On Performance Analysis of Optimal Diversity Combining with Imperfect Channel Estimation

Yong Peng

Center for Communication and Signal Processing Research
New Jersey Institute of Technology

Master Thesis Defense, spring, 2005
Acknowledgement

Thesis Advisor: Dr. Roy You
Committee Member: Dr. Hongya Ge
Committee Member: Dr. Sirin Tekinay
1. Motivation

- Channel fading brings deleterious effects to wireless communication systems.
- Diversity combining is a popular technique in combating with these effects and significantly improve the system bit error performance.
- Optimal Diversity Combining technique with perfect Channel State Information (CSI) for different channel scenarios are well studied. When the CSI is known perfectly at the receiver, Maximal Ratio Combining (MRC) is optimal in BEP sense.
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2. Motivation cont’d

- However, in practice, only an imperfect observation of the channel gain is available at the receiver:
  - What is the optimal diversity combining scheme?
  - Applying the optimal diversity combining scheme, what is the system bit error performance?

- Pilot-Symbol-Assisted-Modulation (PSAM) is widely applied in which pilot channel/symbol is used for the purpose of channel estimation.

- With PSAM, optimal Pilot-to-Data Power Ratio has to be investigated.
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3. Previous Work

- Previous works evaluated the diversity combining techniques in the following cases:
  - MRC with imperfect CSI under correlated Rayleigh fading channel – F. A. Dietrich;
  - Diversity Combining method with imperfect CSI but with uncorrelated diversity branches – P. Schramm;
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- Roy. You and Hong Li’s extended work studied the optimal combining scheme for BPSK modulation over correlated Rayleigh fading channels with MMSE channel estimation.
- This work expanded the model to multiple correlated Ricean fading channel with imperfect channel estimation.
  - The bit error probability (BEP) performance with respect to different Rice K factors;
  - The optimal pilot-to-data symbol power ratio which minimizes the BEP for correlated multi-path fading channel.
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6. System Model

Figure: A general single user diversity combiner.

- $s$: transmitted signal
- $g_i/h_i$: channel gain/observation of channel gain
- $d_i/w_i$: received signal/weighting coefficients
- $n_i$: noise
- $r$: combined output
7. Signal Model

![Diagram of one block of transmitted signals]

**Figure:** One block of the transmitted signals.

The channel is assumed to be block fading, the block length is $L = \lfloor 1/2 f_D T_s \rfloor$, and $M$ pilots are inserted in each block for channel estimation.

- $s \in \{ \sqrt{E_s}, -\sqrt{E_s} \}$: BPSK modulated data symbol with equal *priori* probability;
- $p \in \{ \sqrt{E_p}, -\sqrt{E_p} \}$: BPSK modulated pilot symbol with equal *priori* probability.
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8. Channel Estimation

- The channel has a Ricean fading model with $N$ correlated branches.
  - Channel gain: $g \sim \mathcal{N}(u, C_{gg})$
  - Channel Noise: $n \sim \mathcal{CN}(0, N_0 I_N)$
- The channel observation is
  \[
  h = g + \frac{n}{\sqrt{M E_p}}
  \]  

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9. Channel Estimation
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- Applying *minimum-mean-square-error* (MMSE) estimation
  - Channel estimation: \( \mathbf{m} = \mathbf{u} + C_{gh} C_{hh}^{-1} (\mathbf{h} - \mathbf{u}) \)
  - Channel estimation error \( \mathbf{e} \sim \mathcal{N}(\mathbf{0}, C_{ee}) \), where \( C_{ee} = C_{gg} - C_{mm} \).
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10. Optimal Weighting Coefficients

- Optimal detection rule (minimum distance detection):

\[ |r - w^H m_{s_0}|^2 \begin{cases} \hat{s} = s_0 \\ \hat{s} = s_1 \end{cases} |r - w^H m_{s_1}|^2 \quad (2) \]

- For Binary Modulation with antipodal transmitted signals,

\[ P_b = Q(\sqrt{2\gamma_b}) \]
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- Optimal detection rule (minimum distance detection):

\[ |r - w^H m s_0|^2 \leq |r - w^H m s_1|^2 \]  

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11. Optimal Weighting Coefficients

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- Imperfect channel knowledge (assume $s_1$ transmitted):
  \[
  \Pr(e|s_1, h) = Q \left( \sqrt{\frac{2|w^H m|^2 E_s}{w^H (C_{ee} E_s + N_0 I_N) w}} \right). \tag{3}
  \]

- $w$ which minimizes $\Pr(e|s_1, h)$: $w = (C_{ee} E_s + N_0 I_N)^{-1} m$

- Perfect channel knowledge (assume $s_1$ transmitted):
  \[
  \Pr(e|s_1, h) = Q \left( \sqrt{\frac{2|w^H g|^2 E_s}{w^H w |N_0|}} \right). \tag{4}
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12. Rice Factor

- For a single diversity branch, the link quality is measured by the Rice $K$ Factor,

$$K = \frac{u_i^2}{\sigma^2_{g_i}}$$

which is the ratio between the line of sight (LoS) power and the diffuse power.

