous results, especially when the cooperation link is strong.
- If the cooperation channel gain is infinity, both our TX and RX coop. rates achieve their respective outer bound.
- TX coop. provides larger achievable region than RX coop. even if larger total power constraint is given to RX coop..

Future Works:

- We are currently working on the achievable region where both TX’s and RX’s can cooperate.
- Bridging the gap between the outer bound and achievable region for finite cooperative channel gains should also be considered.
Questions?