Forwarding Strategies for Uplink Gaussian Multi-source Multi-relay Network

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1. Motivation

- Achievable data rates in wireless network can increase significantly when the nodes collaborate with each other.
- **Relay Channel** is a very common channel model in network information theory which features node collaboration.
- The relays concatenate and forward information for the nodes to help them communicate with each other.
- Different forwarding strategies can be used at the relays to exploit the benefits provided by nodes collaboration.
- Which strategy is the best depends on the network topology and the channel conditions.
2. Motivation
Standard Relay Channel

Figure: Structure of a standard 2-sources 2-relays 1-destination network.

Typical forwarding protocols: Decode-and-forward (DF), amplify-and-forward (AF) and compress-and-forward (CF).
3. Motivation
Multi-source Multi-relay Channel (MSMRC)

Figure: Structure of a simple 2-sources 2-relays 1-destination MSMRC. Forwarding protocols: DF, AF.

Compare with standard relay channel:
- No direct link between source and destination;
- No designated relay for each source.
4. Outline

- System model
- Practical schemes to compute the achievable region for both DF and AF
- An capacity outer bound (combining the MIMO and MAC capacity bound)
- Compare DF and AF by numerical simulation
- Conclusions and future work
5. System Model

Revisit:

\[ Y_1 = h_{11}X_1 + h_{21}X_2 + n_1 \]  
\[ Y_2 = h_{12}X_1 + h_{22}X_2 + n_2 \]  
\[ Y = h_3X_3 + h_4X_4 + n \]

Figure: Structure of a simple 2-sources 2-relays 1-destination MSMRC. Forwarding protocols: DF, AF.
6. Forwarding Strategies

- **Decode-and-Forward**: The received signals are first decoded at the relays to retrieve the source messages. Then the decoded messages are re-encoded and forwarded to the receiver through an orthogonal channel.

- **Amplify-and-Forward**: The received signals are simply amplified at the relays and forwarded to the receiver. The amplification factors are optimized within the power constraints to maximize achievable rates.
7. Decode-and-Forward

Using DF, MSMRC can be viewed as a concatenation of a Gaussian soft handover channel (SHC) and a multiple access channel (MAC).

- SHC is introduced by D. Rajan and T. Muharemovic\(^1\).

\[ X_1 \hspace{1cm} h_{11} \hspace{1cm} Y_1 \]
\[ X_2 \hspace{1cm} h_{22} \hspace{1cm} Y_2 \]

- The SHC can be viewed as a combination of the following constituent channels:
  - Two multiple access channel (MAC)
  - Two interference channel (IC)
  - Two Z-channel (Z)
  - Two broadcast channel (BC)

\(^1\)“Bounds on the capacity of the Gaussian soft handover channel” *IEEE Trans. on Vehicular Technology.*
8. Decode-and-Forward
The SHC part

**Goal:** Find the rate pair \((R_1, R_2)\), where \(R_1 = R_{11} + R_{12}\) and \(R_2 = R_{21} + R_{22}\).

For a \(2 \times 2\) SHC, the rate quadruple in 4-D space
\[
\phi_{SHC}(R_{11}, R_{12}, R_{21}, R_{22}) \in \mathcal{R}_{2 \times 2}(h_{11}, h_{12}, h_{21}, h_{22}, N_1, N_2, P_1, P_2),
\]
where

\[
\mathcal{R}_{2 \times 2}(\cdot) = \alpha_1 R_{MAC}(\cdots, P_1^{(1)}, P_2^{(1)}) + \alpha_2 R_{MAC}(\cdots, P_1^{(2)}, P_2^{(2)}) + \alpha_3 R_{IC}(\cdots, P_1^{(3)}, P_2^{(3)}) + \alpha_4 R_{IC}(\cdots, P_1^{(4)}, P_2^{(4)}) + \alpha_5 R_{Z}(\cdots, P_1^{(5)}, P_2^{(5)}) + \alpha_6 R_{Z}(\cdots, P_1^{(6)}, P_2^{(6)}) + \alpha_7 R_{BC}(\cdots, P_1^{(7)}, \cdot) + \alpha_8 R_{BC}(\cdots, \cdot, P_2^{(8)})
\]

(4)

We optimize \(\alpha_i, P_1^i, P_2^i\) to find the optimal rate quadruple of SHC.
9. Decode-and-Forward