- For multiple correlated diversity branches, there is no convenient measurement for the link quality of the channel.

- We assume that all the diversity branches have the same link quality, then the Rice Factor can be written as

$$K = \frac{u^H u}{\text{tr}(C_{gg})} = \frac{u_i^2}{\sigma^2_{g_i}}$$ (5)
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13. Evaluation of BEP

Since the bit error probability can be calculated as

\[ P_e = \int Q(\sqrt{2\gamma_b}) \ p(m) \ dm \]  \hspace{1cm} (6)

Recall:

\[ w = (C_{ee}E_s + N_0 I_N)^{-1}m \]

\[ \Rightarrow \gamma_b = m^H(C_{ee}E_s + N_0 I_N)^{-1}mE_s \]

In order to evaluate the BEP, we have to evaluate \( Q(\cdot) \) function over all channel estimations. We use the following expression of \( Q(\cdot) \) instead

\[ Q(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp \left( -\frac{x^2}{2 \sin^2 \theta} \right) \ d\theta. \]  \hspace{1cm} (7)
14. Evaluation of BEP
Perfect channel estimation

With perfect channel knowledge, the BEP can be derived as

\[ P_e = \frac{1}{\pi} \int_0^{\pi/2} \det \left[ \frac{(E_s/N_0) C_{gg}}{\sin^2 \theta} + I_N \right]^{-1} \cdot \exp \left\{ -u^H [C_{gg} + (E_s/N_0)^{-1} \sin^2 \theta I_N]^{-1} u \right\} d\theta \]  \hspace{1cm} (8)

In Rayleigh fading where \( u = 0 \), the closed-form of BEP is

\[ P_e = \frac{1}{2} \sum_{n=1}^{N} \left( \prod_{\substack{i=1 \atop i \neq n}}^{N} \left( 1 - \frac{\sigma_{g_n}^2}{\sigma_{\tilde{g}_n}^2} \right) \right)^{-1} \left[ 1 - \left( \sqrt{\frac{\sigma_{g_n}^2 (E_s/N_0)}{1 + \sigma_{g_n}^2 (E_s/N_0)}} \right) \right] \]  \hspace{1cm} (9)
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P_e = \frac{1}{2} \sum_{n=1}^{N} \left( \prod_{\substack{i=1 \atop i \neq n}}^{N} \left( 1 - \frac{\lambda_i}{\lambda_n} \right) \right)^{-1} \left[ 1 - \sqrt{\frac{\lambda_n}{1 + \lambda_n}} \right]
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where \( \lambda_n = \frac{\sigma_{g_n}^2 E_s}{N_0} \cdot \frac{M E_p}{E_s + M E_p + N_0/\sigma_{g_n}^2} \)
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16. Pilot-to-Data Power Ratio Optimization

Trade-off between $E_p$ and $E_s$.

- Given total power budget, performance trade-off exists
  - Recall, Observation of channel gain: $h = g + \frac{n}{\sqrt{M E_p}}$, more pilot power allocated will result in better channel estimation and consequently better data detection performance.
  - however, the overhead of pilot symbol will reduce the effective SNR, defined as $\gamma_b = E/N_0$ where $E = \frac{(L-M)E_b+ME_p}{L-M}$.

- The optimization of Pilot-to-Data power Ratio is needed. The goal is to find the optimal pilot-to-data power ratio $\beta = \frac{M E_p}{M E_p+(L-M)E_b}$, $\beta \in (0, 1)$ which optimize the bit error performance.
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  Observation of channel gain: $h = g + \frac{n}{\sqrt{M E_p}}$, more pilot power allocated will result in better channel estimation and consequently better data detection performance.
  
  - however, the overhead of pilot symbol will reduce the effective SNR, defined as $\gamma_b = E/N_0$ where $E = \frac{(L-M)E_b + ME_p}{L-M}$.

- The optimization of Pilot-to-Data power Ratio is needed. The goal is to find the optimal pilot-to-data power ratio $\beta = \frac{ME_p}{ME_p + (L-M)E_b}$, $\beta \in (0, 1)$ which optimize the bit error performance.
17. Optimal Pilot-to-Data Power Ratio $\beta_{opt}$

- Revisit:

$$P_e = \frac{1}{2} \sum_{n=1}^{N} \prod_{i \neq n} \left( 1 - \frac{\lambda_i}{\lambda_n} \right)^{-1} \left( 1 - \sqrt{\frac{\lambda_n}{1 + \lambda_n}} \right)$$

where $\lambda_n = \frac{\sigma_{gn}^2 E_b}{N_0} \cdot \frac{M E_p}{E_b + M E_p + N_0 / \sigma_{gn}^2}$ and $\sigma_{gn}^2$ is the eigenvalues of the channel correlation matrix $C_{gg}$.