The MAC part

The rate quadruple in a 4–D space $\phi_{MAC}(R_{11}, R_{12}, R_{21}, R_{22})$ should also satisfy the Gaussian MAC channel capacity region:

$$R_{11} + R_{21} \leq R_3 \leq C \left( \frac{P_3 h_3^2}{N} \right)$$ (5)

$$R_{12} + R_{22} \leq R_4 \leq C \left( \frac{P_4 h_4^2}{N} \right)$$ (6)

$$R_{11} + R_{21} + R_{12} + R_{22} \leq R_3 + R_4 \leq C \left( \frac{P_3 h_3^2 + P_4 h_4^2}{N} \right)$$ (7)

The overall achievable region using DF can be computed as

$$\phi_{DF}(R_{11}, R_{21}, R_{12}, R_{22}) = \phi_{SHC}(R_{11}, R_{21}, R_{12}, R_{22}) \cap \phi_{MAC}(R_{11}, R_{12}, R_{21}, R_{22})$$ (8)

The achievable region, mapping from 4–D to 2–D space, is then $(R_1, R_2)$ where $R_1 = R_{11} + R_{12}$ and $R_2 = R_{21} + R_{22}$. 
10. Amplify-and-Forward

- Suppose the two relays have power constraints $P_3$ and $P_4$ respectively, the maximum amplification factors are
  
  $$\alpha_1^{\text{max}} = \sqrt{\frac{P_3}{\text{received power}}}, \quad \alpha_2^{\text{max}} = \sqrt{\frac{P_4}{\text{received power}}}.$$  

- However, the maximum amplification factor is not necessarily the optimum.

- Clearly, the optimal amplification factor should satisfy
  
  $$\alpha_1^* \in [0, \alpha_1^{\text{max}}] \quad \text{and} \quad \alpha_2^* \in [0, \alpha_2^{\text{max}}].$$

- Plugging in the amplification factor, the received signal can be written as

  $$Y = (h_3\alpha_1^*h_{11} + h_4\alpha_2^*h_{12})X_1 + (h_3\alpha_1^*h_{21} + h_4\alpha_2^*h_{22})X_2$$
  $$+ (h_3\alpha_1^*n_1 + h_4\alpha_2^*n_2 + n). \quad (9)$$

The MSMRC simplifies to a MAC!
The achievable region for AF is the closure of the convex hull of all \((R_1, R_2)\) satisfying

\[
R_1 \leq C \left( \frac{P_1|h_3\alpha_1^*h_{11} + h_4\alpha_2^*h_{12}|^2}{N_1|h_3\alpha_1^*|^2 + N_2|h_4\alpha_2^*|^2 + N} \right) \tag{10}
\]

\[
R_2 \leq C \left( \frac{P_2|h_3\alpha_1^*h_{21} + h_4\alpha_2^*h_{22}|^2}{N_1|h_3\alpha_1^*|^2 + N_2|h_4\alpha_2^*|^2 + N} \right) \tag{11}
\]

\[
R_1 + R_2 \leq \frac{C \left( P_1|h_3\alpha_1^*h_{11} + h_4\alpha_2^*h_{12}|^2 + P_2|h_3\alpha_1^*h_{21} + h_4\alpha_2^*h_{22}|^2 \right)}{N_1|h_3\alpha_1^*|^2 + N_2|h_4\alpha_2^*|^2 + N} \tag{12}
\]
Recall that the MSMRC can be viewed as a concatenation of a SHC and a MAC.

In SHC, joint decoding at the receivers is not allowed. If we have a central decoder, SHC simplifies to a MIMO channel, whose capacity is well known and is strictly larger than SHC.

A simple capacity outer bound for MSMRC can then be found by finding the intersection of the MIMO and MAC capacity region.
13. Symmetric Channel
Small Relay Power

Figure: $P_1 = P_2 = 6, \ h_3 = h_4 = 1, \ N_1 = N_2 = N = 1$
14. Symmetric Channel
Large Relay Power

Figure: $P_1 = P_2 = 6, \ h_3 = h_4 = 1, \ N_1 = N_2 = N = 1$
15. Asymmetric Channel

Figure: $P_1 = P_2 = 6$, $h_3 = h_4 = 1$, $N_1 = N_2 = N = 1$
16. Asymmetric Channel
Some Properties of AF

Figure: \( P_1 = P_2 = 6, \ h_3 = h_4 = 1, \ N_1 = N_2 = N = 1 \)
17. Conclusions and Future Work

- We derived achievable rates of an uplink Gaussian MSMRC using both DF and AF.
- When the power constraint of the relay is small, we prefer DF, when the power constraint is large, we prefer AF.
- When the power constraint of the relay is small, the sum rate using DF can achieve the capacity outer bound in the symmetric channel case, thus is strictly optimal.

Future works:

- Will the achievable rates increase if we use mixed strategy depend on the channel conditions?
- Can compress-and-forward be applied to MSMRC? If so, will it be better than DF or AF in certain cases?
Questions?