- Expressing $\lambda$ in terms of $\beta$ and optimize $P_e$ with respect to $\beta$, with certain approximation, we arrived at

$$\beta_{opt} = \frac{\sqrt{L-M-1}}{(L-M)-1}$$

- Example: $f_D T_s = .05 \rightarrow L = 10$
  - if we set $E_p = E_s$ then $M \approx 3$;
  - if we set $M = 1$, we should have $E_p = 3E_s$. 

Yong Peng
New Jersey Institute of Technology
Thesis Defense
18. Numerical Results

Frequency Diversity

- Frequency diversity model

\[ C_{gg}(m, n) = \frac{\sigma_g^2}{1 + j2\pi f_d \tau_d (m - n)} \]

- \( f_d \): frequency separation between two adjacent sub-carriers
- \( \tau_d \): channel delay spread
19. Numerical Results
Frequency Diversity – Rayleigh Fading

**Figure:** BEP vs. SNR comparison of optimal combining with perfect and imperfect channel knowledge using different estimation power for $N = 8$ and $N = 16$. (Total transmission bandwidth budget is fixed.)
20. Numerical Results
Frequency Diversity – Rayleigh Fading (cont’d)

Figure: The analytical and simulation results for the perfect channel knowledge, optimal combining with imperfect channel knowledge and the sub-optimal combining BEP performance ($f_d = 1.25$ MHz, $\tau_d = 25$ ns).
21. Numerical Results
Frequency Diversity – Ricean Fading

Figure: Comparison of the BEP $P_e$ vs. SNR $\gamma_b$ with different Rice $K$ factors using Frequency diversity model
($N = 3$, $f_d = 0.55$MHz, $\tau_d = 25$ns, $L = 10$, $M = 1$, $E_p = 3E_s$).
22. Numerical Results
Spatial Diversity

- spatial diversity model

\[ C_{gg}(m, n) = \sigma_g^2 J_0 \left( 2\pi \frac{|m - n| d \cdot f_c}{C} \right) \]

- \( d \): distance between two adjacent antennas
- \( f_c \): carrier frequency
- \( C \): velocity of light
- \( J_0(\cdot) \): zeroth-order Bessel function of the first kind
23. Numerical Results
Spatial Diversity (cont’d)

Figure: Comparison of the analytical and simulation results of BEP $P_e$ vs. SNR $\gamma_b$ with different Rice $K$ factors using spatial diversity model ($N = 3$, $d = .15m$, $f_D T_s = .05$, $L = 10$, $M = 1$, $E_p = 3E_s$).
24. Numerical Results
Spatial Diversity (cont’d)

Figure: Comparison of BEP with imperfect channel estimation $P_e$ vs. SNR $\gamma_b$ with different diversity branches and Rice $K$ factors using spatial diversity model ($d = .15m$, $f_D T_s = .05$, $L = 10$, $M = 1$, $E_p = 3E_s$).
25. Numerical Results
Pilot-to-Data power ratio optimization – Rayleigh Fading

Figure: BEP vs. $\beta$ for $d = .075m$, $f_D T_s = .05$, $L = 10$, $M = 1$. 
26. Numerical Results
Pilot-to-Data power ratio optimization – Ricean Fading

Figure: The optimum value of $\beta$ vs. SNR $\gamma_b$ with Rice factors $K = 0, 1, 2, 5$ ($N = 3$, $d = .15m$, $f_D T_s = .05$, $L = 10$, $M = 1$).
The performance of the Diversity Combining technique is optimal over correlated multi-path fading channel. It is dependent on the Rice Factors and the power ratio between the pilot and data symbols.

When more signal propagate through the line of sight path between transmitter and receiver, there is less channel uncertainty and less resources is needed for channel estimation.
27. Conclusion

▶ The performance of the Diversity Combining technique is optimal over correlated multi-path fading channel. It is dependent on the Rice Factors and the power ratio between the pilot and data symbols.

▶ When more signal propagate through the line of sight path between transmitter and receiver, there is less channel uncertainty and less resources is needed for channel estimation.
27. Conclusion

- The performance of the Diversity Combining technique is optimal over correlated multi-path fading channel. It is dependent on the Rice Factors and the power ratio between the pilot and data symbols.

- When more signal propagate through the line of sight path between transmitter and receiver, there is less channel uncertainty and less resources is needed for channel estimation.
Thank You!

THE END